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Research article

Closed-loop supply chain decisions for power battery with AI technology under government subsidy

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Appendix A

Proofs in Model N, Model C and Model J

Model N is solved by the inverse induction method. The supplier initially determines e, w and p_m, followed the manufacturer decides on p and p_r. The profit function of the manufacturer first takes the derivative of p and p_r, and lets $\frac{\partial \pi_m^{N*}}{\partial p}=0$, $\frac{\partial \pi_m^{N*}}{\partial p_r}=0$.

Solving these yields, $p=\frac{a+bw+e\rho}{2b}$, $p_r=\frac{1}{2}(p_m-p_m\lambda+v\lambda)$.

Substituting p and p_r into the supplier's profit function, and taking the derivative of e, w and p_m, then setting them equal to 0.

Subsequently, the equilibrium solution is obtained as below,

$$e^{N*}=\frac{-a\rho+bc\rho}{4bk(-1+s)+\rho^2},\ \ w^{N*}=\frac{2(a+bc)k(-1+s)+c\rho^2}{4bk(-1+s)+\rho^2},\ \ p^{N*}=\frac{(3a+bc)k(-1+s)+c\rho^2}{4bk(-1+s)+\rho^2},$$

$$p_m^{N*}=\frac{-c+c_r+c\lambda-c_r\lambda+v\lambda}{2(-1+\lambda)},\ \ p_r^{N*}=\frac{1}{4}\left(c+c_r(-1+\lambda)-c\lambda+v\lambda\right).$$

By substituting the equilibrium solutions into the profit functions, the optimal profits can be obtained, as below,

$$\pi_s^{N*}=\frac{1}{8}\left(\left(c+c_r(-1+\lambda)-c\lambda+v\lambda\right)^2+\frac{4(a-bc)^2k(-1+s)}{4bk(-1+s)+\rho^2}\right),$$

$$\pi_m^{N*}=\frac{1}{16}\left(c_r+c(-1+\lambda)-(c_r+v)\lambda\right)^2+\frac{b(a-bc)^2k^2(-1+s)^2}{\left(4bk(-1+s)+\rho^2\right)^2},$$

$$\pi_t^{N*}=\frac{48b^3c^2k^2(-1+s)^2+24bk(-1+s)\left(c-c_r-c\lambda+(c_r+v)\lambda\right)^2\rho^2}{16\left(4bk(-1+s)+\rho^2\right)^2}$$
$$+\frac{3\left(c-c_r-c\lambda+(c_r+v)\lambda\right)^2\rho^4+8a^2k(-1+s)\left(6bk(-1+s)+\rho^2\right)}{16\left(4bk(-1+s)+\rho^2\right)^2}$$
$$+\frac{16abck(-1+s)\left(6bk(-1+s)+\rho^2\right)+8b^2k(-1+s)\left(6k(-1+s)\left(c-c_r-c\lambda+(c_r+v)\lambda\right)^2+c^2\rho^2\right)}{16\left(4bk(-1+s)+\rho^2\right)^2}.$$

The proof processes of Model C and Model J are the same as Model N.

Appendix B

Proof of Proposition 1

We compare the optimal AI technology efforts across the three models,

$$e^{N*}-e^{C*}=\frac{-a\rho+bc\rho}{4bk(-1+s)+\rho^2}-\frac{-a\rho+bc\rho}{4bk(-1+s+\beta)+\rho^2}=\frac{4b(-a+bc)k\beta\rho}{\left(4bk(-1+s)+\rho^2\right)\left(4bk(-1+s+\beta)+\rho^2\right)},$$

$$e^{C*}-e^{J*}=\frac{-a\rho+bc\rho}{4bk(-1+s+\beta)+\rho^2}-\frac{(a-bc)\rho}{2bk(-1+s+\beta)(-2+\gamma)-\rho^2}$$
$$=\frac{2b(-a+bc)k(-1+s+\beta)\gamma\rho}{\left(2bk(-1+s+\beta)(-2+\gamma)-\rho^2\right)\left(4bk(-1+s+\beta)+\rho^2\right)}.$$

Due to $-a+bc<0, 4bk(-1+s)+\rho^2<0, 4bk(-1+s+\beta)+\rho^2<0,$

$-1+s+\beta<0, 2bk(-1+s+\beta)(-2+\gamma)-\rho^2<0$,

we conclude, $e^{N*}<e^{C*}$, $e^{C*}<e^{J*}$,

thus $e^{N*}<e^{C*}<e^{J*}$.

Proof of Corollary 1

We analyze the sensitivity of e^{C*} and e^{J*} to key parameters.

With respect to β, $\dfrac{\partial e^{C*}}{\partial \beta}=\dfrac{4b(a-bc)kp}{\left(4bk(-1+s+\beta)+\rho^2\right)^2}$, $\dfrac{\partial e^{J}}{\partial \beta}=\dfrac{2b(-a+bc)k(-2+\gamma)\rho}{\left(-2bk(-1+s+\beta)(-2+\gamma)+\rho^2\right)^2}$.

With respect to γ, $\dfrac{\partial e^{J}}{\partial \gamma}=\dfrac{2b(-a+bc)k(-1+s+\beta)\rho}{\left(-2bk(-1+s+\beta)(-2+\gamma)+\rho^2\right)^2}$.

Due to $-a+bc<0, -1+s<0, -2+\gamma<0, 4bk(-1+s)+\rho^2<0, 4bk(-1+s+\beta)+\rho^2<0,$

$-1+s+\beta<0, 2bk(-1+s+\beta)(-2+\gamma)-\rho^2<0,$

we obtain, $\dfrac{\partial e^{C}}{\partial \beta}>0$, $\dfrac{\partial e^{J}}{\partial \beta}>0$, $\dfrac{\partial e^{J}}{\partial \gamma}>0.$

The comparative statics with respect to k, ρ and s follow similarly and are summarized below,

$$\frac{\partial e^{N}}{\partial k}=\frac{4b(a-bc)(-1+s)\rho}{\left(4bk(-1+s)+\rho^2\right)^2},\quad \frac{\partial e^{C}}{\partial k}=\frac{4b(a-bc)(-1+s+\beta)\rho}{\left(4bk(-1+s+\beta)+\rho^2\right)^2},$$

$$\frac{\partial e^{J}}{\partial k}=\frac{2b(-a+bc)(-1+s+\beta)(-2+\gamma)\rho}{\left(-2bk(-1+s+\beta)(-2+\gamma)+\rho^2\right)^2},$$

$$\frac{\partial e^{N}}{\partial \rho}=\frac{(a-bc)\left(-4bk(-1+s)+\rho^2\right)}{\left(4bk(-1+s)+\rho^2\right)^2},\quad \frac{\partial e^{C}}{\partial \rho}=-\frac{(a-bc)\left(4bk(-1+s+\beta)-\rho^2\right)}{\left(4bk(-1+s+\beta)+\rho^2\right)^2},$$

$$\frac{\partial e^{J}}{\partial \rho}=\frac{(a-bc)\left(2bk(-1+s+\beta)(-2+\gamma)+\rho^2\right)}{\left(-2bk(-1+s+\beta)(-2+\gamma)+\rho^2\right)^2},$$

$$\frac{\partial e^{N}}{\partial s}=\frac{4b(a-bc)kp}{\left(4bk(-1+s)+\rho^2\right)^2},\quad \frac{\partial e^{C}}{\partial s}=\frac{4b(a-bc)kp}{\left(4bk(-1+s+\beta)+\rho^2\right)^2},$$

$$\frac{\partial e^{J}}{\partial s}=\frac{2b(-a+bc)k(-2+\gamma)\rho}{\left(-2bk(-1+s+\beta)(-2+\gamma)+\rho^2\right)^2}.$$

we conclude, $\dfrac{\partial e^{N}}{\partial k}<0$, $\dfrac{\partial e^{C}}{\partial k}<0$, $\dfrac{\partial e^{J}}{\partial k}<0$; $\dfrac{\partial e^{N}}{\partial \rho}>0$, $\dfrac{\partial e^{C}}{\partial \rho}>0$, $\dfrac{\partial e^{J}}{\partial \rho}>0$;

$\dfrac{\partial e^{N}}{\partial s}>0$, $\dfrac{\partial e^{C}}{\partial s}>0$, $\dfrac{\partial e^{J}}{\partial s}>0$.

Proof of Proposition 2

Comparison of wholesale price w, $w^{N*}-w^{C*}=-\dfrac{2(a-bc)k\beta\rho^2}{\left(4bk(-1+s)+\rho^2\right)\left(4bk(-1+s+\beta)+\rho^2\right)}<0$,

$$w^{J*}-w^{N*}=\frac{2(a-bc)k\left(2bk(-1+s)(-1+s+\beta)\gamma+\left(-\beta+(-1+s+\beta)\gamma\right)\rho^2\right)}{\left(\left(2bk(-1+s+\beta)(-2+\gamma)-\rho^2\right)\left(4bk(-1+s)+\rho^2\right)\right)}.$$

When $0<s<\dfrac{4bk-2bk\beta-\rho^2}{4bk}-\dfrac{\sqrt{4b^2k^2\beta^2\gamma+8bk\beta\rho^2-4bk\beta\gamma\rho^2+\gamma\rho^4}}{4bk\sqrt{\gamma}}$,

we have $w^{J*}<w^{N*}$, thus $w^{J*}<w^{N*}<w^{C*}$.

Comparison of retail price p, $p^{N*}-p^{C*}=-\dfrac{3(a-bc)k\beta\rho^2}{\left(4bk(-1+s)+\rho^2\right)\left(4bk(-1+s+\beta)+\rho^2\right)}<0$,

$$p^{J*}-p^{N*}=\frac{(a-bc)k\left(2bk(-1+s)(-1+s+\beta)\gamma+\left(-3\beta+2(-1+s+\beta)\gamma\right)\rho^2\right)}{\left(2bk(-1+s+\beta)(-2+\gamma)-\rho^2\right)\left(4bk(-1+s)+\rho^2\right)}.$$

When $0<s<\dfrac{2bk-bk\beta-\rho^2}{2bk}-\dfrac{\sqrt{b^2k^2\beta^2\gamma+6bk\beta\rho^2-2bk\beta\gamma\rho^2+\gamma\rho^4}}{2bk\sqrt{\gamma}}$,

we have $p^{J*}<p^{N*}$, so $p^{J*}<p^{N*}<p^{C*}$.

Proof of Proposition 3

Supplier's profit comparison, $\pi_S^{N*}-\pi_S^{C*}=-\dfrac{(a-bc)^2k\beta\rho^2}{2\left(4bk(-1+s)+\rho^2\right)\left(4bk(-1+s+\beta)+\rho^2\right)}<0,$

$$\pi_S^{C*}-\pi_S^{J*}=\frac{b(a-bc)^2k^2(-1+s+\beta)^2\gamma}{\left(2bk(-1+s+\beta)(-2+\gamma)-\rho^2\right)\left(4bk(-1+s+\beta)+\rho^2\right)}<0.$$

Due to $-a+bc<0, -1+s<0, -2+\gamma<0, 4bk(-1+s)+\rho^2<0,$

$4bk(-1+s+\beta)+\rho^2<0, -1+s+\beta<0, 2bk(-1+s+\beta)(-2+\gamma)-\rho^2>0$,

which implies $\pi_S^{N*}<\pi_S^{C*}<\pi_S^{J*}$.

Manufacturer's profit comparison,

$$\pi_m^{N*}-\pi_m^{C*}=-\frac{(a-bc)^2k\beta\rho^2\left(16b^2k^2(-1+s)\beta+2bk(2-2s+\beta)\rho^2-\rho^4\right)}{2\left(4bk(-1+s)+\rho^2\right)^2\left(4bk(-1+s+\beta)+\rho^2\right)^2},$$

$$\pi_m^{C*} - \pi_m^{J*} =$$

$$\frac{b(a-bc)^2 k^2(-1+s+\beta)\gamma\left(4b^2k^2(-1+s+\beta)^3\gamma + 2bk(-1+s+\beta)(-2+2s+6\beta-\beta\gamma)\rho^2 + (-1+s+3\beta)\rho^4\right)}{\left(4bk(-1+s+\beta)+\rho^2\right)^2\left(-2bk(-1+s+\beta)(-2+\gamma)+\rho^2\right)^2},$$

when $0<\beta<\dfrac{-4bk\rho^2+4bks\rho^2+\rho^4}{-16b^2k^2+16b^2k^2s+2bk\rho^2}$, $0<\gamma<-\dfrac{(-1+s+3\beta)\rho^2\left(4bk(-1+s+\beta)+\rho^2\right)}{2bk(-1+s+\beta)\left(2bk(-1+s+\beta)^2-\beta\rho^2\right)}$ we

obtain $\pi_m^{N*} < \pi_m^{C*} < \pi_m^{J*}$.

Proof of Corollary 2

Sensitivity of supplier's profit to subsidy rate s,

$$\frac{\partial \pi_S^N}{\partial s} = \frac{(a-bc)^2 k\rho^2}{2\left(4bk(-1+s)+\rho^2\right)^2} > 0, \quad \frac{\partial \pi_S^C}{\partial s} = \frac{(a-bc)^2 k\rho^2}{2\left(4bk(-1+s+\beta)+\rho^2\right)^2} > 0,$$

$$\frac{\partial \pi_S^J}{\partial s} = \frac{(a-bc)^2 k\rho^2}{2\left(-2bk(-1+s+\beta)(-2+\gamma)+\rho^2\right)^2} > 0.$$

Sensitivity of manufacturer's profit to subsidy rate s, $\dfrac{\partial \pi_m^N}{\partial s} = \dfrac{2b(a-bc)^2 k^2(-1+s)\rho^2}{\left(4bk(-1+s)+\rho^2\right)^3} > 0,$

when $0<s<1-3\beta$, $\dfrac{\partial \pi_m^C}{\partial s} = \dfrac{2b(a-bc)^2 k^2(-1+s+3\beta)\rho^2}{\left(4bk(-1+s+\beta)+\rho^2\right)^3} > 0,$

when $0<s<\dfrac{3\beta-2\beta\gamma+\gamma-1}{\gamma-1}$, $\dfrac{\partial \pi_m^J}{\partial s} = \dfrac{2b(a-bc)^2 k^2\left(1-s-3\beta+(-1+s+2\beta)\gamma\right)\rho^2}{\left(2bk(-1+s+\beta)(-2+\gamma)-\rho^2\right)^3} > 0.$

Proof of Proposition 4

Emissions comparison, $E^{N*} - E^{C*} = \dfrac{b(-a+bc)k\beta\rho^2(\tau_c+\tau_m+\tau_u)}{\left(4bk(-1+s)+\rho^2\right)\left(4bk(-1+s+\beta)+\rho^2\right)},$

$$E^{C*} - E^{J*} = \frac{2b^2(a-bc)k^2(-1+s+\beta)^2\gamma(\tau_c+\tau_m+\tau_u)}{\left(2bk(-1+s+\beta)(-2+\gamma)-\rho^2\right)\left(4bk(-1+s+\beta)+\rho^2\right)},$$

Due to $\tau_c>0, \tau_m>0, \tau_u>0$ and $-a+bc<0, -1+s<0, -2+\gamma<0, 4bk(-1+s)+\rho^2<0,$

$4bk(-1+s+\beta)+\rho^2<0, -1+s+\beta<0, 2bk(-1+s+\beta)(-2+\gamma)-\rho^2>0.$

We conclude $E^{N*} < E^{C*}$, $E^{C*} < E^{J*}$

Thus $E^{N*} < E^{C*} < E^{J*}$.

Proof of Corollary 3

Emissions sensitivity to subsidy rate s,

$$\frac{\partial E^N}{\partial s}=\frac{b(a-bc)k\rho^2(\tau_c+\tau_m+\tau_u)}{\left(4bk(-1+s)+\rho^2\right)^2},\quad \frac{\partial E^C}{\partial s}=\frac{b(a-bc)k\rho^2(\tau_c+\tau_m+\tau_u)}{\left(4bk(-1+s+\beta)+\rho^2\right)^2},$$

$$\frac{\partial E^J}{\partial s}=\frac{b(a-bc)k\rho^2(\tau_c+\tau_m+\tau_u)}{\left(-2bk(-1+s+\beta)(-2+\gamma)+\rho^2\right)^2}.$$

Due to $\tau_c>0,\tau_m>0,\tau_u>0$ and $a-bc>0$,

so $\frac{\partial E^N}{\partial s}>0$, $\frac{\partial E^C}{\partial s}>0$, $\frac{\partial E^J}{\partial s}>0$.

Proof of Proposition 5

Social welfare comparison, when $0<\beta<-\frac{2(2+b)(-1+s)\left(4bk(-1+s)+\rho^2\right)}{8b(4+b)k(-1+s)+(6+b)\rho^2}$,

$0<\gamma<\frac{\left(4bk(-1+s+\beta)+\rho^2\right)\left(b(2+b)k(-1+s+\beta)^2-\beta\rho^2\right)}{bk(-1+s+\beta)\left(b(6+b)k(-1+s+\beta)^2+(-1+s)\rho^2\right)}$, we obtain,

$$SW^N-SW^C=$$

$$-\frac{b(a-bc)^2k^2\beta\rho^2\left(8bk(-1+s)\left((2+b)(-1+s)+(4+b)\beta\right)+\left(2(2+b)(-1+s)+(6+b)\beta\right)\rho^2\right)}{2\left(4bk(-1+s)+\rho^2\right)^2\left(4bk(-1+s+\beta)+\rho^2\right)^2}<0,$$

$$SW^C-SW^J=$$

$$\frac{2b(a-bc)^2k^2(-1+s+\beta)\gamma b^2k^2(-1+s+\beta)^3\left(-8+b(-4+\gamma)+6\gamma\right)}{\left(-8b^2k^2(-1+s+\beta)^2(-2+\gamma)-2bk(-1+s+\beta)(-4+\gamma)\rho^2+\rho^4\right)^2}$$

$$-\frac{2b(a-bc)^2k^2(-1+s+\beta)\gamma(bk(-1+s+\beta)\left(-2-2\beta+b(-1+s+\beta)-s(-2+\gamma)+\gamma\right)\rho^2+\beta\rho^4)}{\left(-8b^2k^2(-1+s+\beta)^2(-2+\gamma)-2bk(-1+s+\beta)(-4+\gamma)\rho^2+\rho^4\right)^2}<0,$$

we conclude, $SW^N<SW^C<SW^J$.

Proof of Proposition 6

Optimal Environmental impact:

$$E^{N*}=\left(e\rho+\frac{(a-bc)\left(bk(-1+s)+\rho^2\right)}{4bk(-1+s)+\rho^2}\right)\left(-\sqrt{e}\eta+\tau_c\right)+\left(\frac{1}{4}\left(cr+c(-1+\lambda)-(cr+v)\lambda+4e\rho\right)+\frac{(a-bc)\left(bk(-1+s)+\rho^2\right)}{4bk(-1+s)+\rho^2}\right)\tau_u$$
$$+\left(-\frac{1}{4}(-1+\lambda)\left(cr+c(-1+\lambda)-(cr+v)\lambda\right)+e\rho+\frac{(a-bc)\left(bk(-1+s)+\rho^2\right)}{4bk(-1+s)+\rho^2}\right)\left(-\sqrt{e}\eta+\tau_m\right)$$
$$+\frac{1}{4}(-1+\lambda)\left(c+cr(-1+\lambda)-c\lambda+v\lambda\right)\left(\sqrt{e}\eta-\tau_r\right)+\frac{1}{4}\lambda\left(c+cr(-1+\lambda)-c\lambda+v\lambda\right)\tau_t$$

$$E^{C*}=\left(e\rho+\frac{(a-bc)\left(bk(-1+s+\beta)+\rho^2\right)}{4bk(-1+s+\beta)+\rho^2}\right)\left(-\sqrt{e}\eta+\tau_c\right)$$
$$+\left(-\frac{1}{4}(-1+\lambda)\left(cr+c(-1+\lambda)-(cr+v)\lambda\right)+e\rho+\frac{(a-bc)\left(bk(-1+s+\beta)+\rho^2\right)}{4bk(-1+s+\beta)+\rho^2}\right)\left(-\sqrt{e}\eta+\tau_m\right)$$
$$+\frac{1}{4}(-1+\lambda)\left(c+cr(-1+\lambda)-c\lambda+v\lambda\right)\left(\sqrt{e}\eta-\tau_r\right)+\frac{1}{4}\lambda\left(c+cr(-1+\lambda)-c\lambda+v\lambda\right)\tau_t$$
$$+\left(\frac{1}{4}\left(cr+c(-1+\lambda)-(cr+v)\lambda+4e\rho\right)+\frac{(a-bc)\left(bk(-1+s+\beta)+\rho^2\right)}{4bk(-1+s+\beta)+\rho^2}\right)\tau_u$$

$$E^{J*}=\left(e\rho-\frac{(a-bc)\left(bk(-1+s+\beta)+\rho^2\right)}{2bk(-1+s+\beta)(-2+\gamma)-\rho^2}\right)\left(-\sqrt{e}\eta+\tau_c\right)$$
$$+\left(-\frac{1}{4}(-1+\lambda)\left(cr+c(-1+\lambda)-(cr+v)\lambda\right)+e\rho-\frac{(a-bc)\left(bk(-1+s+\beta)+\rho^2\right)}{2bk(-1+s+\beta)(-2+\gamma)-\rho^2}\right)\left(-\sqrt{e}\eta+\tau_m\right)$$
$$+\frac{1}{4}(-1+\lambda)\left(c+cr(-1+\lambda)-c\lambda+v\lambda\right)\left(\sqrt{e}\eta-\tau_r\right)+\frac{1}{4}\lambda\left(c+cr(-1+\lambda)-c\lambda+v\lambda\right)\tau_t$$
$$+\frac{1}{4}\left(cr+c(-1+\lambda)-(cr+v)\lambda+4e\rho-\frac{4(a-bc)\left(bk(-1+s+\beta)+\rho^2\right)}{2bk(-1+s+\beta)(-2+\gamma)-\rho^2}\right)\tau_u$$

$$E^{N*}-E^{C*}=\frac{3b(-a+bc)k\beta\rho^2\left(2\sqrt{e}\eta-\tau_c-\tau_m-\tau_u\right)}{\left(4bk(-1+s)+\rho^2\right)\left(4bk(-1+s+\beta)+\rho^2\right)},$$

$$E^{C*}-E^{J*}=\frac{2b(-a+bc)k(-1+s+\beta)\gamma\left(bk(-1+s+\beta)+\rho^2\right)\left(2\sqrt{e}\eta-\tau_c-\tau_m-\tau_u\right)}{\left(2bk(-1+s+\beta)(-2+\gamma)-\rho^2\right)\left(4bk(-1+s+\beta)+\rho^2\right)},$$

Due to $\tau_c>0,\tau_m>0,\tau_u>0$ and $-a+bc<0,-1+s<0,-2+\gamma<0,4bk(-1+s)+\rho^2<0,$

$4bk(-1+s+\beta)+\rho^2<0,-1+s+\beta<0,2bk(-1+s+\beta)(-2+\gamma)-\rho^2>0$.

And when $\eta>\dfrac{\tau_c+\tau_m+\tau_u}{2\sqrt{e}}$,we conclude $E^{N*}<E^{C*}$, $E^{C*}<E^{J*}$

Thus $E^{N*} < E^{C*} < E^{J*}$.

***Proof of Proposition** 7*

Optimal Social welfare:

$$SW^{N*} = \frac{\begin{array}{l} 48b^3c^2k^2(-1+s)^2 + 8b^4c^2k^2(-1+s)^2 + 24bk(-1+s)\left(c-cr-c\lambda+(cr+v)\lambda\right)^2\rho^2 \\ +3\left(c-cr-c\lambda+(cr+v)\lambda\right)^2\rho^4 + 8a^2k\left(b(6+b)k(-1+s)^2-2\rho^2\right) \\ +16abck\left(-b(6+b)k(-1+s)^2+2\rho^2\right)+16b^2k\left(3k(-1+s)^2\left(c-cr-c\lambda+(cr+v)\lambda\right)^2-c^2\rho^2\right) \end{array}}{16\left(4bk(-1+s)+\rho^2\right)^2}$$

$$SW^{C*} = \frac{\begin{array}{l} 48b^3c^2k^2(-1+s+\beta)^2 + 8b^4c^2k^2(-1+s+\beta)^2 \\ +24bk(-1+s+\beta)\left(c-cr-c\lambda+(cr+v)\lambda\right)^2\rho^2 \\ +3\left(c-cr-c\lambda+(cr+v)\lambda\right)^2\rho^4 + 8a^2k\left(b(6+b)k(-1+s+\beta)^2-2\rho^2\right) \\ +16abck\left(-b(6+b)k(-1+s+\beta)^2+2\rho^2\right)+ \\ 16b^2k\left(3k(-1+s+\beta)^2\left(c-cr-c\lambda+(cr+v)\lambda\right)^2-c^2\rho^2\right) \end{array}}{16\left(4bk(-1+s+\beta)+\rho^2\right)^2}$$

$$SW^{J*} = \frac{\begin{array}{l} 8b^4c^2k^2(-1+s+\beta)^2 - 16b^3c^2k^2(-1+s+\beta)^2(-3+2\gamma) \\ -12bk(-1+s+\beta)(-2+\gamma)\left(c-cr-c\lambda+(cr+v)\lambda\right)^2\rho^2 \\ +3\left(c-cr-c\lambda+(cr+v)\lambda\right)^2\rho^4 + 8a^2k\left(bk(-1+s+\beta)^2(6+b-4\gamma)-2\rho^2\right) \\ -16abck\left(bk(-1+s+\beta)^2(6+b-4\gamma)-2\rho^2\right) \\ +4b^2k\left(3k(-1+s+\beta)^2(-2+\gamma)^2\left(c-cr-c\lambda+(cr+v)\lambda\right)^2-4c^2\rho^2\right) \end{array}}{16\left(-2bk(-1+s+\beta)(-2+\gamma)+\rho^2\right)^2}$$

Social welfare comparison,

when $0 < \beta < -\dfrac{2(2+b(-1+s)+6s)(4bk(-1+s)+\rho^2)}{8bk(-2+b(-1+s)+6s)+(6+b)\rho^2}$,

$0 < \gamma < \dfrac{4b^2(2+b)k^2(-1+s+\beta)^3 + bk(-1+s+\beta)(b(-1+s+\beta)-2(3+s+\beta))\rho^2 - (1+s+\beta)\rho^4}{bk(-1+s+\beta)(b(6+b)k(-1+s+\beta)^2-2\rho^2)}$, we

obtain,

$$SW^N - SW^C = -\frac{\begin{array}{l} b(a-bc)^2k^2\beta\rho^2(8bk(b(-1+s)(-1+s+\beta)+6s(s+\beta) \\ -2(1+2s+\beta))+(4+12s+6\beta+b(-2+2s+\beta))\rho^2) \end{array}}{2(4bk(-1+s)+\rho^2)^2(4bk(-1+s+\beta)+\rho^2)^2} < 0,$$

$$SW^C - SW^J =$$
$$\frac{2b(a-bc)^2 k^2(-1+s+\beta)\gamma(b^2k^2(-1+s+\beta)^3(-8+b(-4+\gamma)+6\gamma) -bk(-1+s+\beta)(b(-1+s+\beta)-2(3+s+\beta-\gamma))\rho^2+(1+s+\beta)\rho^4)}{(-8b^2k^2(-1+s+\beta)^2(-2+\gamma)-2bk(-1+s+\beta)(-4+\gamma)\rho^2+\rho^4)^2}<0,$$

we conclude, $SW^N < SW^C < SW^J$.

Proof of Corollary 4

Social welfare sensitivity to subsidy rate s,

$$\frac{\partial SW^N}{\partial s}=\frac{b(a-bc)^2k^2(2+b(-1+s)+6s)\rho^2}{(4bk(-1+s)+\rho^2)^3},\quad \frac{\partial SW^C}{\partial s}=\frac{b(a-bc)^2k^2(2+6s+6\beta+b(-1+s+\beta))\rho^2}{(4bk(-1+s+\beta)+\rho^2)^3},$$

$$\frac{\partial SW^J}{\partial s}=\frac{b(a-bc)^2k^2(2+6s+6\beta+b(-1+s+\beta)-4(s+\beta)\gamma)\rho^2}{(2bk(-1+s+\beta)(-2+\gamma)+\rho^2)^3}.$$

Due to $-a+bc<0, -1+s<0, -2+\gamma<0, 4bk(-1+s)+\rho^2<0,$

$4bk(-1+s+\beta)+\rho^2<0, -1+s+\beta<0, 2bk(-1+s+\beta)(-2+\gamma)-\rho^2>0$,

So, when $s<\dfrac{-2+b}{6+b}$, $\dfrac{\partial SW^N}{\partial s}>0$,

when $s<\dfrac{-2+b-6\beta-b\beta}{6+b}$, $\dfrac{\partial SW^C}{\partial s}>0$,

When $s<\dfrac{-2+b-6\beta-b\beta+4\beta\gamma}{6+b-4\gamma}$, $\dfrac{\partial SW^J}{\partial s}>0$.

The proof processes of Proposition 8 are the same as Proposition 1 and 3.