Journal of Industrial and Management Optimization

JIMO, 22(9): 4258–4283.
DOI: 10.3934/jimo.2026149
Received: 05 May 2026
Revised: 17 July 2026
Accepted: 29 July 2026
Published: 12 August 2026

https://www.aimspress.com/journal/jimo

Research article

Supply chain decision making under government subsidies: Considering CSR and consumer fairness concerns

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Appendix

Illustration of Lemma1

In case 1, suppose the government subsidizes to the retailer: (1) When $2p-w+s-e_r>1$, $v\le 1$ is satisfied, the consumer demand takes the value interval on: $v\in[p-e_r+\frac{\lambda(p+s-w)}{1+\lambda},1]$, the demand can be expressed as $D=1-p+e_r-\frac{\lambda(p+s-w)}{1+\lambda}$; (2) when $2p-w+s-e_r\le 1$, the consumer's demand takes the value interval on $v\in[p-e_r+\frac{\lambda(p+s-w)}{1+\lambda},2p+s-w-e_r]$ and the demand can be expressed as $D=\frac{(p+s-w)}{1+\lambda}$. In case 1-(2), we can address the market demand is not corresponding with e_r, and the retailer's optimal effort $e_r=0$. At this time, the government's subsidies to enterprises cannot provide incentives. Regime validity proof: Substituting the optimal equilibrium solutions in Tables 4 and 6 back into the threshold conditions in Lemma 1, we can verify that the optimal subsidy coefficients s^{RS} and s^{MS} derived in Propositions 2 and 4 always satisfy the threshold constraints within the parameter ranges defined in this paper. This confirms that the optimal equilibrium always

falls in the effective regime where CSR efforts are positive and consumer fairness concerns are active, justifying our focus on Case 1-(1) throughout the equilibrium analysis.

We focus on Case 1-(1) for two reasons. First, mathematically, Case 1-(2) yields zero CSR effort in equilibrium, which contradicts the core research focus of this paper on the CSR incentive effects of subsidies. Second, practically, the pandemic context is characterized by supply shortages and high consumer demand for essential goods, where firms have sufficient incentive to invest in CSR to boost demand, and consumer fairness concerns remain active. Case 1-(2) corresponds to a scenario where subsidies are too low to stimulate any CSR effort, which is not the focus of our policy analysis. Then, the government subsidy needs to consider the value range: $s>1-2p+w+e_r$, and the market demand under RS-strategy is $D^{RS}=1-p+e_r-\frac{\lambda(p+s-w)}{1+\lambda}$.

In case 2, suppose the government subsidizes to the manufacturer, the derivation is similar, and the value range of s is $s>1-p-w-s+e_m+c$: (1) When $p+w+s-e_m-c>1$, $D=1-p+e_m-\frac{\lambda}{1+\lambda}(w+s-c)$; (2)When $p+w+s-e_m-c<1$, $D=\frac{w+s-c}{1+\lambda}$. We consider the case 2-(1). In case 2-(2), we can address that the market demand is not corresponding with e_m, and the manufacturer's optimal effort $e_m=0$. Therefore, the market demand under the MS-strategy is: $D^{MS}=1-p+e_m-\frac{\lambda}{1+\lambda}(w+s-c)$.

Illustration of Assumption 1

(1) When we consider the efforts cost factor θ, the retailer's profit Π_r^{RS} can be expressed as

$$\Pi_r^{RS}(p,e_r)=(p-w)[1-p+e_r-\frac{\lambda(p+s-w)}{\lambda+1}]-\frac{\theta e_r^{\,2}}{2}+se_r \tag{A1}$$

The Hessian matrix of Equation (A1) can be expressed as

$$H_{RS}(e_r,p)=\begin{bmatrix}\frac{\partial^2\Pi_r^{RS}}{\partial e_r^{\,2}} & \frac{\partial^2\Pi_r^{RS}}{\partial e_r\partial p}\\ \frac{\partial^2\Pi_r^{RS}}{\partial p\partial e_r} & \frac{\partial^2\Pi_r^{RS}}{\partial p^2}\end{bmatrix}=\begin{bmatrix}-\theta & 1\\ 1 & -\frac{4\lambda+2}{\lambda+1}\end{bmatrix}=\frac{4\lambda+2}{\lambda+1}\theta-1 \tag{A2}$$

When $\theta>\frac{\lambda+1}{4\lambda+2}$, $H_{RS}(e_r,p)>0$, the Hessian matrix is a negative definite matrix, the function concaves down. There exists a uniquely determined $[e_r^{RS*},p^{RS*}]$ that maximizes the value of Π_r^{RS}, and if e_r^{RS*} and p^{RS*} are specific values of e and p, the equilibrium solution for the retailer can be obtained at this point.

Since $\frac{\lambda+1}{4\lambda+2}$ is a monotonically decreasing function, the maximum value $\frac{1}{2}$ is obtained at $\lambda=0$, so $H_{RS}(e_r,p)>0$ holds when $\theta>\frac{1}{2}$. $\Pi_r^{RS}(p;e_r)$ has a maximum value.

Suppose $\theta\le\frac{1}{2}$, the Hessian matrix $H_{RS}(e_r,p)\le 0$, the original equation**(A1)** has no optimal

solution.

(2) When we consider the efforts cost factor θ , the manufacturer's profit Π_m^{MS} can be expressed as

$$\Pi_m^{RS}(w,e_m)=[1-p+e_m-\frac{\lambda}{1+\lambda}(w+s-c)](w-c)-\frac{\theta e_m^2}{2}+se_m \tag{A3}$$

Substitute **(A14)** into **(A3)**. The Hessian matrix of Equation (A3) can be expressed as

$$H_{MS}(e_m,w)=\begin{bmatrix}\frac{\partial^2\Pi_m^{MS}}{\partial e_m{}^2} & \frac{\partial^2\Pi_m^{MS}}{\partial e_m\partial w}\\ \frac{\partial^2\Pi_m^{MS}}{\partial w\partial e_m} & \frac{\partial^2\Pi_m^{MS}}{\partial w^2}\end{bmatrix}=\begin{bmatrix}-\theta & \frac{1}{2}\\ \frac{1}{2} & -\frac{1+2\lambda}{1+\lambda}\end{bmatrix}=\frac{\theta+2\theta\lambda}{1+\lambda}-\frac{1}{4} \tag{A4}$$

When $\theta>\frac{\lambda+1}{4(2\lambda+1)}$, $H_{MS}(e_m,p)>0$, the function concaves down. There exists a uniquely determined $[e_m^{MS*},w^{MS*}]$ that maximizes the value of Π_m^{MS} . Since $\frac{\lambda+1}{4(2\lambda+1)}$ is a monotonically decreasing function, the maximum value $\frac{1}{4}$ is obtained at $\lambda=0$, so $H_{MS}(e_m,w)>0$ holds when $\theta>\frac{1}{4}$.

Suppose $\theta\le\frac{1}{4}$, and the Hessian matrix $H_{MS}(e_r,w)\le 0$, so the original equation **(A3)** has no optimal solution.

The government will consider the enterprises' product prices and social responsibility efforts when deciding on his subsidy strategy. Therefore, the government will not subsidize the manufacturer when $\theta<\frac{1}{4}$ and will not subsidize the retailer when $\theta<\frac{1}{2}$. Moreover, the costs borne by enterprises during the epidemic period are relatively large to avoid trivial cases, so we assume θ=1 .

Proof of Proposition 1

(1) According to **Illustration of Assumption 1-(1)**, it is known that there exists the optimal $[e_r,p]$ such that the retailer's profit is maximized.

Equation **(7)** for e and p are used to find the partial derivative, respectively, and letting the partial derivative be 0 can obtain: $e=p+s-w$, $p=\frac{1+e+w+(1+e-s+3w)\lambda}{4\lambda+2}$, i.e.,

$$e=\frac{(1-w)(1+\lambda)+s(3\lambda+2)}{3\lambda+1} \tag{A5}$$

$$p=\frac{1+s+\lambda+2w\lambda}{1+3\lambda} \tag{A6}$$

Substituting **Eq. (A5)** and **(A6)** into Eq. **(6)** and taking the partial derivatives of w yields

$$\frac{\partial\Pi_m^{RS}}{\partial w}=\frac{(1+2\lambda)(1+c+s-2w+\lambda+c\lambda-2w\lambda)}{(1+\lambda)(1+3\lambda)} \tag{A7}$$

Since the second partial derivative $\frac{\partial^2 \Pi_m^{RS}}{\partial w^2} = -\frac{2+4\lambda}{1+3\lambda} < 0$, the function concaves down, and there exists a w such that Π_m^{RS} has a maximum value.

When the partial derivative $\frac{\partial \Pi_m^{RS}}{\partial w}$ is $0, \Pi_m^{RS}$ obtains its maximum value, which gives

$$w^{RS*} = \frac{1}{2}(c+1+\frac{\lambda s}{1+\lambda}) \tag{A8}$$

Bringing equations (A5), (A6), and (A8) into equations (6) and (7) yields the results shown in **Table 3**.

Proof of Proposition 2

The maximum price affordable to the consumer is available when the government subsidizes the demand $D^{RS} = 0$.

$$p^{RS}{}_{\max} = \frac{(\lambda+1)(1+e)+\lambda(w-s)}{2\lambda+1} = \frac{(1+s+\lambda)(3+5\lambda)+c(-1+\lambda^2)}{2(1+\lambda)(1+3\lambda)}$$

Bringing $p_{\max}$ and D^{RS} into **Assumption 4** yields

$$CS^{RS} = \int_p^{p_{\max}} D^{RS} dp = \int_p^{\frac{(\lambda+1)(1+e)+\lambda(w-s)}{2\lambda+1}} 1-p+e-\frac{\lambda(p+s-w)}{\lambda+1} dp = \frac{(1+e-p+\lambda+e\lambda-2p\lambda-s\lambda+w\lambda)^2}{2+6\lambda+4\lambda^2} \tag{A9}$$

First, according to **(10)**, the government considers a nonlinear programming problem with constraints. It is simple to demonstrate the problem is a convex programming. We construct the Lagrangian function with respect to $\Pi_g^{RS}(s)$. Here, $\mu_1 \geq 0$ is the generalized Lagrange multiplier of constraint $s.t. \frac{s+2s\lambda+(-1+c)\lambda(1+\lambda)}{(1+\lambda)(1+3\lambda)} > 0$. We propose the K-K-T condition as:

$$L(s,\mu_1) = \Pi_g^{RS}(s) + \mu_1 \frac{s+2s\lambda+(-1+c)\lambda(1+\lambda)}{(1+\lambda)(1+3\lambda)} \tag{A10}$$

$$\frac{\partial L(s,\mu_1)}{\partial s} = \frac{(1+2\lambda)(2-4s+5\lambda+c(-2-5\lambda+3\lambda^3)-3\lambda(\lambda^2+3s(1+\lambda)(3+2\lambda))+4(1+\lambda)(1+3\lambda)\mu_1)}{4(1+\lambda)^2(1+3\lambda)^2} = 0$$

(A11)

$$\frac{\partial L(s,\mu_1)}{\partial \mu_1} = \frac{s+2s\lambda+(-1+c)\lambda(1+\lambda)}{(1+\lambda)(1+3\lambda)} \geq 0 \tag{A12}$$

$$\mu_1 \frac{s+2s\lambda+(-1+c)\lambda(1+\lambda)}{(1+\lambda)(1+3\lambda)}=0 \tag{A13}$$

(1) When $\mu_1=0$, $\frac{\partial^2\Pi_g^{RS}(s)}{\partial s^2}=-\frac{(1+2\lambda)(4+9\lambda(1+\lambda)(3+2\lambda))}{4(1+\lambda)^2(1+3\lambda)^2}<0$, the function concaves down. We set $\frac{\partial\Pi_g^{RS}(s)}{\partial s}=0$, and get the optimal solution:

$$\begin{cases} s_1^{SR}=\dfrac{(1-c)(2+5\lambda-3\lambda^3)}{4+9\lambda(1+\lambda)(3+2\lambda)} \\ \Pi_{g1}^{RS}=\dfrac{(1-c)^2(1+\lambda)^2(5+9\lambda)}{8+18\lambda(1+\lambda)(3+2\lambda)} \end{cases}$$

(2) When $\mu_1>0$. Since the value of (A11) is 0, $s_2^{SR}=\frac{(1-c)\lambda(1+\lambda)}{1+2\lambda}$, substitute it into (A9). Thus, we get $\mu_1=\frac{(1-c)(-2+3\lambda(1+\lambda(5+2\lambda)))}{4+8\lambda}$.

The government's utility is: $\Pi_{g2}^{RS}=\frac{(1-c)^2(4+5\lambda-5\lambda^2-2\lambda^3)}{8(1+2\lambda)}$.

Since $\Pi_{g1}^{RS}-\Pi_{g2}^{RS}=\frac{(1-c)^2(2-3\lambda(1+\lambda(5+2\lambda)))^2}{8(1+2\lambda)(4+9\lambda(1+\lambda)(3+2\lambda))}>0$, case (1) is the optimal solution. Substituting **(A5), (A6), (A8), and Eq. (10)** into **(6), (7),** and **(9)**, we obtain the optimal decision under RS-strategy, as shown in **Table 4**.

Proof of Proposition 3

Same as Proposition 1, we use backward induction to solve the optimal solutions of the manufacturer and the retailer under the MS-strategy.

Since $\frac{\partial^2\Pi_r^{MS}}{\partial p^2}=-2<0$, the function concaves down, and there exists a p such that Π_r^{MS} has a maximum value. Let $\frac{\partial\Pi_r^{MS}}{\partial p}=1-2p+w-\frac{(-c+s+w)\lambda}{1+\lambda}+e_m=0$ and address

$$p=\frac{1+w+\lambda+c\lambda-s\lambda+e_m+\lambda e_m}{2+2\lambda} \tag{A14}$$

According to **Illustration of Assumption 1**-(2), the manufacturer has the optimal solution $[e_m, w]$. The first order condition of the optimal solution is

$$\begin{cases} \dfrac{\partial \Pi_m^{MS}}{\partial w} = -\dfrac{-1-c+2w-\lambda-3c\lambda+s\lambda+4w\lambda-e_m-\lambda e_m}{2(1+\lambda)} = 0 \\ \dfrac{\partial \Pi_m^{MS}}{\partial e_m} = -\dfrac{c-2s-w+c\lambda-2s\lambda-w\lambda+2e_m+2\lambda e_m}{2(1+\lambda)} = 0 \end{cases} \tag{A15}$$

The optimal decisions of the manufacturer and the retailer are expressed as follows

$$w^{MS*} = \frac{c+5c\lambda+2(1+s+\lambda)}{3+7\lambda} \tag{A16}$$

$$e_m^{MS*} = \frac{1+\lambda-c(1+\lambda)+s(4+7\lambda)}{3+7\lambda} \tag{A17}$$

Substituting equations **(A14), (A16)**, and **(A17)** into equations **(12)** and **(13)** yields the results shown in **Table 5**.

Proof of Proposition 4

The proof process is similar to that of Proposition 2. According to **(15)**, we propose the Lagrangian function with respect to $\Pi_g^{MS}(s)$. Here, $\mu_2 \geq 0$ is the generalized Lagrange multiplier of constraint $s.t.\dfrac{1-c+7s+(-1+c+15s)\lambda+(-2+2c+7s)\lambda^2}{(1+\lambda)(3+7\lambda)} > 0$. We address the K-K-T condition as:

$$L(s,\mu_2) = \Pi_g^{MS}(s) + \mu_2 \frac{1-c+7s+(-1+c+15s)\lambda+(-2+2c+7s)\lambda^2}{(1+\lambda)(3+7\lambda)} \tag{A18}$$

$$\frac{\partial L(s,\mu_2)}{\partial s} = \frac{1}{(1+\lambda)^2(3+7\lambda)^2}\begin{pmatrix} -(1+\lambda)(-3+\lambda(1+\lambda)(-9+7\lambda)) + c(1+\lambda)(-3+\lambda(1+\lambda)(-9+7\lambda)) \\ -s(9+\lambda(64+\lambda(157+7\lambda(22+7\lambda)))) + (1+\lambda)(3+7\lambda)(7+\lambda(15+7\lambda))\mu_2 \end{pmatrix} = 0 \tag{A19}$$

$$\frac{\partial L(s,\mu_2)}{\partial \mu_2} = \frac{1-c+7s+(-1+c+15s)\lambda+(-2+2c+7s)\lambda^2}{(1+\lambda)(3+7\lambda)} \geq 0 \tag{A20}$$

$$\mu_2 \frac{1-c+7s+(-1+c+15s)\lambda+(-2+2c+7s)\lambda^2}{(1+\lambda)(3+7\lambda)} = 0 \tag{A21}$$

(1) When $\mu_2 = 0$, $\dfrac{\partial^2 \Pi_g^{MS}(s)}{\partial s^2} = -\dfrac{9+\lambda(64+\lambda(157+7\lambda(22+7\lambda)))}{(1+\lambda)^2(3+7\lambda)^2} < 0$, the function concaves down. We set $\dfrac{\partial \Pi_g^{MS}(s)}{\partial s} = 0$, and get the optimal solution

$$\begin{cases} s_1^{MS} = \dfrac{(1-c)(1+\lambda)\left(3+\lambda(1+\lambda)(9-7\lambda)\right)}{9+\lambda\left(64+\lambda\left(157+7\lambda(22+7\lambda)\right)\right)} \\ \Pi_{g1}^{MS} = \dfrac{(-1+c)^2(1+\lambda)^2(1+2\lambda)(7+10\lambda)}{2\left(9+\lambda\left(64+\lambda\left(157+7\lambda(22+7\lambda)\right)\right)\right)} \end{cases}.$$

(2) When $\mu_2 > 0$, since the value of **(A21)** is 0, then $s_2^{MR} = \dfrac{(1-c)(1+\lambda)(-1+2\lambda)}{7+\lambda(15+7\lambda)}$.

Substituting this into **(A19)** we can get $\mu_2 = \dfrac{(1-c)\left(-10-28\lambda-\lambda^2+41\lambda^3+21\lambda^4\right)}{\left(7+15\lambda+7\lambda^2\right)^2}$.

The government's utility is: $\Pi_{g2}^{MS} = \dfrac{(1-c)^2\left(27+\lambda\left(116+\lambda\left(161+\lambda(80+11\lambda)\right)\right)\right)}{2\left(7+\lambda(15+7\lambda)\right)^2}$.

Since $\Pi_{g1}^{MS} - \Pi_{g2}^{MS} = \dfrac{(1-c)^2\left(10+\lambda\left(28+\lambda-41\lambda^2-21\lambda^3\right)\right)^2}{2\left(7+\lambda(15+7\lambda)\right)^2\left(9+\lambda\left(64+\lambda\left(157+7\lambda(22+7\lambda)\right)\right)\right)} > 0$, case (1) is the optimal solution. Substituting **(A14), (A16), (A17), and Eq. (16)** into **(12), (13),** and **(15),** we can obtain the optimal decision under the MS-strategy as shown in **Table 6**.

Proof of Corollary 1

Since $0 < c < 1, 1-c > 0$.

$$\frac{\partial e_r^{RS}}{\partial \lambda} = -\frac{(1-c)\left(79+9\lambda\left(42+\lambda\left(73+2\lambda(28+9\lambda)\right)\right)\right)}{\left(4+9\lambda(1+\lambda)(3+2\lambda)\right)^2} < 0;$$

$$\frac{\partial e_m^{MS}}{\partial \lambda} = -\frac{(1-c)\left(169+\lambda\left(1406+\lambda\left(4745+\lambda\left(8360+\lambda\left(8193+196\lambda(22+5\lambda)\right)\right)\right)\right)\right)}{\left(9+\lambda\left(64+\lambda\left(157+7\lambda(22+7\lambda)\right)\right)\right)^2} < 0;$$

$$\frac{\partial p^{RS}}{\partial \lambda} = -\frac{6(1-c)(1+\lambda)\left(11+39\lambda+36\lambda^2\right)}{\left(4+9\lambda(1+\lambda)(3+2\lambda)\right)^2} < 0;$$

$$\frac{\partial p^{MS}}{\partial \lambda} = -\frac{(1-c)(1+\lambda)\left(219+\lambda\left(1497+\lambda\left(3817+\lambda\left(4411+35\lambda(61+7\lambda)\right)\right)\right)\right)}{\left(9+\lambda\left(64+\lambda\left(157+7\lambda(22+7\lambda)\right)\right)\right)^2} < 0;$$

$$\frac{\partial s^{RS}}{\partial \lambda} = -\frac{(1-c)\left(34+9\lambda\left(20+\lambda\left(41+\lambda(38+15\lambda)\right)\right)\right)}{\left(4+9\lambda(1+\lambda)(3+2\lambda)\right)^2} < 0;$$

$$\frac{\partial s^{MS}}{\partial \lambda} = -\frac{(1-c)\left(84+\lambda\left(744+\lambda\left(2701+\lambda\left(5176+\lambda\left(5587+7\lambda(468+119\lambda)\right)\right)\right)\right)\right)}{\left(9+\lambda\left(64+\lambda\left(157+7\lambda(22+7\lambda)\right)\right)\right)^2} < 0$$

$$\frac{\partial CS^{RS}}{\partial \lambda} = -\frac{9(1-c)^2(1+\lambda)^4\left(26+3\lambda\left(35+39\lambda+6\lambda^2\right)\right)}{2\left(4+9\lambda(1+\lambda)(3+2\lambda)\right)^3} < 0;$$

$$\frac{\partial w^{MS}}{\partial \lambda}=-\frac{2(1-c)(1+\lambda)^2\left(85+\lambda\left(492+\lambda\left(975+7\lambda(106+21\lambda)\right)\right)\right)}{\left(9+\lambda\left(64+\lambda\left(157+7\lambda(22+7\lambda)\right)\right)\right)^2}<0.$$

Proof of Corollary 2

$$\frac{\partial e_r^{RS*}}{\partial s}=\frac{3+6\lambda}{2+6\lambda},\frac{\partial e_m^{MS*}}{\partial s}=1+\frac{1}{3+7\lambda};$$

$$\frac{\partial e_r^{RS*}}{\partial s}-\frac{\partial e_m^{MS*}}{\partial s}=\frac{1+\lambda}{6+32\lambda+42\lambda^2}>0.$$

Proof of Corollary 3

$$\frac{\partial \Pi_g^{RS}}{\partial \lambda}=-\frac{(1-c)^2(1+\lambda)\left(59+9\lambda\left(23+\lambda(21+\lambda)\right)\right)}{2\left(4+9\lambda(1+\lambda)(3+2\lambda)\right)^2}<0;$$

$$\frac{\partial \Pi_g^{MS}}{\partial \lambda}=-\frac{(1-c)^2(1+\lambda)\left(53+\lambda\left(371+\lambda\left(965+\lambda\left(1117+\lambda(507+28\lambda)\right)\right)\right)\right)}{\left(9+\lambda\left(64+\lambda\left(157+7\lambda(22+7\lambda)\right)\right)\right)^2}<0;$$

$$\Pi_g^{RS}-\Pi_g^{MS}=\frac{(1-c)^2(1+\lambda)^4\left(17+\lambda\left(82+\lambda(137+81\lambda)\right)\right)}{2\left(4+9\lambda(1+\lambda)(3+2\lambda)\right)\left(9+\lambda\left(64+\lambda\left(157+7\lambda(22+7\lambda)\right)\right)\right)}>0.$$

Proof of Corollary 4

Since $0<c<1, c-1<0$.

$$S_r^{RS*}=se_r^{RS*}=\frac{s\left(1+\lambda-c(1+\lambda)+s(3+6\lambda)\right)}{2+6\lambda}, S_m^{MS*}=se_m^{MS*}=\frac{s\left(1+\lambda-c(1+\lambda)+s(4+7\lambda)\right)}{3+7\lambda}.$$

Then we address

$$S_m^{MS*}-S_r^{RS*}=-\frac{s(1+\lambda)\left(1+s+\lambda-c(1+\lambda)\right)}{2(1+3\lambda)(3+7\lambda)}. \tag{A22}$$

To establish formula $S_m^{MS*}-S_r^{RS*}<0$, it is necessary to meet the following conditions: $\left(1+s+\lambda-c(1+\lambda)\right)>0$, i.e., $s>(c-1)(1+\lambda)$. Additionally, we know that $(c-1)(1+\lambda)<0$, so $S_m^{MS*}-S_r^{RS*}<0$ is always true when condition $s>0$ is satisfied.

$$D^{RS*}=1+e_r^{RS*}-p^{RS*}-\frac{\lambda\left(p^{RS*}+s-w^{RS*}\right)}{1+\lambda}=\frac{(1+2\lambda)(1-c+s+(1-c)\lambda)}{2(1+\lambda)(1+3\lambda)},$$

$$D^{MS*}=1+e_m^{MS*}-p^{MS*}-\frac{\lambda\left(w^{MS*}+s-c\right)}{1+\lambda}=\frac{(1+2\lambda)\left(1-c+s+(1-c)\lambda\right)}{(1+\lambda)(3+7\lambda)}.$$

Then we address

$$D^{RS*}-D^{MS*}=\frac{(1+2\lambda)\left(1-c+s+(1-c)\lambda\right)}{2(1+3\lambda)(3+7\lambda)}>0$$

Proof of Proposition 5

$$\Pi_g^{RS*}=\Pi_r^{RS*}+\Pi_m^{RS*}+CS^{RS*}-S^{RS*}\ ;\ \Pi_g^{MS*}=\Pi_r^{MS*}+\Pi_m^{MS*}+CS^{MS*}-S^{MS*}$$

$$\Pi_g^{RS*}-\Pi_g^{MS*}=\frac{(-1+c-s+(-1+c)\lambda)\begin{pmatrix}-12+c(1+\lambda)\left(12+\lambda\left(89+\lambda\left(243+\lambda\left(299+149\lambda\right)\right)\right)\right)\\+\lambda\left(-101-\lambda\left(332+\lambda\left(542+\lambda\left(448+149\lambda\right)\right)\right)+s\left(11+\lambda\left(69+\lambda\left(157+\lambda\left(167+84\lambda\right)\right)\right)\right)\right)\end{pmatrix}}{8(1+\lambda)^2(1+3\lambda)^2(3+7\lambda)^2} \quad \text{(A23)}$$

We solve inequality $\Pi_g^{RS*}-\Pi_g^{MS*}>0$ and address

$$(c-1)(1+\lambda)<0<s<\frac{(1-c)(1+\lambda)\left(12+\lambda\left(89+\lambda\left(243+\lambda\left(299+149\lambda\right)\right)\right)\right)}{\lambda\left(11+\lambda\left(69+\lambda\left(157+\lambda\left(167+84\lambda\right)\right)\right)\right)} \quad \text{(A24)}$$

Where $s*=\frac{(1-c)(1+\lambda)\left(12+\lambda\left(89+\lambda\left(243+\lambda\left(299+149\lambda\right)\right)\right)\right)}{\lambda\left(11+\lambda\left(69+\lambda\left(157+\lambda\left(167+84\lambda\right)\right)\right)\right)}$.

Therefore, when $s<s*, \Pi_g^{RS}>\Pi_g^{MS}$; when $s<s*, \Pi_g^{RS}<\Pi_g^{MS}$.

Then we proof $s_2<s_1<s^{RS}<s*$:

(1) $s_1-s_2=\frac{(1-c)\lambda(1+\lambda)}{1+2\lambda}-\frac{(1-c)(1+\lambda)(-1+2\lambda)}{7+\lambda(15+7\lambda)}=\frac{(1-c)(1+\lambda)\left(1+\lambda\left(7+\lambda(11+7\lambda)\right)\right)}{(1+2\lambda)\left(7+\lambda(15+7\lambda)\right)}>0$.

(2) s^{SR} is the optimal solution that satisfies the constraint $s>s_1$, so $s_1<s^{RS}$.

(3) $s*-s^{RS}=\frac{2(1-c)(1+\lambda)^3(1+3\lambda)\left(24+\lambda\left(209+\lambda\left(659+915\lambda+489\lambda^2\right)\right)\right)}{\lambda\left(4+9\lambda(1+\lambda)(3+2\lambda)\right)\left(11+\lambda\left(69+\lambda\left(157+\lambda\left(167+84\lambda\right)\right)\right)\right)}>0$.

As a consequence, we conclude government's subsidy strategy options in **Proposition 5**.

Illustration of Figure 5

The profit of the supply chain under centralized decision-making without government subsidies is expressed as

$$\Pi_t^N(p,e)=(p-c)\left(1-p+e-\frac{\lambda(p+e-c)}{1+\lambda}\right)-\frac{e^2}{2} \quad \text{(A25)}$$

$$H_t^N(p,e)=\begin{bmatrix}\frac{\partial^2\Pi_t^N}{\partial p} & \frac{\partial^2\Pi_t^N}{\partial p\partial e}\\ \frac{\partial^2\Pi_t^N}{\partial e\partial p} & \frac{\partial^2\Pi_t^N}{\partial e^2}\end{bmatrix}=\begin{bmatrix}-\frac{2+4\lambda}{1+\lambda} & \frac{1}{1+\lambda}\\ \frac{1}{1+\lambda} & -1\end{bmatrix}=\frac{1+6\lambda+4\lambda^2}{(1+\lambda)^2}>0 \quad \text{(A26)}$$

Thus, the function concaves down. There exists a uniquely determined $[e, p]$ that maximizes the value of Π_t^N. Solve the following system of equations:

$$\begin{cases} \dfrac{\partial \Pi_t^N}{\partial e} = (p-c)\left(1-\dfrac{\lambda}{1+\lambda}\right) - e = 0 \\ \dfrac{\partial \Pi_t^N}{\partial p} = \dfrac{1+c+e-2p+\lambda+3c\lambda-4p\lambda}{1+\lambda} = 0 \end{cases} \tag{A27}$$

We conclude the optimal solution as follows

$$e^N = \frac{(1-c)(1+\lambda)}{1+6\lambda+4\lambda^2} \tag{A28}$$

$$p^N = \frac{1+\lambda\left(2+\lambda+c\left(4+3\lambda\right)\right)}{1+6\lambda+4\lambda^2} \tag{A29}$$

$$\Pi_t^N = \frac{(1-c)^2(1+\lambda)^2}{2+4\lambda(3+2\lambda)} \tag{A30}$$

$$\Pi_r^{MS} + \Pi_m^{MS} - \Pi_t^N = \frac{(-1+c)^2\left(-28+\lambda\left(-92+\lambda\left(-34+\lambda\left(180+\lambda\left(255+\lambda(134+27\lambda)\right)\right)\right)\right)\right)}{2\left(1+6\lambda+4\lambda^2\right)\left(7+\lambda(15+7\lambda)\right)^2} \tag{A31}$$

Solving $\Pi_r^{MS} + \Pi_m^{MS} - \Pi_t^N = 0$， we can address $\lambda^* = 0.628435$.

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