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Research article

Bidirectional calibration-robust booking control under heterogeneous cancellation risk

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A. Proofs

A.1. Proof of Theorem 4.1

For every fixed z, the mapping

$$m \mapsto c_o\,(m + \sigma z - C)^+ + c_u\,(C - m - \sigma z)^+$$

is the maximum of affine functions and is therefore convex. Expectation preserves convexity, proving the convexity of G.

Because Z has a continuous distribution, the event $m + \sigma Z = C$ has a probability of zero. Differentiation under the expectation gives

$$G'(m) = c_o\mathbb{P}(m + \sigma Z > C) - c_u\mathbb{P}(m + \sigma Z < C).$$

Since

$$\mathbb{P}(m + \sigma Z < C) = F_Z\left(\frac{C - m}{\sigma}\right),$$

we obtain Equation (18). Differentiating once more yields Equation (19). □

A.2. Proof of Corollary 4.2

Strict positivity of the density on the relevant support implies strict convexity. The unique minimizer satisfies $G'(m^\star) = 0$. Substitution into Equation (18) gives Equation (20); inversion gives Equation (21). □

A.3. Proof of Proposition 4.3

The derivative with respect to capacity follows directly from Equation (21). The critical ratio $c_o/(c_o + c_u)$ increases with c_o and decreases with c_u. Since F_Z^{-1} is increasing, the claimed directions follow. $\square$

A.4. Proof of Proposition 4.4

The defining equation is

$$F_{\alpha,\ell}(z_u) = u.$$

Implicit differentiation gives

$$\frac{\partial z_u}{\partial \alpha} = -\frac{\partial F_{\alpha,\ell}(z_u)/\partial \alpha}{f_{\alpha,\ell}(z_u)}.$$

Because $m^\star = C - \sigma z_u$, Equation (23) follows. $\square$

A.5. Proof of Theorem 4.6

The mappings $d \mapsto (d - C)^+$ and $d \mapsto \mathcal{L}(d; C)$ are convex. The defining expectation inequality for convex order therefore gives both conclusions [44]. $\square$

A.6. Proof of Theorem 4.7

Under the differentiability assumption in Theorem 4.7, differentiating $R(x, q)$ with respect to q_j gives $x_j(r_j - f_j)$, while $m(x, q)$ changes at the rate $a_j x_j$. The ordinary chain rule therefore gives a capacity loss derivative $a_j x_j H'_\varepsilon(m(x, q)) = a_j x_j g$, and subtracting the two terms yields Equation (29). The differentiability assumption also gives the first-order expansion in Equation (30). If its coefficient is positive, local minimization moves q_j downward toward the feasible endpoint $\underline{q}_j$; if it is negative, the adverse move is upward toward $\overline{q}_j$. Dividing by $a_j > 0$ yields the threshold comparison with τ_j. For the global coordinatewise statement, $q_j \mapsto \Phi(x, q)$ is concave because R and m are affine in q_j and H_ε is convex. Hence, a minimum over the compact clipped interval $I_j = [\underline{q}_j, \overline{q}_j]$ is reached at an endpoint. A non-negative derivative throughout the interval identifies the lower endpoint, a non-positive derivative throughout identifies the upper endpoint, and a sign change requires a direct comparison of the two endpoint values. Nondifferentiable load points are not covered by the local derivative identity; they are handled later through the full subdifferential in the active scenario KKT certificate. $\square$

A.7. Proof of Theorem 4.8

By Theorem 4.7, the local adverse direction is determined by the sign of $\tau_j - g(\theta)$. The continuity of $g(\theta)$ implies that this sign can change only at a root of $g(\theta) = \tau_j$. If the sign is uniform over the full calibration interval on each side, the globally adverse coordinate endpoint changes accordingly. $\square$

A.8. Proof of Theorem 4.9

The revenue function is affine in q, and $m(x, q)$ is affine in q. Because H_ε is convex, $-H_\varepsilon(m(x, q))$ is concave. Hence $\Phi(x, q)$ is concave. A concave function minimized over a nonempty compact polytope reaches a minimum at an extreme point. $\square$

A.9. Proof of Propositio 4.10

After the locally adverse sign has been selected, let $\bar{u}_j \in [0, 1]$ denote the feasible perturbation magnitude cap implied by $q_j \in [0, 1]$ in that direction. The linearized loss from perturbing segment j by a magnitude of $|u_j| \leq \bar{u}_j$ is $s_j(x, g)|u_j|$, while it consumes the weighted budget $\omega_j|u_j|$. The inner problem is therefore a continuous knapsack problem with item-specific upper caps. The greedy rule allocates the budget in nonincreasing order of value per unit of weight $s_j(x, g)/\omega_j$, filling each segment up to its feasible cap before moving to the next ratio. Consequently, an optimal allocation can be chosen with at most one segment fractional relative to its feasible cap, apart from equivalent redistributions among tied scores. □

A.10. Proof of Proposition 4.11

The expression inside Equation (36) is affine in q_j with a slope $(r_j - f_j) - a_j g$, so its exact coordinatewise minimum is reached at one of the clipped endpoints in $[\widehat{q}_j - \delta_j, \widehat{q}_j + \delta_j] \cap [0, 1]$. When probability clipping is inactive, the two endpoints are symmetric about $\widehat{q}_j$, and the minimum subtracts δ_j times the absolute slope, yielding Equation (37). When clipping is active, Equation (36) remains the exact definition, and the appropriate clipped endpoint is used directly. Under a locally unique active box scenario, $M_j^{\text{box}}(g)$ is the one-sided marginal contribution of segment j, giving the stated local sign interpretation. The argument does not extend to a shared weighted budget without accounting for active scenario coupling. □

A.11. Proof of Theorem 4.12

Under the fixed-scale specification, each extreme calibration scenario defines a concave scenario value function of x. If H_ε is nondifferentiable, any $g(x, q) \in \partial H_\varepsilon(m(x, q))$ generates the corresponding scenario supergradient component in Equation (39); no derivative through $\sigma(x)$ is present because the theorem fixes the residual scale. By Theorem 4.9, the robust objective is the pointwise minimum of finitely many such concave scenario functions. Danskin-type value function calculus and standard variational analysis imply that a supergradient of the robust objective at $x^\star$ can be represented as a convex combination of active scenario supergradients [45, 46]. Hence the weights $\lambda_q \geq 0$ summing to one exist such that the aggregate supergradient has components Equation (40). Applying the normal-cone optimality condition for the box $[0, N]$ gives Equation (41). If a coupling constraint, the robust CVaR constraint, or the support compatibility constraint is active, its multiplier-weighted subgradient must also enter stationarity, so the reduced sign certificate alone is insufficient. If the portfolio-dependent scale extension is used, the additional derivative or subgradient contribution through $\sigma(x)$ must likewise be included. □

A.12. Proof of Theorem 4.13

If $\Gamma_2 \geq \Gamma_1$, then $\mathcal{U}_{\text{cal}}(\Gamma_1) \subseteq \mathcal{U}_{\text{cal}}(\Gamma_2)$. Hence, for every fixed portfolio x, $\Psi_{\Gamma_2,\varepsilon}(x) \leq \Psi_{\Gamma_1,\varepsilon}(x)$. The same set inclusion also makes the robust CVaR and support constraints weakly more restrictive, so $\mathcal{X}_{\Gamma_2,\varepsilon} \subseteq \mathcal{X}_{\Gamma_1,\varepsilon}$. Maximizing a weakly smaller objective over a weakly smaller feasible set proves $V(\Gamma_2, \varepsilon) \leq V(\Gamma_1, \varepsilon)$ and therefore non-negativity and monotonicity of PCU whenever the corresponding robust problems are feasible. For the fixed-portfolio bound, $\Gamma = 0$ fixes $q = \widehat{q}$. For any $q \in \mathcal{U}_{\text{cal}}(\Gamma)$, $|q_j - \widehat{q}_j| \leq \delta_j|u_j|$. Thus the contractual value change is at most $x_j|r_j - f_j|\delta_j|u_j|$ in coordinate j, while the L_H-Lipschitz property of H_ε bounds the robust capacity loss change by $a_j x_j L_H \delta_j |u_j|$. Summing and maximizing over $\sum_j \omega_j |u_j| \le \Gamma$ gives the first inequality in Equation(43); using $x_j \le N_j$ gives the second. This argument concerns the fixed portfolio objective exposure only and therefore does not assert the same upper bound for the global PCU when the robust feasible set changes with Γ. □

A.13. Proof of Theorem 4.14

Under the fixed-scale specification, divide the objective by $c_u C$ and all load quantities by C. The normalized realized load is $D/C = m/C + (\sigma/C)Z$, while the asymmetric loss is scaled by $c_u C$ and the robust CVaR bound by C, which yields the normalized parameters in Equation (12). The Wasserstein ambiguity set in Equatin (12) is defined on the standardized residual Z, not on the dimensional load D, so its radius remains ε under capacity normalization rather than becoming ε/C. After these substitutions, the two normalized problems have identical feasible sets, objectives, ambiguity sets, and CVaR constraints, so their normalized solution sets and normalized optimal values coincide. If $\sigma = \sigma(x)$ is used, the same conclusion requires the equality of the entire normalized scale map $x \mapsto \sigma(x)/C$. □

A.14. Proof of Theorem 4.15

Because $\Gamma_2 \ge \Gamma_1$ and $\varepsilon_2 \ge \varepsilon_1$, we have $\mathcal{U}_{\text{cal}}(\Gamma_1) \subseteq \mathcal{U}_{\text{cal}}(\Gamma_2)$ and $\mathcal{P}_{\varepsilon_1} \subseteq \mathcal{P}_{\varepsilon_2}$. Therefore $\Psi_{\Gamma_2,\varepsilon_2}(x) \le \Psi_{\Gamma_1,\varepsilon_1}(x)$ for every portfolio x. The larger adversarial sets also make the robust CVaR and support constraints weakly more restrictive, implying $\mathcal{X}_{\Gamma_2,\varepsilon_2} \subseteq \mathcal{X}_{\Gamma_1,\varepsilon_1}$. Consequently,we have

$$\max_{x \in \mathcal{X}_{\Gamma_2,\varepsilon_2}} \Psi_{\Gamma_2,\varepsilon_2}(x) \le \max_{x \in \mathcal{X}_{\Gamma_2,\varepsilon_2}} \Psi_{\Gamma_1,\varepsilon_1}(x) \le \max_{x \in \mathcal{X}_{\Gamma_1,\varepsilon_1}} \Psi_{\Gamma_1,\varepsilon_1}(x),$$

which is exactly Equation (46). □

A.15. Proof of Proposition 4.17

The revenue difference is bounded by

$$|R(x,q) - R(x,q')| \le \sum_j x_j |r_j - f_j| |q_j - q'_j|.$$

The mean load difference is bounded by

$$|m(x,q) - m(x,q')| \le \sum_j a_j x_j |q_j - q'_j|.$$

Because $\mathcal{G}_Q$ is $L_{\mathcal{G}}$-Lipschitz in m,

$$|\mathcal{G}_Q(m(x,q)) - \mathcal{G}_Q(m(x,q'))| \le L_{\mathcal{G}} \sum_j a_j x_j |q_j - q'_j|.$$

Combining the revenue and capacity loss bounds gives

$$|\Pi(x;q,Q) - \Pi(x;q',Q)| \le \sum_j \left[x_j |r_j - f_j| + a_j x_j L_{\mathcal{G}}\right] |q_j - q'_j|,$$

and bounding the coefficients by their maximum yields Equation (48). □

A.16. Proof of Theorem 5.1

The result follows from strong duality for distributionally robust expectations over a 1-Wasserstein ball under the upper semicontinuity and linear growth of $h_{x,q}$. The support restriction is incorporated in the inner supremum $z \geq \ell$. □

A.17. Proof of Corollary 5.2

By the Kantorovich–Rubinstein duality, the expectation of an L-Lipschitz function can increase by at most $L\varepsilon$ over a 1-Wasserstein ball of radius ε. Substitute $L = \sigma(x) \max\{c_o, c_u\}$. □

A.18. Proof of Proposition 5.3

For the fixed m, the mismatch loss as a function of z is piecewise linear with a lower tail slope $-\sigma c_u$ and upper tail slope σc_o. Because $c_o \geq c_u$ and the support is unbounded above, the maximal Lipschitz slope is σc_o and can be approached by transporting an arbitrarily small probability mass sufficiently far into the upper tail. Kantorovich–Rubinstein duality therefore gives Equation (55) under the finite first moment condition [20, 22]. For the CVaR representation, the hinge $((m + \sigma z - C)^+ - \tau)_+$ has the upper-tail slope σ for every $\tau \geq 0$. The Rockafellar–Uryasev formula and the assumed minimax interchange then permit the transport argument to be applied inside the auxiliary minimization, adding $\varepsilon\sigma/(1-\beta)$ and yielding Equation (56) [27]. □

A.19. Proof of Proposition 5.4

The function $h_{x,q}$ is affine on either side of $z_C(x,q)$, while the transportation penalty is affine on either side of $\widehat{z}_s$. Their difference is piecewise affine with breakpoints at $\ell, \widehat{z}_s$, and $z_C(x,q)$. Under the stated slope bound, the function cannot diverge upward as $z \to \infty$. A finite maximum is therefore reached at a breakpoint. □

A.20. Proof of Theorem 5.5

Fix $q \in \mathcal{U}_{\mathrm{cal}}(\Gamma)$ and define

$$F_q(Q,\tau) = \tau + \frac{1}{1-\beta}\mathbb{E}_Q[(O(x,q,Z) - \tau)^+].$$

The Rockafellar–Uryasev representation gives $\mathrm{CVaR}_\beta^Q(O) = \min_{\tau\geq 0} F_q(Q,\tau)$. Under the assumed minimax interchange for this fixed q, we have

$$\sup_{Q\in\mathcal{P}_\varepsilon} \mathrm{CVaR}_\beta^Q(O) = \min_{\tau\geq 0} \sup_{Q\in\mathcal{P}_\varepsilon} F_q(Q,\tau).$$

Hence the fixed-q robust CVaR value is at most κ if and only if there is a threshold $\tau_q \geq 0$ satisfying Equation (62). The outer supremum over q is at most κ if and only if this fixed-q condition holds for every $q \in \mathcal{U}_{\mathrm{cal}}(\Gamma)$. Because the minimizing threshold may vary with q, replacing the family $\{\tau_q\}$ by one common threshold would generally impose a stronger condition. The inner Wasserstein expectation is then dualized as in Theorem 5.1. □

A.21. Proof of Proposition 5.6

In the interior band case, write the adverse contribution as $-\sum_j \delta_j|\varphi_j|v_j$ with $0 \le v_j \le 1$ and $\sum_j \omega_j v_j \le \Gamma$. Maximizing the magnitude of this adverse deviation is a continuous knapsack linear program. Its dual is

$$\min_{\pi\ge0,\, s_j\ge0} \left\{ \Gamma\pi + \sum_j s_j :\ \omega_j\pi + s_j \ge \delta_j|\varphi_j| \right\}.$$

Subtracting this worst deviation from $\varphi^\top\widehat{q}$ yields Equation (64). If a probability boundary clips one direction, the primal simply replaces the symmetric unit bound by the corresponding directional cap, and standard LP duality applies to that modified cap. □

A.22. Proof of Proposition 5.7

The full finite master consists of the value cut family indexed by $\mathcal{E}\times\mathcal{P}_L$, the robust CVaR family indexed by $\mathcal{E}\times\mathcal{P}_C$, and the support family indexed by $\mathcal{E}$. Equation (69) computes the smallest admissible of the value right-hand side at x^K, so Δ^K_{val} is exactly the largest value cut violation. Equation (70) computes the largest robust-CVaR left-hand side minus κ, so its positive part is exactly the largest CVaR violation. Equation (71) is the positive part of the largest support violation. Hence the three residuals are simultaneously zero if and only if no omitted full master constraint is violated. Replacing zero by ϵ_{alg} gives the stated residual certificate. □

A.23. Proof of Theore 5.8

Let $\mathcal{E}$, $\mathcal{P}_L$, and $\mathcal{P}_C$ be the finite sets underlying the conceptual full master Equations (65)–(68). There are at most $|\mathcal{E}||\mathcal{P}_L|$ value cuts, $|\mathcal{E}||\mathcal{P}_C|$ CVaR cuts, and $|\mathcal{E}|$ support constraints. The restricted master contains only a subset of these constraints and is therefore a relaxation of the full finite model.

At each nonterminating iteration, at least one of Δ^K_{val}, Δ^K_{cvar}, or Δ^K_{sup} exceeds the algorithmic tolerance. Exact weighted budget separation identifies an extreme-point calibration scenario and, when relevant, an affine piece that violates a full-model constraint. That constraint cannot already be present because the incumbent is feasible for the restricted master. Hence, every nonterminating iteration adds at least one previously absent constraint from a finite pool.

After finitely many iterations, no residual exceeds the tolerance. In the exact case $\epsilon_{\text{alg}} = 0$, the incumbent is feasible for the full finite model. Because the restricted master is a relaxation, its optimal value is no smaller than the full finite optimum; because the incumbent is full-model feasible at termination, the full finite optimum is no smaller than the incumbent value. The two inequalities imply equality, so the incumbent is globally optimal for the finite piecewise-linear robust model. For positive tolerance, the same argument yields the reported robust value, CVaR, and support residual certificate. □

A.24. Proof of Theorem 5.9

By the union bound,

$$\mathbb{P}\left(q^0 \in \mathcal{U}_{\text{cal}}(\Gamma), P^0 \in \mathcal{P}_\varepsilon\right) \ge 1-\eta-\zeta.$$

In this event, the true parameters are feasible adversarial selections in the robust model. Therefore the robust objective is a lower bound on true expected profit and the robust CVaR constraint dominates the true-distribution CVaR. □

B. Numerical certification diagnostics

The following table reports the detailed numerical checks for the 35-case uncertainty grid. The monotonicity tests correspond to Theorem 4.15; the PCU diagnostic checks the global non-negativity implication of Theorem 4.13 while the fixed portfolio sensitivity bound in Equation (43) is not treated as a global optimal value certificate; and the feasibility and gap checks audit the finite cutting-plane implementation in Theorem 5.8. The certificate concerns the finite quadrature and piecewise-linear model solved in computation.

Table A1. Numerical certification checks for the 35-case uncertainty grid.

Diagnostic	Worst observed value	Status
Violation of robust value monotonicity in Γ for each tested ε	0	Pass
Violation of robust value monotonicity in ε for each tested Γ	0	Pass
Violation of non negativity of the price of calibration uncertainty	0	Pass
Number of master problems not solved to optimality	0	Pass
Maximum robust CVaR constraint residual	2.84×10^{-14}	Pass
Maximum support constraint residual	0	Pass
Maximum final finite-model optimality gap	3.64×10^{-12}	Pass

Note: A positive residual denotes a constraint or theoretical property violation. Values below the numerical tolerance of 10^{-8} are treated as zero. The finite-model optimality gap is reported in absolute objective value units.

C. Real-world-calibrated illustration audit

Table A2 reports the split-sample audit for the five representative public data segments used in Table 11. The audit is included only to document the construction of the calibration bands used in the illustration. It is not evidence of implemented robust booking control performance.

Table A2. Split-sample calibration audit for the real-world-calibrated illustration

Segment	Train N	Cal. N	Test N	Train show-up	Cal. show-up	Test show-up	Cal. error	Δ_j
Online TA	33,858	11,392	11,227	0.631	0.638	0.631	0.007	0.020
Offline TA/TO	14,560	4,779	4,880	0.659	0.648	0.658	0.011	0.024
Groups	11,833	3,989	3,989	0.390	0.395	0.382	0.005	0.020
Direct	7,624	2,404	2,578	0.851	0.830	0.849	0.021	0.036
Corporate	3,175	1,115	1,005	0.814	0.799	0.825	0.014	0.038

Notes: For all five representative segments, the held-out test show-up rate lies within the corresponding calibration band. The audit concerns only the descriptive calibration illustration and does not evaluate realized overbooking decisions.

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