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Research article

Stackelberg game analysis of carbon reduction label standards in agricultural supply chains: Impacts of consumer low-carbon preference and label recognition

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Appendix A

A1 Proof of Theorem 1

From Eq. (6), the Hessian matrix of the processor's profit is $\begin{pmatrix} -2b & \theta_2 \\ \theta_2 & -\beta_1 \end{pmatrix}$, the first principal minor is -2b < 0, the second principal minor is $-\theta_2^2 + 2b\beta_1 > 0$ as b and β_1 become sufficiently large, so the Hessian matrix of the processor's profit is negative definite, the processor's profit is concave in both p and m_r, solving $\begin{cases} \frac{\partial \pi_r^{NS}}{\partial p} = 0 \\ \frac{\partial \pi_r^{NS}}{\partial m_r} = 0 \end{cases}$. The optimal retail price and the carbon reduction efforts of processors can be expressed as $-\frac{1}{2b\beta_1 - \theta_2^2}\left(-a\beta_1 - bw\beta_1 + l_0\beta_1\theta_1 - l_m\beta_1\theta_1 - l_0\beta_1\theta_2 + w\theta_2^2 + \beta_1\theta_2\lambda_r + \beta_1\theta_2\lambda_s\right)$ and $-\frac{\theta_2\left(-a + bw + l_0\theta_1 - l_m\theta_1 - l_0\theta_2 + \theta_2\lambda_r + \theta_2\lambda_s\right)}{2b\beta_1 - \theta_2^2}$. We put those two expressions into Eq (7), then

$\pi_s^{NS} = \dfrac{bw\beta_1\left(a - bw + l_m\theta_1 + l_0\left(-\theta_1 + \theta_2\right) - \theta_2\left(\lambda_r + \lambda_s\right)\right)}{2b\beta_1 - \theta_2^2}$, solving $\dfrac{\partial \pi_s^{NS}}{\partial w} = 0$, and the optimal wholesale price is $w^{NS*} = \dfrac{a - l_0\theta_1 + l_m\theta_1 + l_0\theta_2 - \theta_2\lambda_r - \theta_2\lambda_s}{2b}$. If we put the expressions back into the abovementioned optimal retail price and the processor's carbon reduction effort, we can get the following expressions:

$p^{NS*} = \dfrac{\left(3b\beta - \theta_2^2\right)\left[a + \theta_1\left(l_m - l_0\right) + \theta_2\left(l_0 - \lambda_s - \lambda_r\right)\right]}{2b\left(2b\beta - \theta_2^2\right)}$, $m_r^{NS*} = \dfrac{\theta_2\left[a + \theta_1\left(l_m - l_0\right) + \theta_2\left(l_0 - \lambda_s - \lambda_r\right)\right]}{2\left(2b\beta - \theta_2^2\right)}$,

$D^{NS*} = \dfrac{b\beta_1\left[a + \theta_1\left(l_m - l_0\right) + \theta_2\left(l_0 - \lambda_s - \lambda_r\right)\right]}{2\left(2b\beta - \theta_2^2\right)}$, $\pi_r^{NS*} = \dfrac{\beta\left[a + \theta_1\left(l_m - l_0\right) + \theta_2\left(l_0 - \lambda_s - \lambda_r\right)\right]^2}{8\left(2b\beta - \theta_2^2\right)}$, and

$\pi_s^{NS*} = \dfrac{\beta\left[a + \theta_1\left(l_m - l_0\right) + \theta_2\left(l_0 - \lambda_s - \lambda_r\right)\right]^2}{4\left(2b\beta - \theta_2^2\right)}$.

Putting the optimal decision into Eqs (2–4) can yield E^{NS*}, CS^{NS*} and S^{NS*}.To ensure that the price and profit are greater than 0, it is necessary to satisfy $\left[a + \theta_1\left(l_m - l_0\right) + \theta_2\left(l_0 - \lambda_s - \lambda_r\right)\right] > 0$ and $\left(2b\beta - \theta_2^2\right) > 0$.

In addition, $\dfrac{\partial w^{NS*}}{\partial l_0} = \dfrac{-\theta_1 + \theta_2}{2b}$, $\dfrac{\partial p^{NS*}}{\partial l_0} = \dfrac{\left(-\theta_1 + \theta_2\right)\left(3b\beta_1 - \theta_2^2\right)}{2b\left(2b\beta_1 - \theta_2^2\right)}$, $\dfrac{\partial m^{NS*}}{\partial l_0} = \dfrac{\theta_2\left(-\theta_1 + \theta_2\right)}{2\left(2b\beta_1 - \theta_2^2\right)}$,

$\dfrac{\partial \pi_r^{NS*}}{\partial l_0} = \dfrac{\beta_1\left(-\theta_1 + \theta_2\right)\left(a + \theta_1\left(l_m - l_0\right) + \theta_2\left(l_0 - \lambda_s - \lambda_r\right)\right)}{4\left(2b\beta_1 - \theta_2^2\right)}$, $\dfrac{\partial \pi_s^{NS*}}{\partial l_0} = \dfrac{\beta_1\left(-\theta_1 + \theta_2\right)\left(a + \theta_1\left(l_m - l_0\right) + \theta_2\left(l_0 - \lambda_s - \lambda_r\right)\right)}{2\left(2b\beta_1 - \theta_2^2\right)}$,

$\dfrac{\partial E^{NS*}}{\partial l_0} = \dfrac{2p_{carbon}b\beta_1\theta_2\left(-\theta_1 + \theta_2\right)\left(a + \theta_1\left(l_m - l_0\right) + \theta_2\left(l_0 - \lambda_s - \lambda_r\right)\right)}{\left(4b\beta_1 - 2\theta_2^2\right)^2}$,

$\dfrac{\partial CS^{NS*}}{\partial l_0} = \dfrac{b^2\beta_1^2\theta_2\left(-\theta_1 + \theta_2\right)\left(a + \theta_1\left(l_m - l_0\right) + \theta_2\left(l_0 - \lambda_s - \lambda_r\right)\right)}{4\left(2b\beta_1 - \theta_2^2\right)^2}$,

$\dfrac{\partial S^{NS*}}{\partial l_0} = \dfrac{\beta_1\left(-\theta_1 + \theta_2\right)\left(6b\beta_1 - 3\theta_2^2 + 2p_{carbon}b\theta_2 + b^2\beta_1\right)\left(a + \theta_1\left(l_m - l_0\right) + \theta_2\left(l_0 - \lambda_s - \lambda_r\right)\right)}{4\left(2b\beta_1 - \theta_2^2\right)^2}$.

When $\theta_2 < \theta_1$, the first-order derivatives mentioned above are all less than 0.

A2 Proof of Theorem 2

Referring to the proof of Theorem 1, we have $\dfrac{\partial w^{TS*}}{\partial \theta_1} = -\dfrac{\left(l_0 - l_{\mathrm{m}}\right)\beta_2\left(2b\beta_1 - \theta_2^2\right)}{b\left(4b\beta_1\beta_2 - \left(\beta_1 + 2\beta_2\right)\theta_2^2\right)}$, $\dfrac{\partial p^{TS*}}{\partial \theta_1} = -\dfrac{\left(l_0 - l_{\mathrm{m}}\right)\beta_2\left(3b\beta_1 - \theta_2^2\right)}{b\left(4b\beta_1\beta_2 - \left(\beta_1 + 2\beta_2\right)\theta_2^2\right)}$, and $\dfrac{\partial D^{TS*}}{\partial \theta_1} = \dfrac{\left(l_m - l_0\right)b\beta_1\beta_2}{\left(4b\beta_1\beta_2 - \left(\beta_1 + 2\beta_2\right)\theta_2^2\right)}$. Because $0 < l_0 < l_{\mathrm{m}}$, $\dfrac{\partial w^{TS*}}{\partial \theta_1} > 0$, $\dfrac{\partial p^{TS*}}{\partial \theta_1} > 0$, and $\dfrac{\partial D^{TS*}}{\partial \theta_1} > 0$. Furthermore, we have

$\dfrac{\partial \pi_r^{TS*}}{\partial \theta_1} = \dfrac{\left(\beta_1\beta_2^2\left(2b\beta_1 - \theta_2^2\right)\left(l_{\mathrm{m}} - l_0\right)\left(a + \theta_1\left(l_{\mathrm{m}} - l_0\right) + \theta_2\left(l_0 - \lambda_r - \lambda_s\right)\right)\right)}{\left(-4b\beta_1\beta_2 + \left(\beta_1 + 2\beta_2\right)\theta_2^2\right)^2} > 0$,

$$\frac{\partial \pi_s^{TS*}}{\partial \theta_1}=\frac{\left(2\beta_1\beta_2\left(l_{\mathrm{m}}-l_0\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\right)}{2\left(4b\beta_1\beta_2-\left(\beta_1+2\beta_2\right)\theta_2^{\ 2}\right)}>0 \quad ,$$

$$\frac{\partial E^{TS*}}{\partial \theta_1}=\frac{\left(2p_{carbon}b\left(l_{\mathrm{m}}-l_0\right)\beta_1\beta_2\left(\beta_1+\beta_2\right)\theta_2\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\right)}{\left(-4b\beta_1\beta_2+\left(\beta_1+2\beta_2\right)\theta_2^{\ 2}\right)^2}>0 \quad ,$$

$$\frac{\partial CS^{TS*}}{\partial \theta_1}=\frac{\left(b^2\beta_1^{\ 2}\beta_2^{\ 2}\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)^2\right)}{2\left(4b\beta_1\beta_2-\left(\beta_1+2\beta_2\right)\theta_2^{\ 2}\right)^2}>0 \quad , \quad \text{and}$$

$$\frac{\partial S^{TS*}}{\partial \theta_1}=\frac{\left(\left(l_{\mathrm{m}}-l_0\right)\beta_1\beta_2\left(b\beta_1\beta_2\left(6+b\right)+2p_{carbon}b\theta_2\left(\beta_1+\beta_2\right)-\theta_2^{\ 2}\left(\beta_1+3\beta_2\right)\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\right)}{\left(4b\beta_1\beta_2-\left(\beta_1+2\beta_2\right)\theta_2^{\ 2}\right)^2}>0 .$$

A3 Proof of Proposition 1

Because $\frac{\partial w^{TS*}}{\partial \theta_2}=\frac{\beta_2((8b^2\beta_1^2\beta_2+(\beta_1+2\beta_2)\theta_2^4)(l_0-\lambda_s-\lambda_r)+2b\beta_1\theta_2(2\beta_1(a+\theta_1(l_m-l_0)+\theta_2(\beta_1-4\beta_2)(l_0-\lambda_s-\lambda_r))))}{b(4b\beta_1\beta_2-(\beta_1+2\beta_2)\theta_2^2)^2}$,

if $\frac{\partial w^{TS*}}{\partial \theta_2}>0$, the molecules $4b\beta_1\beta_2\left(l_0-\lambda_s-\lambda_r\right)+\left(\beta_1+2\beta_2\right)\theta_2\left(2a+2\theta_1\left(l_m-l_0\right)+\theta_2\left(l_0-\lambda_s-\lambda_r\right)\right)>0$. Therefore, the solution is $0<l_0<l_m$. $\frac{\partial p^{TS*}}{\partial \theta_2}=\frac{\beta_2\begin{pmatrix}\left(12b^2\beta_1^2\beta_2+(\beta_1+2\beta_2)\theta_2^4\right)(l_0-\lambda_s-\lambda_r)\\+2b\beta_1\theta_2\left(3\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_m-l_0\right)+3\theta_2(\beta_1-2\beta_2)(l_0-\lambda_s-\lambda_r)\right)\end{pmatrix}}{b(4b\beta_1\beta_2-(\beta_1+2\beta_2)\theta_2^2)^2}$. If $\frac{\partial p^{TS*}}{\partial \theta_2}>0$, the molecules $\beta_2\begin{pmatrix}\left(12b^2\beta_1^2\beta_2+(\beta_1+2\beta_2)\theta_2^4\right)(l_0-\lambda_s-\lambda_r)\\+2b\beta_1\theta_2\left(3\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_m-l_0\right)+3\theta_2(\beta_1-2\beta_2)(l_0-\lambda_s-\lambda_r)\right)\end{pmatrix}>0$. It can be seen that $l_0>\frac{\left(12b^2\beta_1^2\beta_2+(\beta_1+2\beta_2)\theta_2^4+6b\beta_1\theta_2^2\left(3\beta_1+2\beta_2\right)(\beta_1-2\beta_2)\right)(\lambda_s+\lambda_r)-2b\beta_1\theta_2\left(3\beta_1+2\beta_2\right)\left(a+\theta_1 l_m\right)}{\left(12b^2\beta_1^2\beta_2+(\beta_1+2\beta_2)\theta_2^4+6b\beta_1\theta_2^2\left(3\beta_1+2\beta_2\right)(\beta_1-2\beta_2)-2b\beta_1\theta_1\theta_2\left(3\beta_1+2\beta_2\right)\right)}\cdot$ We make $l_1=\frac{\left(12b^2\beta_1^2\beta_2+(\beta_1+2\beta_2)\theta_2^4+6b\beta_1\theta_2^2\left(3\beta_1+2\beta_2\right)(\beta_1-2\beta_2)\right)(\lambda_s+\lambda_r)-2b\beta_1\theta_2\left(3\beta_1+2\beta_2\right)\left(a+\theta_1 l_m\right)}{\left(12b^2\beta_1^2\beta_2+(\beta_1+2\beta_2)\theta_2^4+6b\beta_1\theta_2^2\left(3\beta_1+2\beta_2\right)(\beta_1-2\beta_2)-2b\beta_1\theta_1\theta_2\left(3\beta_1+2\beta_2\right)\right)}$. Therefore, the solution is $l_1<l_0<l_m$.

Because $\frac{\partial m^{TS*}}{\partial \theta_2}=\frac{\left(\beta_1+\beta_2\right)\begin{pmatrix}4ab\beta_1\beta_2+a\left(\beta_1+2\beta_2\right)\theta_2^{\ 2}-\left(l_0-l_m\right)\left(\beta_1+2\beta_2\right)\theta_1\theta_2^{\ 2}\\-4b\beta_1\beta_2\left(-l_m\theta_1+l_0\left(\theta_1-2\theta_2\right)+2\theta_2\left(\lambda_r+\lambda_s\right)\right)\end{pmatrix}}{\left(-4b\beta_1\beta_2+\left(\beta_1+2\beta_2\right)\theta 2^2\right)^2}>0$ and $\frac{\partial D^{TS*}}{\partial \theta_2}=\frac{\left(b\beta_1\beta_2\left(4b\beta_1\beta_2\left(l_0-\lambda_s-\lambda_r\right)+(\beta_1+2\beta_2)\theta_2\left(2a+2\theta_1\left(l_m-l_0\right)+\theta_2\left(l_0-\lambda_s-\lambda_r\right)\right)\right)\right)}{(4b\beta_1\beta_2-(\beta_1+2\beta_2)\theta_2^2)^2}$. When $\frac{\partial D^{TS*}}{\partial \theta_2}>0$, the molecules $4b\beta_1\beta_2\left(l_0-\lambda_s-\lambda_r\right)+\left(\beta_1+2\beta_2\right)\theta_2\left(2a+2\theta_1\left(l_m-l_0\right)+\theta_2\left(l_0-\lambda_s-\lambda_r\right)\right)>0$. It can be seen that $l_0>\frac{\left(4b\beta_1\beta_2+\theta_2^2\left(\beta_1+2\beta_2\right)\right)(\lambda_s+\lambda_r)-2\theta_2\left(a+\theta_1 l_m\right)\left(\beta_1+2\beta_2\right)}{4b\beta_1\beta_2+\theta_2^2\left(\beta_1+2\beta_2\right)\left(\theta_2^2-2\theta_1\theta_2\right)}$. We take $l_2=\frac{\left(4b\beta_1\beta_2+\theta_2^2\left(\beta_1+2\beta_2\right)\right)(\lambda_s+\lambda_r)-2\theta_2\left(a+\theta_1 l_m\right)\left(\beta_1+2\beta_2\right)}{4b\beta_1\beta_2+\theta_2^2\left(\beta_1+2\beta_2\right)\left(\theta_2^2-2\theta_1\theta_2\right)}$, then the solution is $l_2<l_0<l_m$.

A4 Proof of Proposition 2

Because $\frac{\partial \pi_r^{TS*}}{\partial \theta_2} = \frac{\beta_1\beta_2^2\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\begin{pmatrix}-\left(\left(\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)\theta_2^3\right)\\+8b^2\beta_1^2\beta_2\left(l_0-\lambda_r-\lambda_s\right)\\+2b\beta_1\theta_2\left(2\left(\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)+\left(\beta_1-2\beta_2\right)\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\end{pmatrix}}{\left(4b\beta_1\beta_2-\left(\beta_1+2\beta_2\right)\theta_2^2\right)^3}$,

if $\frac{\partial \pi_r^{TS*}}{\partial \theta_2}>0$, then $\beta_1\beta_2^2\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\begin{pmatrix}-\left(\left(\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)\theta_2^3\right)\\+8b^2\beta_1^2\beta_2\left(l_0-\lambda_r-\lambda_s\right)\\+2b\beta_1\theta_2\left(2\left(\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)+\left(\beta_1-2\beta_2\right)\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\end{pmatrix}>0$.

It can be seen that $l_0 > \frac{\left(a+\theta_1\theta_2{}^3 l_m-4b\beta_1\theta_2\left(a+\theta_1 l_m\right)\right)\left(\beta_1+2\beta_2\right)+\left(8b^2\beta_1\beta_2+2b\beta_1\theta_2{}^2\left(\beta_1-2\beta_2\right)\right)\left(\lambda_r+\lambda_s\right)}{\left(\theta_1\theta_2{}^3-4b\beta_1\theta_2\right)\left(\beta_1+2\beta_2\right)+\left(8b^2\beta_1^2\beta_2+2b\beta_1\theta_2{}^2\left(\beta_1-2\beta_2\right)\right)}$. If we take $l_3 = \frac{\left(a+\theta_1\theta_2{}^3 l_m-4b\beta_1\theta_2\left(a+\theta_1 l_m\right)\right)\left(\beta_1+2\beta_2\right)+\left(8b^2\beta_1\beta_2+2b\beta_1\theta_2{}^2\left(\beta_1-2\beta_2\right)\right)\left(\lambda_r+\lambda_s\right)}{\left(\theta_1\theta_2{}^3-4b\beta_1\theta_2\right)\left(\beta_1+2\beta_2\right)+\left(8b^2\beta_1^2\beta_2+2b\beta_1\theta_2{}^2\left(\beta_1-2\beta_2\right)\right)}$, then the solution is $l_3<l_0<l_m$.

Because $\frac{\partial \pi_s^{TS*}}{\partial \theta_2} = \frac{\beta_1\beta_2\left(\left(\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)\theta_2+4b\beta_1\beta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)}{\left(4b\beta_1\beta_2-\left(\beta_1+2\beta_2\right)\theta_2^2\right)^2}$,

if $\frac{\partial \pi_s^{TS*}}{\partial \theta_2}>0$, then $\beta_1\beta_2\left(\left(\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)\theta_2+4b\beta_1\beta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)>0$.

We obtain $l_0 > \frac{4b\beta_1\beta_2\left(\lambda_r+\lambda_s\right)-\theta_2\left(\beta_1+2\beta_2\right)\left(a+\theta_1 l_{\mathrm{m}}\right)}{4b\beta_1\beta_2-\theta_1\theta_2\left(\beta_1+2\beta_2\right)}$, then take $l_4 = \frac{4b\beta_1\beta_2\left(\lambda_r+\lambda_s\right)-\theta_2\left(\beta_1+2\beta_2\right)\left(a+\theta_1 l_{\mathrm{m}}\right)}{4b\beta_1\beta_2-\theta_1\theta_2\left(\beta_1+2\beta_2\right)}$, so the solution is $l_4<l_0<l_m$.

Because $\frac{\partial E^{TS*}}{\partial \theta_2} = \frac{\begin{pmatrix}p_{carbon}b\beta_1\beta_2\left(\beta_1+\beta_2\right)\left(a+\theta_1\left(l_m-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\\\begin{pmatrix}4ab\beta_1\beta_2+3a\left(\beta_1+2\beta_2\right)\theta_2^2\\+\left(\beta_1+2\beta_2\right)\theta_2^2\left(3l_m\theta_1+l_0\left(-3\theta_1+\theta_2\right)-\theta_2\left(\lambda_r+\lambda_s\right)\right)\\-4b\beta_1\beta_2\left(-l_m\theta_1+l_0\left(\theta_1-3\theta_2\right)+3\theta_2\left(\lambda_r+\lambda_s\right)\right)\end{pmatrix}\end{pmatrix}}{\left(4b\beta_1\beta_2+\left(\beta_1+2\beta_2\right)\theta 2^2\right)^3}>0$

$b^2\beta 1^2\beta 2^2\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)$

and $\frac{\partial CS^{TS*}}{\partial \theta_2} = \frac{\left(4b\beta_1\beta_2\left(l_0-\lambda_r-\lambda_s\right)+\left(\beta_1+2\beta_2\right)\theta_2\left(2a+2\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\right)}{\left(4b\beta_1\beta_2-\left(\beta_1+2\beta_2\right)\theta_2^2\right)^3}$, if $\frac{\partial CS^{TS*}}{\partial \theta_2}>0$, then $\left(4b\beta_1\beta_2\left(l_0-\lambda_r-\lambda_s\right)+\left(\beta_1+2\beta_2\right)\theta_2\left(2a+2\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\right)>0$, it can be seen that $l_0 > \frac{4b\beta_1\beta_2\left(\lambda_r+\lambda_s\right)-\left(\beta_1+2\beta_2\right)\theta_2\left(2a+2\theta_1 l_{\mathrm{m}}-\theta_2\left(\lambda_r+\lambda_s\right)\right)}{4b\beta_1\beta_2-\left(\beta_1+2\beta_2\right)\theta_2\left(\theta_2-2\theta_1\right)}$. If we take $l_5 = \frac{4b\beta_1\beta_2\left(\lambda_r+\lambda_s\right)-\left(\beta_1+2\beta_2\right)\theta_2\left(2a+2\theta_1 l_{\mathrm{m}}-\theta_2\left(\lambda_r+\lambda_s\right)\right)}{4b\beta_1\beta_2-\left(\beta_1+2\beta_2\right)\theta_2\left(\theta_2-2\theta_1\right)}$, then the solution is $l_5<l_0<l_m$.

In addition,

$$\frac{\partial S^{TS*}}{\partial \theta_2}=\frac{\beta_1\beta_2\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)}{\left(4b\beta_1\beta_2-\left(\beta_1+2\beta_2\right)\theta_2^{\,2}\right)^3}$$

$$\begin{pmatrix}-\left(\left(\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)\theta_2^3\right)+8b^2\beta_1^{\,2}\beta_2\left(l_0-\lambda_r-\lambda_s\right)+2b\beta_1\theta_2\left(2\left(\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)+\left(\beta_1-2\beta_2\right)\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\\+\beta_2\left(\beta_1+2\beta_2\right)\left(4b\beta_1\beta_2-\left(\beta_1+2\beta_2\right)\theta_2^{\,2}\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)\theta_2+4b\beta_1\beta_2\left(l_0-\lambda_r-\lambda_s\right)\\p_{carbon}b\left(\beta_1+\beta_2\right)\left(\left(\beta_1+2\beta_2\right)\theta_2^2\left(3a+3\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)+4b\beta_1\beta_2\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)+3\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\right)\\b^2\beta_1\beta_2\left(4b\beta_1\beta_2\left(l_0-\lambda_r-\lambda_s\right)+\left(\beta_1+2\beta_2\right)\theta_2\left(2a+2\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\right)\end{pmatrix},$$

if $\frac{\partial S^{TS*}}{\partial \theta_2}>0$, then

$$\begin{pmatrix}-\left(\left(\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)\theta_2^3\right)+8b^2\beta_1^{\,2}\beta_2\left(l_0-\lambda_r-\lambda_s\right)+2b\beta_1\theta_2\left(2\left(\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)+\left(\beta_1-2\beta_2\right)\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\\+\beta_2\left(\beta_1+2\beta_2\right)\left(4b\beta_1\beta_2-\left(\beta_1+2\beta_2\right)\theta_2^{\,2}\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)\theta_2+4b\beta_1\beta_2\left(l_0-\lambda_r-\lambda_s\right)\\p_{carbon}b\left(\beta_1+\beta_2\right)\left(\left(\beta_1+2\beta_2\right)\theta_2^2\left(3a+3\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)+4b\beta_1\beta_2\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)+3\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\right)\\b^2\beta_1\beta_2\left(4b\beta_1\beta_2\left(l_0-\lambda_r-\lambda_s\right)+\left(\beta_1+2\beta_2\right)\theta_2\left(2a+2\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\right)\end{pmatrix}>0.$$

Let $\Omega(l_0)=\begin{pmatrix}-\left(\left(\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)\theta_2^3\right)+8b^2\beta_1^{\,2}\beta_2\left(l_0-\lambda_r-\lambda_s\right)+2b\beta_1\theta_2\left(2\left(\beta_1+2\beta_2\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)+\left(\beta_1-2\beta_2\right)\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\\+\beta_2\left(\beta_1+2\beta_2\right)\left(4b\beta_1\beta_2-\left(\beta_1+2\beta_2\right)\theta_2^{\,2}\right)\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)\right)\theta_2+4b\beta_1\beta_2\left(l_0-\lambda_r-\lambda_s\right)\\p_{carbon}b\left(\beta_1+\beta_2\right)\left(\left(\beta_1+2\beta_2\right)\theta_2^2\left(3a+3\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)+4b\beta_1\beta_2\left(a+\theta_1\left(l_{\mathrm{m}}-l_0\right)+3\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\right)\\b^2\beta_1\beta_2\left(4b\beta_1\beta_2\left(l_0-\lambda_r-\lambda_s\right)+\left(\beta_1+2\beta_2\right)\theta_2\left(2a+2\theta_1\left(l_{\mathrm{m}}-l_0\right)+\theta_2\left(l_0-\lambda_r-\lambda_s\right)\right)\right)\end{pmatrix}$,

The function $\Omega(l_0)$ is linear with respect to l_0. Let l_6 denote the value of l_0 when $\Omega(l_0)=0$. Therefore, the solution is $l_6<l_0<l_m$.

A5 Proof of Proposition 3

Because $\Delta w=\frac{\beta_1\theta_2^2\left[a+\theta_1\left(l_m-l_0\right)+\theta_2\left(l_0-\lambda_s-\lambda_r\right)\right]}{2b\left[2\beta_2\left(2b\beta_1-\theta_2^2\right)-\beta_1\theta_2^2\right]}>0$, $\Delta p=\frac{\left(3b\beta_1-\theta_2^2\right)\beta_1\theta_2^2\left[a+\theta_1\left(l_m-l_0\right)+\theta_2\left(l_0-\lambda_s-\lambda_r\right)\right]}{2b\left(2b\beta_1-\theta_2^2\right)\left[2\beta_2\left(2b\beta_1-\theta_2^2\right)-\beta_1\theta_2^2\right]}>0$, $\Delta m=\frac{\beta_1\theta_2\left(4b\beta_1-\theta_2^2\right)\left[a+\theta_1\left(l_m-l_0\right)+\theta_2\left(l_0-\lambda_s-\lambda_r\right)\right]}{2\left(2b\beta_1-\theta_2^2\right)\left[2\beta_2\left(2b\beta_1-\theta_2^2\right)-\beta_1\theta_2^2\right]}>0$, and $\Delta D=\frac{b\beta_1^2\theta_2^2\left[a+\theta_1\left(l_m-l_0\right)+\theta_2\left(l_0-\lambda_s-\lambda_r\right)\right]}{2\left(2b\beta_1-\theta_2^2\right)\left[2\beta_2\left(2b\beta_1-\theta_2^2\right)-\beta_1\theta_2^2\right]}>0$, then the solution is $w^{TS*}>w^{NS*}$, $p^{TS*}>p^{NS*}$, $m_r^{TS*}+m_s^{TS*}>m^{NS*}$, and $D^{TS*}>D^{NS*}$.

A6 Proof of Proposition 4

We have $\frac{\partial \Delta m}{\partial \theta_1}=\frac{\beta_1\theta_2\left(4b\beta_1-\theta_2^2\right)\left(l_m-l_0\right)}{2\left(2b\beta_1-\theta_2^2\right)\left[2\beta_2\left(2b\beta_1-\theta_2^2\right)-\beta_1\theta_2^2\right]}>0$ and $\frac{\partial \Delta D}{\partial \theta_1}=\frac{b\beta_1^2\theta_2^2\left(l_m-l_0\right)}{2\left(2b\beta_1-\theta_2^2\right)\left[2\beta_2\left(2b\beta_1-\theta_2^2\right)-\beta_1\theta_2^2\right]}>0$.

A7 Proof of Proposition 5

We have $\frac{\partial \Delta m}{\partial \theta_2}=\frac{\beta_1\begin{pmatrix}\left((l_m-l_0)(\beta_1+2\beta_2)\theta_1\theta_2^6\right)+32b^3\beta_1^3\beta_2\left(l_m\theta_1-l_0(\theta_1-2\theta_2)-2(\lambda_r+\lambda_s)\theta_2\right)+\\ a\left(32b^3\beta_1^3\beta_2+8b^2\beta_1^2(\beta_1+\beta_2)\theta_2^2-2b\beta_1(5\beta_1+8\beta_2)\theta_2^4+(\beta_1+2\beta_2)\theta_2^6\right)+\\ 2b\beta_1\theta_2^4\left(-l_m(5\beta_1+8\beta_2)\theta_1+2(\lambda_r+\lambda_s)\beta_1\theta_2+l_0(5\beta_1\theta_1+8\beta_2\theta_1-2\beta_1\theta_2)\right)+\\ 8b^2\beta_1^2\theta_2^2\left(l_m(\beta_1+\beta 2)\beta_2\theta_1+4(\lambda_r+\lambda_s)\beta_2\theta_2-l_0(\beta_1\theta_1+\beta_2(\theta_1+4\theta_2))\right)\end{pmatrix}}{2\left(2b\beta_1-\theta_2^2\right)^2\left[2\beta_2\left(2b\beta_1-\theta_2^2\right)-\beta_1\theta_2^2\right]^2}>0$ and $\frac{\partial \Delta D}{\partial \theta_2}=\frac{b\beta_1^2\theta_2\begin{pmatrix}16b^2\beta_1^2\beta_2\left(a+(1_m-l_0)\theta_1\right)+24b^2(l_0-\lambda_r-\lambda_s)\beta_1^2\beta_2\theta_2-2b(l_0-\lambda_r-\lambda_s)\beta_1(\beta_1+4\beta_2)\theta_2^3\\ -2(\beta_1+2\beta_2)\left(a+(1_m-l_0)\theta_1\right)\theta_2^4-(l_0-\lambda_r-\lambda_s)(\beta_1+2\beta_2)\theta_2^5\end{pmatrix}}{2\left(2b\beta_1-\theta_2^2\right)^2\left[2\beta_2\left(2b\beta_1-\theta_2^2\right)-\beta_1\theta_2^2\right]^2}>0$.

A8 Proof of Proposition 6

If we let $\Delta\pi_r=\pi_r^{TS*}-\pi_r^{NS*}$, $\Delta\pi_s=\pi_s^{TS*}-\pi_s^{NS*}$, $\Delta E=E^{TS*}-E^{NS*}$, $\Delta CS=CS^{TS*}-CS^{NS*}$, and $\Delta S=S^{TS*}-S^{NS*}$, we obtain $\Delta\pi_r=\frac{\left[a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_s-\lambda_r)\right]^2\beta_1^2\left(8b\beta_1\beta_2\theta_2^2-(\beta_1+4\beta_2)\theta_2^4\right)}{8\left(2b\beta_1-\theta_2^2\right)\left[2\beta_2\left(2b\beta_1-\theta_2^2\right)-\beta_1\theta_2^2\right]^2}$,

$\Delta\pi_s=\frac{\beta_1^2\theta_2^2\left[a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_s-\lambda_r)\right]^2}{4\left(2b\beta_1-\theta_2^2\right)\left[2\beta_2\left(2b\beta_1-\theta_2^2\right)-\beta_1\theta_2^2\right]}$,

$\Delta E=\frac{p_{carbon}}{4}b\beta_1\theta_2\left[a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_s-\lambda_r)\right]^2\times\left(-\frac{1}{\left(2b\beta_1-\theta_2^2\right)^2}+\frac{4\beta_2(\beta_1+\beta_2)}{\left[2\beta_2\left(2b\beta_1-\theta_2^2\right)-\beta_1\theta_2^2\right]^2}\right)$,

$\Delta CS=\frac{b^2\beta_1^3\left(8b\beta_1\beta_2\theta_2^2-(\beta_1+4\beta_2)\theta_2^4\right)\left[a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_s-\lambda_r)\right]^2}{8\left(-2b\beta_1+\theta_2^2\right)^2\left(-4b\beta_1\beta_2+(\beta_1+2\beta_2)\theta_2^2\right)^2}$, and

$\Delta S=\frac{\beta_1\left[a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_s-\lambda_r)\right]^2}{8}\times\left(\begin{array}{l}\frac{-b(6+b)\beta_1-2p_{carbon}b\theta_2+3\theta_2^2}{\left(-2b\beta_1+\theta_2^2\right)^2}\\ +\frac{4\beta_2\left(b\beta_1\beta_2(6+b)+2p_{carbon}b(\beta_1+\beta_2)\theta_2-(\beta_1+3\beta_2)\theta_2^2\right)}{\left[2\beta_2\left(2b\beta_1-\theta_2^2\right)-\beta_1\theta_2^2\right]^2}\end{array}\right)$.

A9 Proof of Proposition 7

We have $\frac{\partial \Delta\pi_r}{\partial \theta_1}=\frac{(l_m-l_0)\beta_1\left(a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_r-\lambda_s)\right)\left(1-\frac{4\beta_2\left(-2b\beta_1+\theta_2^2\right)^2}{\left(-4b\beta_1\beta_2+(\beta_1+2\beta_2)\theta_2^2\right)^2}\right)}{4\left(2b\beta_1-\theta_2^2\right)}>0$,

$\frac{\partial \Delta\pi_s}{\partial \theta_1}=\frac{(l_m-l_0)\beta_1^2\theta_2^2\left(a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_r-\lambda_s)\right)}{2\left(2b\beta_1-\theta_2^2\right)\left(4b\beta_1\beta_2-(\beta_1+2\beta_2)\theta_2^2\right)}>0$,

$\frac{\partial \Delta E}{\partial \theta_1}=\frac{1}{2}p_{carbon}b(l_m-l_0)\beta_1\theta_2\left(a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_r-\lambda_s)\right)\left(-\frac{1}{\left(-2b\beta_1+\theta_2^2\right)^2}+\frac{4\beta_2(\beta_1+\beta_2)}{\left(-4b\beta_1\beta_2+(\beta_1+2\beta_2)\theta_2^2\right)^2}\right)>0$,

$\frac{\partial \Delta CS}{\partial \theta_1}=\frac{b^2(l_m-l_0)\beta_1^3\left(a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_r-\lambda_s)\right)\left(8b\beta_1\beta_2\theta_2^2-(\beta_1+4\beta_2)\theta_2^4\right)}{4\left(-2b\beta_1+\theta_2^2\right)^2\left(-4b\beta_1\beta_2+(\beta_1+2\beta_2)\theta_2^2\right)^2}<0$, and

$$\frac{\partial \Delta S}{\partial \theta_1}=\frac{1}{4}(l_m-l_0)\beta_1\left(a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_r-\lambda_s)\right)\left(\begin{array}{l}\dfrac{-b(6+b)\beta_1-2p_{carbon}b\theta_2+3\theta_2^2}{\left(2b\beta_1-\theta_2^2\right)^2}\\ +\dfrac{4\beta_2\left(b(6+b)\beta_1\beta_2+2p_{carbon}b(\beta_1+\beta_2)\theta_2-(\beta_1+3\beta_2)\theta_2^2\right)}{\left(4b\beta_1\beta_2-(\beta_1+2\beta_2)\theta_2^2\right)^2}\end{array}\right)<0\,.$$

A10 Proof of Proposition 8

We have $\dfrac{\partial \Delta\pi_r}{\partial \theta_2}=\dfrac{1}{4}\beta_1\left(a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_r-\lambda_s)\right)\left(\begin{array}{l}-\dfrac{2b\beta_1(l_0-\lambda_r-\lambda_s)+(l_m-l_0)\theta_1\theta_2+a\theta_2}{\left(2b\beta_1-\theta_2^2\right)^2}\\ +\dfrac{16b\beta_1^2\beta_2\left(4b\beta_1\beta_2(l_0-\lambda_r-\lambda_s)+(\beta_1+2\beta_2)\left(a+(l_m-l_0)\theta_1\right)\theta_2\right)}{(\beta_1+2\beta_2)\left(4b\beta_1\beta_2-(\beta_1+2\beta_2)\theta_2^2\right)^3}\\ +\dfrac{-8b(l_0-\lambda_r-\lambda_s)\beta_1(\beta_1-2\beta_2)\beta_2+4\beta_2(\beta_1+2\beta_2)\left(a+(l_m-l_0)\theta_1\right)\theta_2}{(\beta_1+2\beta_2)\left(4b\beta_1\beta_2-(\beta_1+2\beta_2)\theta_2^2\right)^2}\end{array}\right)>0$,

$$\frac{\partial \Delta\pi_s}{\partial \theta_2}=\frac{\beta_1^2\theta_2\left(a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_r-\lambda_s)\right)\left(\begin{array}{l}8b^2\beta_1^2\beta_2\left(a+\theta_1(l_m-l_0)\right)+16b^2\beta_1^2\beta_2\theta_2(l_0-\lambda_r-\lambda_s)\\ -2b(l_0-\lambda_r-\lambda_s)\beta_1(\beta_1+4\beta_2)\theta_2^3\\ -(\beta_1+2\beta_2)\left(a+(l_m-l_0)\theta_1\right)\theta_2^4\end{array}\right)}{2\left(2b\beta_1-\theta_2^2\right)^2\left(4b\beta_1\beta_2-(\beta_1+2\beta_2)\theta_2^2\right)^2}>0\quad,$$

$$\frac{\partial \Delta E}{\partial \theta_2}=\frac{1}{4}p_{carbon}b\beta_1\left(a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_r-\lambda_s)\right)$$

$$\left(\begin{array}{l}4\theta_2^2\left(a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_r-\lambda_s)\right)\left(\dfrac{1}{\left(-2b\beta_1+\theta_2^2\right)^3}-\dfrac{4\beta_2(\beta_1+\beta_2)(\beta_1+2\beta_2)}{\left(-4b\beta_1\beta_2+(\beta_1+2\beta_2)\theta_2^2\right)^3}\right)\\ +2\theta_2(l_0-\lambda_r-\lambda_s)\left(-\dfrac{1}{\left(-2b\beta_1+\theta_2^2\right)^2}+\dfrac{4\beta_2(\beta_1+\beta_2)}{\left(-4b\beta_1\beta_2+(\beta_1+2\beta_2)\theta_2^2\right)^2}\right)\\ +\left(a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_r-\lambda_s)\right)\left(-\dfrac{1}{\left(-2b\beta_1+\theta_2^2\right)^2}+\dfrac{4\beta_2(\beta_1+\beta_2)}{\left(-4b\beta_1\beta_2+(\beta_1+2\beta_2)\theta_2^2\right)^2}\right)\end{array}\right)>0\quad\text{, and}$$

$$\frac{\partial \Delta CS}{\partial \theta_2}=\frac{b^2\beta_1^3\theta_2\left(a+\theta_1(l_m-l_0)+\theta_2(l_0-\lambda_r-\lambda_s)\right)\left(\begin{array}{l}64b^3\beta_1^3\beta_2^2\left(a+\theta_1(l_m-l_0)\right)\\ +128b^3(l_0-\lambda_r-\lambda_s)\beta_1^3\beta_2^2\theta_2\\ -24b^2(l_0-\lambda_r-\lambda_s)\beta_1^2\beta_2(\beta_1+4\beta_2)\theta_2^3\\ -24b\beta_1\beta_2(\beta_1+2\beta_2)\left(a+\theta_1(l_m-l_0)\right)\theta_2^4\\ +2b(l_0-\lambda_r-\lambda_s)\beta_1^3\theta_2^5\\ +2(\beta_1+2\beta_2)(\beta_1+4\beta_2)\left(a+\theta_1(l_m-l_0)\right)\theta_2^6\\ +(l_0-\lambda_r-\lambda_s)(\beta_1+2\beta_2)(\beta_1+4\beta_2)\theta_2^7\end{array}\right)}{4\left(2b\beta_1-\theta_2^2\right)^3\left(4b\beta_1\beta_2-(\beta_1+2\beta_2)\theta_2^2\right)^3}>0\,.$$

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