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Research article

Green subsidy strategies in capital-constrained supply chains with manufacturer fairness concerns

Fei Pan[1,2,*], **Ming Liu**[1] **and Yanqiu Zhu**[3]

[1] Business School, University of Shanghai for Science and Technology, Shanghai 200093, China

[2] School of Intelligent Emergency Management, University of Shanghai for Science and Technology, Shanghai 200093, China

[3] Business School, Suzhou University, Suzhou 234000, China

* **Correspondence:** Email: feipan@usst.edu.cn.

Appendix

Part A (Solution of equilibrium results under different strategies)

Proof of Lemma 1

Proof. Substituting $D_{PL} = 1 - p_{PL}$ into Eq. (4.2) and solving the first-order condition yields $w_{PL} = \frac{1}{2}(c - y + cr + 1)$. Substituting this into the retailer's profit function and solving the first-order condition gives $y^*_{PL} = \frac{1}{2} - cr - \frac{c}{2}$. Substituting back, we obtain $w^*_{PL} = \frac{3c}{4} + cr + \frac{1}{4}$. The equilibrium profits are $\pi^{\mathrm{PL}}_{\mathrm{R}} = \frac{(c-1)^2}{8}$, $\pi^{\mathrm{PL}}_{\mathrm{M}} = \frac{(c-1)^2}{16}$. □

Proof of Lemma 2

Proof. We substitute $D_{GL} = 1 - p_{GL} + e_{GL}$ into Eq. (4.3). The first-order condition yields $e_{GL} = -\frac{c+y+cr-1}{2k(1+r)-1}$ and $w_{GL} = \frac{k(1+c+r-y+2cr-ry+cr^2)-c}{2k(1+r)-1}$. We then plug w_{GL} and e_{GL} into Eq. (4.3), and obtain $y_{GL} = \frac{c(1+4r+3r^2)-(1+2r)+2k(1+2r+r^2)-2ck(1+4r+5r^2+2r^3)}{4k(1+r)^2-(3r+2)}$ by solving the first-order optimality condition. Substituting back, we obtain $e^*_{GL} = \frac{(1-c)(r+1)}{4k(r+1)^2-3(r+1)+1}$ and $w^*_{GL} = \frac{4ck(r+1)^3+\left(kc(k+3)\right)(r+1)^2+c(r+1)}{4k(r+1)^2-3(r+1)+1}$. In accordance with the above results, we can obtain $\pi^{\mathrm{GL}}_R = \frac{k(c-1)^2(r+1)^2}{2\left[4k(r+1)^2-3(r+1)+1\right]}$ and $\pi^{\mathrm{GL}}_M = \frac{k(c-1)^2(r+1)^3\left(2k(r+1)-1\right)}{2\left[4k(r+1)^2-3(r+1)+1\right]^2}$. The second-order conditions hold when $k > \frac{1}{2(1+r)}$. □

Proof of Lemma 3

Proof. We first solve the manufacturer's profit function. We substitute $D_{CL} = 1 - p_{CL} + e_{CL}$ into Eq. (4.5). The first-order condition yields $w_{CL} = \frac{c+k(-1-r+t+y)+ck(-1-2r+t-r^2)+k\left[rt+ry-ty+c(2rt-rty+r^2t)\right]}{1-2k(1+r)(1-t)}$ and $e_{CL} = -\frac{c+y+cr-1}{1-2k(1+r)(1-t)}$. We then plug w_{CL} and e_{CL} into Eq. (4.6), and obtain

$$y_{CL} = -\frac{(1-c)+(2r-2t-4cr-2rt-3cr^2+3cr^2t+5crt+2ct)+k(-2-4r+4t-2r^2-2t^2)+ck(2+8r+10r^2+4r^3-4t+2t^2)+ckrt(-16+8t-20r-8r^2+10rt+4r^2t)}{4k(1+2r-2t+r^2+t^2+2rt^2-2r^2t+r^2t^2-4rt)-(3r-3t-3rt+2)}$$

by solving the first-order optimality condition. Substituting y_{CL} into w_{CL} and e_{CL}, we obtain $e^*_{CL} = \frac{(1-c)(r+1)(1-t)}{4k(r+1)^2(1-t)^2+3(r+1)(1-t)-3(1-t)+1}$ and $w^*_{CL} = \frac{(1+r)\left[(t-1)^2\left(k(1+r)+c\left(-2-3r+k(1+r)(3+4r)\right)\right)+3(1+r)t\right]}{(3t-2)+3r(t-1)+4k(1+r)^2(t-1)^2}$. In accordance with the above results, we obtain $\pi_R^{CL} = \frac{(-1+c)^2k(1+r)^2(-1+t)^2}{-4+6r(-1+t)+8k(1+r)^2(-1+t)^2+6t}$ and $\pi_M^{CL} = \frac{(-1+c)^2k(1+r)^3(1+2k(1+r)(-1+t))(-1+t)^3}{2\left(-2+3r(-1+t)+4k(1+r)^2(-1+t)^2+3t\right)^2}$. The second-order conditions hold when $k > \frac{1}{2(1+r)(1-t)}$. □

Part B (Proof)

Proof of Proposition 1

Proof. According to the equilibrium degree of product greening in Strategies GL and CL, we have $e_{CL} - e_{GL} = \frac{t(r+1)(c-1)(1+4k(1+r)^2(t-1))}{(4k(1+r)^2-(3r+2))(4k(1+r)^2(t-1)^2+(3t-3r+3rt-2))}$. Since $0 < c < 1, 0 < t < 1$, the numerator is negative. We derive the thresholds $\hat{k}_3 = \dfrac{3r+2}{4(1+r)^2}$, $\hat{k}_5 = \dfrac{3r-3t-3rt+2}{4(1+r)^2(t-1)^2}$, $\hat{t}_1 = \dfrac{3r+1}{3r+2}$, and $\hat{t}_2 = \dfrac{3r+2}{3r+3}$.

When $0 < t < \hat{t}_1$, if $\hat{k}_2 < k < \hat{k}_3$ or $\hat{k}_5 < k < 2$, the denominator is greater than zero, and if $\hat{k}_3 < k < \hat{k}_5$, the denominator is less than zero. When $\hat{t}_1 < t < \hat{t}_2$, if $\hat{k}_5 < k < \hat{k}_3$, the denominator is less than zero, and if $\hat{k}_3 < k < 2$, the denominator is greater than zero. When $\hat{t}_2 < t < 1$, if $\hat{k}_2 < k < \hat{k}_3$, the denominator is less than zero, and if $\hat{k}_3 < k < 2$, the denominator is greater than zero. □

Proof of Proposition 2

According to the equilibrium wholesale prices in Strategies PL and GL, we have $w_{GL} - w_{PL} = \frac{(3r+2)(1-c)}{4(4k(1+r)^2-(3r+2))}$. We derive a threshold $\hat{k}_3 = \dfrac{3r+2}{4(1+r)^2}$. When $\hat{k}_3 < k$, then $w_{GL} - w_{PL} > 0$. However, when $k < \hat{k}_3$, $w_{GL} - w_{PL} < 0$.

According to the equilibrium wholesale prices in Strategies GL and CL, we have

$$w_{CL} - w_{GL} = \frac{kt(1+r)^2(1-c)(1+3r-2t-3rt)}{(4k(1+r)^2-(3r+2))(4k(1+r)^2(1-t)^2+(3t-3r+3rt-2))}.$$

We derive the thresholds $\hat{k}_3 = \dfrac{3r+2}{4(1+r)^2}$, $\hat{k}_5 = \dfrac{3r-3t-3rt+2}{4(1+r)^2(t-1)^2}$, $\hat{t}_1 = \dfrac{3r+1}{3r+2}$, $\hat{t}_2 = \dfrac{3r+2}{3r+3}$, $\hat{t}_3 = \dfrac{r}{3r+2}$, and $\hat{t}_4 = \dfrac{r}{1+r}$. When $0 < t < \hat{t}_3$, if $\hat{k}_2 < k < \hat{k}_3$ or $\hat{k}_5 < k$, we have $w_{CL} - w_{GL} > 0$, and if $\hat{k}_3 < k < \hat{k}_5$, we have $w_{CL} - w_{GL} < 0$. When $\hat{t}_3 < t < \hat{t}_4$, and if $\hat{k}_2 < k < \hat{k}_5$, we have $w_{CL} - w_{GL} < 0$, if $\hat{k}_5 < k$, we have $w_{CL} - w_{GL} > 0$. When $\hat{t}_4 < t < \hat{t}_1$ and $\hat{k}_2 < k$, we have $w_{CL} - w_{GL} > 0$. When $\hat{t}_1 < t < 1$ and $\hat{k}_2 < k$, we have $w_{CL} - w_{GL} < 0$.

According to the equilibrium wholesale prices in Strategies PL and CL, we have

$$w_{CL} - w_{PL} = \frac{(1-c)(2-3t+3r(1-t))}{4(4k(1+r)^2-(3r+2))(4k(1+r)^2(1-t)^2+(3t-3r+3rt-2))}.$$

When $0 < t < \hat{t}_3$, if $\hat{k}_2 < k < \hat{k}_5$, we have $w_{CL} - w_{PL} < 0$, and if $\hat{k}_5 < k$, we have $w_{CL} - w_{PL} > 0$. When $\hat{t}_3 < t < \hat{t}_4$, ans if $\hat{k}_2 < k < \hat{k}_5$, we have $w_{CL} - w_{PL} < 0$, if $\hat{k}_5 < k$, we have $w_{CL} - w_{PL} > 0$. When $\hat{t}_4 < t < \hat{t}_2$ and $\hat{k}_2 < k$, we have $w_{CL} - w_{PL} > 0$. When $\hat{t}_2 < t < 1$ and $\hat{k}_2 < k$, we have $w_{CL} - w_{PL} < 0$.

Proof of Proposition 3

Proof. According to the equilibrium retail prices in Strategies PL and GL, we have $p_{GL} - p_{PL} = \frac{(r+2)(1-c)}{4(4k(1+r)^2-(3r+2))}$. We derive a threshold $\hat{k}_3 = \dfrac{3r+2}{4(1+r)^2}$. When $\hat{k}_3 < k$, then $p_{GL} - p_{PL} > 0$. However, when $k < \hat{k}_3$, $p_{GL} - p_{PL} < 0$.

According to the equilibrium retail prices in Strategies GL and CL, we have

$$p_{CL} - p_{GL} = \frac{t(1+r)(c-1)(1+k(t(2+3r+r^2)-(3+4r+r^2))}{(4k(1+r)^2-(3r+2))(4k(1+r)^2(1-t)^2+(3t-3r+3rt-2))}.$$

Since $0 < c < 1, 0 < t < 1$, the numerator is greater than zero. We derive these thresholds $\hat{k}_2 = \dfrac{1}{2(1+r)(1-t)}$, $\hat{k}_3 = \dfrac{3r+2}{4(1+r)^2}$, $\hat{k}_5 = \dfrac{3r-3t-3rt+2}{4(1+r)^2(t-1)^2}$, $\hat{t}_1 = \dfrac{3r+1}{3r+2}$, $\hat{t}_2 = \dfrac{3r+2}{3r+3}$, and $\hat{t}_3 = \dfrac{r}{3r+2}$. When $0 < t < \hat{t}_3$, if $\hat{k}_2 < k < \hat{k}_3$ or $\hat{k}_5 < k < 2$, the denominator is greater than zero, ans if $\hat{k}_3 < k < \hat{k}_5$, the denominator is less than zero. When $\hat{t}_3 < t < \hat{t}_1$, if $\hat{k}_2 < k < \hat{k}_5$, the denominator is less than zero, if $\hat{k}_5 < k < 2$, the denominator is greater than zero. If $\hat{t}_1 < t < 1$ and $\hat{k}_2 < k < 2$, the denominator is greater than zero.

According to the equilibrium retail prices in Strategies PL and CL, we have

$$p_{CL} - p_{PL} = \frac{(1-c)(r(1-t)+2-t)}{4(4k(1+r)^2(1-t)^2+(3t-3r+3rt-2))}.$$

Since $0 < c < 1, 0 < t < 1$, the numerator is greater than zero. We derive a threshold $\hat{k}_5 = \dfrac{3r-3t-3rt+2}{4(1+r)^2(t-1)^2}$. When $\hat{k}_5 < k$, $p_{CL} - p_{PL} > 0$. However, when $k < \hat{k}_5$, $p_{CL} - p_{PL} < 0$. □

Proof of Proposition 4

Proof. According to the retailer's profits in Strategies PL and GL, we have $\pi_r^{GL} - \pi_r^{PL} = \frac{(3r+2)(c-1)^2}{8(4k(1+r)^2-(3r+2))}$. Obviously, the numerator is greater than zero. We derive a threshold $\hat{k}_3 = \dfrac{3r+2}{4(1+r)^2}$. When $k < \hat{k}_3$, we have $\pi_r^{GL} - \pi_r^{PL} < 0$. However, when $\hat{k}_3 < k$, $\pi_r^{GL} - \pi_r^{PL} > 0$.

According to the retailer's profits in Strategies GL and CL, we have

$$\pi_r^{CL} - \pi_r^{GL} = \frac{kt(r+1)^2(c-1)^2(3r-2t-3rt+1)}{2(4k(1+r)^2-(3r+2))(4(4k(1+r)^2-(3r+2))4k(1+r)^2(1-t)^2+(3t-3r+3rt-2))}.$$

We derive the thresholds $\hat{k}_3 = \dfrac{3r+2}{4(1+r)^2}$, $\hat{k}_5 = \dfrac{3r-3t-3rt+2}{4(1+r)^2(t-1)^2}$, $\hat{t}_1 = \dfrac{3r+1}{3r+2}$, and $\hat{t}_3 = \dfrac{r}{3r+2}$. When $0 < t < \hat{t}_3$, we derive that $\frac{kt(r+1)^2(c-1)^2(3r-2t-3rt+1)}{2} > 0$. If $\hat{k}_2 < k < \hat{k}_3$, we derive that $\frac{1}{4k(1+r)^2-(3r+2)} < 0$ and $\frac{1}{4(4k(1+r)^2-(3r+2))4k(1+r)^2(1-t)^2+(3t-3r+3rt-2)} < 0$. If $\hat{k}_3 < k < \hat{k}_5$, we derive that $\frac{1}{4k(1+r)^2-(3r+2)} > 0$ and $\frac{1}{4(4k(1+r)^2-(3r+2))4k(1+r)^2(1-t)^2+(3t-3r+3rt-2)} < 0$. If $\hat{k}_5 < k < 2$, we derive that $\frac{1}{4k(1+r)^2-(3r+2)} > 0$ and $\frac{1}{4(4k(1+r)^2-(3r+2))4k(1+r)^2(1-t)^2+(3t-3r+3rt-2)} > 0$. When $\hat{t}_3 < t < \hat{t}_1$, we derive that $\frac{kt(r+1)^2(c-1)^2(3r-2t-3rt+1)}{2} > 0$ and $\frac{1}{4k(1+r)^2-(3r+2)} > 0$. If $\hat{k}_2 < k < \hat{k}_5$, we derive that $\frac{1}{4(4k(1+r)^2-(3r+2))4k(1+r)^2(1-t)^2+(3t-3r+3rt-2)} < 0$. If $\hat{k}_5 < k < 2$, we derive that $\frac{1}{4(4k(1+r)^2-(3r+2))4k(1+r)^2(1-t)^2+(3t-3r+3rt-2)} > 0$. When $\hat{t}_1 < t < 1$, we derive that $\frac{kt(r+1)^2(c-1)^2(3r-2t-3rt+1)}{2} < 0$ and $\frac{1}{4k(1+r)^2-(3r+2)} \times \frac{1}{4(4k(1+r)^2-(3r+2))4k(1+r)^2(1-t)^2+(3t-3r+3rt-2)} > 0$. □

Proof of Proposition 5

Proof. According to the manufacturer's profits in Strategies PL and GL, we have

$$\pi_m^{GL} - \pi_m^{PL} = \frac{\left[8k(1+r)^2(2r+1) - (3r+2)^2\right](c-1)^2}{16\left[4k(1+r)^2 - (3r+2)\right]^2}.$$

Obviously, the denominator is greater than zero. We derive a threshold $\hat{k}_4 = \dfrac{(3r+2)^2}{8(1+2r)(1+r)^2}$. when $k > \hat{k}_4$, we have $\pi_m^{GL} - \pi_m^{PL} > 0$. However, $\hat{k}_4 < \hat{k}_2$. Thus, $\pi_m^{GL} > \pi_m^{PL}$ is firmly established.

According to the manufacturer's profits in Strategies PL and CL, we have

$$\pi_m^{CL} - \pi_m^{PL} = \frac{(1-c)^2[8k(1+r)^2(1-t)^2(2(1+r)(1-t)-1) - (3(1+r)(1-t)-1)^2]}{16(4k(1+r)^2(1-t)^2 + (3t-3r+3rt-2))}.$$

Obviously, the denominator is greater than zero. We derive a threshold $\hat{k}_7 = \frac{3(1+r)(1-t)-1}{8(1+r)^2(t-1)^2\left[(2r+1)-2t(1+r)\right]}$. when $k < \hat{k}_7$, the denominator is less than zero. However, when $\hat{k}_7 < k$, the numerator is greater than zero.

According to the manufacturer's profits in Strategies GL and CL, we have

$$\pi_m^{CL} - \pi_m^{GL} = \frac{k(1-c)^2(1+r)^3}{2(4k(1+r)^2 - 3(1+r)+1)^2(4k(1+r)^2(1-t)^2 - 3(1+r)(1-t)+1)^2}\,\Omega(k;r,t),$$

where $\Omega(k;r,t) = (1+r)(1-t)^3(2k(1+r)(1-t)-1)(4k(1+r)^2 - 3(1+r)+1)^2 - (1+r)(2k(1+r)-1)(4k(1+r)^2(1-t)^2 - 3(1+r)(1-t)+1)^2$.

Since $k > 0$, $(1-c)^2 > 0$, $(1+r) > 0$, $(4k(1+r)^2(1-t)^2 - 3(1+r)(1-t)+1)^2 > 0$, and $(4k(1+r)^2 - 3(1+r)+1)^2 > 0$, it follows that the sign of $\pi_m^{CL} - \pi_m^{GL}$ is fully characterized by $\Omega(k;r,t)$.

Noting that $\Omega(k;r,t)$ is quadratic in k, we mark $\Omega(k;r,t) = Ak^2 + Bk + C$, where $A = 16(1+r)^4(1-t)^2(1-(1-t))(2(1+r)(1-t) - (1-t) - 1)$, $B = -2(1+r)^2(1-(1-t))\Big(9(1+r)^2(1-t)^3 + 9(1+r)^2(1-t)^2 - 6(1+r)(1-t)^3 - 10(1+r)(1-t)^2 - 6(1+r)(1-t) + (1-t)^3 + (1-t)^2 + (1-t) + 1\Big)$, $C = (1+r)(1-(1-t))\Big(9(1+r)^2(1-t)^2 - 6(1+r)(1-t)^2 - 6(1+r)(1-t) + (1-t)^2 + (1-t) + 1\Big)$. Let $\hat{k}_8(r,t)$ denote the economically relevant root of $\Omega(k;r,t) = 0$, given by $\hat{k}_8(r,t) = \frac{-B+\sqrt{B^2-4AC}}{2A}$. Then, $\pi_m^{CL} \le \pi_m^{GL}$ if and only if $k \le \hat{k}_8(r,t)$, and $\pi_m^{CL} > \pi_m^{GL}$ if and only if $k > \hat{k}_8(r,t)$. □

Proof of Proposition 6

Proof. According to the equilibrium degree of product greening with fairness concern in Strategies GL and CL, we have

$$\tilde{e}_{CL} - \tilde{e}_{GL} = \frac{t(c-1)(1+r)(2a^2+3a+1)\Big(1+3a^2-4k((2a+1)(1+r)-a)((2a+1)(1+r)(1-t)-a)\Big)}{\Big(4k((2a+1)(1+r)-a)^2-(a+1)[(6a+3)(1+r)-(4a+1)]\Big)}$$

$$\times \frac{1}{\Big(4k((2a+1)(1+r)(1-t)-a)^2-(a+1)[(6a+3)(1+r)(1-t)-(4a+1)]\Big)}.$$

We derive the thresholds $\hat{k}_{10} = \frac{(a+1)(2a+3r-3t+6ar-6at-3rt-6art+2)}{4((2a+1)(1+r)(1-t)-a)^2}$, $\hat{k}_2 = \frac{1}{2(1+r)(1-t)}$, $\hat{k}_9 = \frac{(a+1)(2a+3r+6ar+2)}{4((2a+1)(1+r)-a)^2}$, $\hat{t}_5 = \frac{r(a+r+ar-2a^2r+1)}{(a+1)(r+1)(2a+3r+6ar+2)}$, $\hat{t}_6 = \frac{a+r+ar-2a^2r+1}{(r+1)(-2a^2+a+1)}$, and $\hat{t}_4 = \frac{2r+1}{2r+2}$. When $0 < t < \hat{t}_5$, if $\hat{k}_2 < k < \hat{k}_9$ or $\hat{k}_{10} < k$, we have $\tilde{e}_{GL} < \tilde{e}_{CL}$, and if $\hat{k}_9 < k < \hat{k}_{10}$, we have $\tilde{e}_{CL} < \tilde{e}_{GL}$. When $\hat{t}_5 < t < \hat{t}_4$, if $\hat{k}_2 < k < \hat{k}_{10}$, we have $\tilde{e}_{CL} < \tilde{e}_{GL}$, and if $\hat{k}_{10} < k$, we have $\tilde{e}_{GL} < \tilde{e}_{CL}$. If $\hat{t}_4 < t < \hat{t}_6$ and $\hat{k}_2 < k$, we have $\tilde{e}_{GL} < \tilde{e}_{CL}$. If $\hat{t}_6 < t < 1$ and $\hat{k}_2 < k$, we have $\tilde{e}_{CL} < \tilde{e}_{GL}$. □

Proof of Proposition 7

Proof. According to the equilibrium degree of product greening with and without fairness concern under Strategy GL, we have

$$\tilde{e}_{GL} - e_{GL} = \frac{ar(c-1)(4k(1+r)(a+r+2ar+1)-(a+1))}{(4k(1+r)^2-3(1+r)+1)(4k(a+r+2ar+1)^2-(a+1)(2a+3r+6ar+2))}.$$

We derive the thresholds $\hat{k}_1 = \frac{1}{2(1+r)}$, $\hat{k}_3 = \frac{3r+2}{4(1+r)^2}$, and $\hat{k}_9 = \frac{(a+1)(2a+3r+6ar+2)}{4(a+r+2ar+1)^2}$. When $k > \hat{k}_1$, $4k(1+r)(a+r+2ar+1)-(a+1) > 0$. When $\hat{k}_3 < k$, $4k(1+r)^2-3(1+r)+1 > 0$. However, when $k < \hat{k}_3$, $4k(1+r)^2-3(1+r)+1 < 0$ When $\hat{k}_9 < k$, $4k(a+r+2ar+1)^2-(a+1)(2a+3r+6ar+2) > 0$. However, when $k < \hat{k}_9$, it is negative. □

Proof of Proposition 8

Proof. According to the equilibrium degree of product greening with and without fairness concern under Strategy CL, we have

$$\tilde{e}_{CL} - e_{CL} = \frac{a(1-c)(1-(1+r)(1-t))(4k(1+r)(1-t)((2a+1)(1+r)(1-t)-a)-(a+1))}{((2a+1)(1+r)(1-t)-a)(4k(1+r)^2(1-t)^2-3(1+r)(1-t)+1)}.$$

We derive the thresholds $\hat{t}_4 = \frac{r}{1+r}$, $\hat{t}_9 = \frac{a+2r+4ar+1}{4a+2r+4ar+2}$, $\hat{k}_2 = \frac{1}{2(1+r)(1-t)}$, $\hat{k}_5 = \frac{3(1+r)(1-t)-1}{4(1+r)^2(1-t)^2}$, $\hat{k}_9 = \frac{(a+1)(2a+3r+6ar+2)}{4(a+r+2ar+1)^2}$, and $\hat{k}_{10} = \frac{(a+1)(2a+3r+2-3t(1+r+2ar)-6at)}{4(a+r-t+2ar-2at-tr-2art+1)^2}$. When $0 < t < \hat{t}_4$, we have $1-(1+r)(1-t) < 0$. If $\hat{k}_2 < k < \hat{k}_{10}$, then $(4k(1+r)(1-t)((2a+1)(1+r)(1-t)-a)-(a+1)) < 0$ and $(4k(1+r)^2(1-t)^2-3(1+r)(1-t)+1) < 0$. If $\hat{k}_{10} < k < \hat{k}_5$, then $(4k(1+r)(1-t)((2a+1)(1+r)(1-t)-a)-(a+1))$ and $(4k(1+r)^2(1-t)^2-3(1+r)(1-t)+1) < 0$. If $\hat{k}_5 < k$, then $(4k(1+r)(1-t)((2a+1)(1+r)(1-t)-a)-(a+1)) > 0$ and $(4k(1+r)^2(1-t)^2-3(1+r)(1-t)+1) > 0$. When $\hat{t}_4 < t < \hat{t}_9$ and $\hat{k}_2 < k$, we have

$1-(1+r)(1-t)>0$ and $(4k(1+r)^2(1-t)^2-3(1+r)(1-t)+1)>0$, while the sign of $(4k(1+r)(1-t)((2a+1)(1+r)(1-t)-a)-(a+1))$ remains positive. When $\hat{t}_9<t<1$, we have $1-(1+r)(1-t)>0$ and $(4k(1+r)(1-t)((2a+1)(1+r)(1-t)-a)-(a+1))>0$. If $\hat{k}_2<k<\hat{k}_{13}$, then $(4k(1+r)^2(1-t)^2-3(1+r)(1-t)+1)<0$. If $\hat{k}_{13}<k$, then $(4k(1+r)^2(1-t)^2-3(1+r)(1-t)+1)>0$. □

Proof of Proposition 9

Proof. According to the retailer's utility in Strategies PL and GL, we have

$$U_R^{GL}-U_R^{PL}=\frac{(a+1)^2(1-c)^2(2a+3r+6ar+2)}{8(2a+1)(4k(a+r+2ar+1)^2-(a+1)(2a+3r+6ar+2))}.$$

We derive a threshold $\hat{k}_9=\frac{(a+1)(2a+3r+6ar+2)}{4(a+r+2ar+1)^2}$. When $k>\hat{k}_9$, we have $U_R^{PL}<U_R^{GL}$, and when $k<\hat{k}_9$, we have $U_R^{GL}<U_R^{PL}$.

According to the retailer's utility in Strategies PL and CL, we have

$$U_R^{CL}-U_R^{PL}=\frac{(a+1)^2(1-c)^2\left[2(a+1)+3(2a+1)(r-t-rt)\right]}{8(2a+1)\left\{4k\left[(a+1)+(2a+1)(r-t-rt)\right]^2-(a+1)\left[2(a+1)+3(2a+1)(r-t-rt)\right]\right\}}.$$

We derive a threshold $\hat{k}_{10}=\frac{(a+1)(2a+3r-3t+6ar-6at-3rt-6art+2)}{4(a+r-t+2ar-2at-tr-2art+1)^2}$. When $0<t<\hat{t}_8$, if $\hat{k}_2<k<\hat{k}_9$ or $\hat{k}_{10}<k$, then $U_R^{CL}<U_R^{PL}$, and if $\hat{k}_9<k<\hat{k}_{10}$, then $U_R^{PL}<U_R^{CL}$. When $\hat{t}_8<t<1$ and $\hat{k}_2<k$, we have $U_R^{CL}<U_R^{PL}$.

According to the retailer's utility in Strategies GL and CL, we have

$$\begin{aligned}U_R^{CL}-U_R^{GL}=\frac{\begin{aligned}&kt(a+1)^2(1-c)^2(1+r)\Big(2a+4r-2t+12ar-6at-5rt\\&\quad+12ar^2+8a^2r-4a^2t-3r^2t+a^2+3r^2\\&\quad+12a^2r^2-12ar^2t-16a^2rt-12a^2r^2t\\&\quad-18art+1\Big)\end{aligned}}{\begin{aligned}&2\Big(4k(a+r+2ar+1)^2\\&\quad-(a+1)(2a+3r+6ar+2)\Big)\\&\times\Big(4k(a+r-t+2ar-2at-tr-2art+1)^2\\&\quad-(a+1)(2a+3r-3t+6ar-6at-3rt-6art+2)\Big)\end{aligned}}.\end{aligned}$$

We derive the thresholds $\hat{t}_7=\frac{2a+3r+6ar+2}{6a+3r+6ar+3}$ and $\hat{t}_8=\frac{2a+4r+12ar+12ar^2+8a^2r+a^2+3r^2+12a^2r^2}{(2a+1)(r+1)(2a+3r+6ar+2)}$. When $0<t<\hat{t}_8$, if $\hat{k}_2<k<\hat{k}_9$ or $\hat{k}_{10}<k$, we have $U_R^{GL}<U_R^{CL}$, and if $\hat{k}_9<k<\hat{k}_{10}$, we have $U_R^{CL}<U_R^{GL}$. When $\hat{t}_8<t<1$ and $\hat{k}_2<k$, we have $U_R^{CL}<U_R^{GL}$. □

Proof of Proposition 10

Proof. According to the manufacturer's utility in Strategies PL and GL, we have

$$U_m^{GL}-U_m^{PL}=\frac{(a+1)^2(c-1)^2}{16}\times$$
$$\frac{\begin{array}{l}8k-12a-12r+24ak-48ar+32kr+24a^2k+8a^3k-45ar^2\\-60a^2r-24a^3r+40kr^2+16kr^3-12a^2-4a^3-9r^2\\-72a^2r^2-36a^3r^2+320a^2kr^2+192a^2kr^3+160a^3kr^2\\+128a^3kr^3+128akr+200akr^2+160a^2kr+96akr^3\\+64a^3kr-4\end{array}}{\left(\begin{array}{l}4k-4a-3r+8ak-9ar+8kr+4a^2k-6a^2r+4kr^2\\-2a^2+16a^2kr^2+24akr+16akr^2+16a^2kr-2\end{array}\right)^2}.$$

Give that $k_2<k$, the numerator is greater than zero. Thus, $U_m^{GL}>U_m^{PL}$ is firmly established.

According to the manufacturer's utility in Strategies GL and CL, we have

$$U_m^{CL}-U_m^{GL}=-\frac{(1+a)^2(1+2a)^2(-1+c)^2k(1+r)t}{2D_1^2D_2^2}\cdot\Bigg[32(1+2a)^5k^2r^6(-1+t)^3$$
$$+2(1+2a)^4kr^5(-1+t)^2\big(-2(1+a)(-9+40k)+(-9+88k+a(-9+136k))t\big)$$
$$+(1+2a)^3r^4(-1+t)M_1+(1+a)(1+2a)r^2M_2+(1+a)^2rM_3$$
$$+(1+a)^3(-1+2k)tM_4+2r^3M_5\Bigg],$$

where $D_1=2(1+a)^2(-1+2k)+(1+a)(1+2a)(-3+8k)r+4k(r+2ar)^2$, $D_2=-2-3r+4k(1+r)^2(-1+t)^2+3(1+r)t+2a^2(-1-3r+3(1+r)t+2k(1+2r-2(1+r)t)^2)+a(-4-9r+9(1+r)t+8k(1+r)(-1+t)(-1-2r+2(1+r)t))$, $M_1=(1+a)^2(9+8k(-17+40k))-(1+a)(9+9a-226k-316ak+80(9+14a)k^2)t+2k(-3(1+a)(13+22a)+8(25+a(77+58a))k)t^2$, $M_2=-(1+a)^3(21+40k(-3+4k))+(1+a)^2\Big(57+8k(-51+80k)+a(93+8k(-87+140k))\Big)t-(1+a)\Big(37+116a+88a^2-8(51+a(170+139a))k+32(20+a(62+47a))k^2\Big)t^2+2(1+2a)k(-3(19+8a(7+5a))+8(20+a(62+47a))k)t^3$, $M_3=-2(1+a)^3(-1+2k)(-3+8k)+(1+a)^2\Big(24+2k(-67+88k)+a(45-254k+336k^2)\Big)t-4(1+a)(1+2a)\Big(5+8k(-5+8k)+4a(2+k(-17+28k))\Big)t^2+8(1+2a)^2k(-6+14k+a(-9+22k))t^3$, $M_4=(1+a)^2(-3+8k)-4(1+a)(1+2a)(-1+4k)t+8k(t+2at)^2$, $M_5=-4(1+a)^3(1+2a)^2(3+8k(-3+5k))+(1+a)^2(1+2a)^2\Big(27+280k(-1+2k)+4a(9+4k(-26+55k))\Big)t+(1+2a)^2k\Big(-67+240k+a\Big(3(-97+368k)+8a(-51+207k+a(-23+101k))\Big)\Big)t^3-(1+a)\Big(3(1+a)(5+8a)-4(63+2a(92+65a))k+32(20+a(62+47a))k^2\Big)(t+2at)^2$.

We derive a threshold $\hat{k}_{12}(a,r,t)=2(1+a)\Big[-2r(1+a+r+2ar)^3(7+9r+a(7+18r))+(1+r)(1+a+r+2ar)^2(7(1+a)^2+46(1+a)(1+2a)r+45(r+2ar)^2)t-4(1+2a)(1+r)^2(1+a+r+2ar)(3(1+a)^2+11(1+a)(1+2a)r+9(r+2ar)^2)t^2+(1+2a)^2(1+r)^3(2+3r+a(2+6r))^2t^3\Big]$. When $\hat{k}_{12}(a,r,t)<k$, we have $U_m^{GL}<U_m^{CL}$. When $k<\hat{k}_{12}(a,r,t)$, we have $U_m^{CL}<U_m^{GL}$.

According to the manufacturer's utility in Strategies PL and CL, we have

$$U_m^{CL}-U_m^{PL}=-\frac{(a+1)^2(c-1)^2}{16}\times$$

$$
\frac{\begin{array}{c}
12a-8k+12r-12t+9r^2t^2-24ak+48ar-48at-32kr+32kt-30rt \\
-24a^2k-8a^3k+45ar^2+60a^2r+24a^3r+45at^2-60a^2t-24a^3t \\
-40kr^2-16kr^3-40kt^2+16kt^3+18rt^2-18r^2t+12a^2+4a^3+9r^2+9t^2 \\
+72a^2r^2+36a^3r^2+72a^2t^2+36a^3t^2+90art^2-90ar^2t-204a^2rt-96a^3rt \\
-128krt^2+128kr^2t+48krt^3+48kr^3t-320a^2kr^2-192a^2kr^3-160a^3kr^2 \\
-128a^3kr^3-320a^2kt^2+192a^2kt^3-160a^3kt^2+128a^3kt^3+45ar^2t^2 \\
+144a^2rt^2-144a^2r^2t+72a^3rt^2-72a^3r^2t-136kr^2t^2+48kr^2t^3 \\
-48kr^3t^2+16kr^3t^3-128akr+128akt-138art+112krt+72a^2r^2t^2 \\
+36a^3r^2t^2-200akr^2-160a^2kr-96akr^3-64a^3kr-200akt^2+160a^2kt \\
+96akt^3+64a^3kt+528akrt-1472a^2kr^2t^2+576a^2kr^2t^3-576a^2kr^3t^2 \\
-928a^3kr^2t^2+192a^2kr^3t^3+384a^3kr^2t^3-384a^3kr^3t^2+128a^3kr^3t^3 \\
-688akrt^2+688akr^2t+800a^2krt+288akrt^3+288akr^3t+384a^3krt \\
-776akr^2t^2-1216a^2krt^2+1216a^2kr^2t+288akr^2t^3-288akr^3t^2 \\
+576a^2krt^3+576a^2kr^3t-704a^3krt^2+704a^3kr^2t+96akr^3t^3 \\
+384a^3krt^3+384a^3kr^3t+4
\end{array}}{\left[\begin{array}{c}
4k-4a-3r+3t+8ak-9ar+9at+8kr-8kt+3rt+4a^2k-6a^2r+6a^2t \\
+4kr^2+4kt^2-2a^2+6a^2rt+8krt^2-8kr^2t+16a^2kr^2+16a^2kt^2+4kr^2t^2 \\
+24akr-24akt+9art-16krt+16akr^2+16a^2kr+16akt^2-16a^2kt-56akrt \\
+16a^2kr^2t^2+32akrt^2-32akr^2t-48a^2krt+16akr^2t^2+32a^2krt^2 \\
-32a^2kr^2t-2
\end{array}\right]^2}.
$$

We derive $\hat{k}_{11} = \dfrac{(a+1)(2a+3r-3t+6ar-6at-3rt-6art+2)^2}{8(a+r-t+2ar-2at-rt-2art+1)^2(a+2r-2t+4ar-4at-2rt-4art+1)}$. When $\hat{k}_{11} < k$, we can demonstrate that $U_m^{PL} < U_m^{CL}$. When $k < \hat{k}_{11}$, we have $U_m^{CL} < U_m^{PL}$. □



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