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Research article

Hybrid financing strategies for capital-constrained 3D printing technology vendors: a game-theoretic approach to supply chain coordination

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Appendix A. The proof of proposition 1

Taking the partial derivative of the retail price with respect to equation (5) in the main text, and knowing that $\frac{\partial^2 \pi_{r,1}}{\partial^2 p} = -2\beta < 0$, it can be concluded that the retailer's profit function is concave in terms of p, therefore, for any given w and h, the retailer has a unique optimal retail price p. Setting

$$\frac{\partial \pi_{r,1}}{\partial p} = a - 2\beta p + \tau\delta + \beta w = 0, \tag{A1}$$

gives the retail price corresponding to the maximum profit:

$$p^* = \frac{a + \tau\delta + w\beta}{2\beta}. \tag{A2}$$

Substitute the retailer's best response into the manufacturer's profit function in equation (4), and calculate the partial derivative of equation (4) with respect to the wholesale price. Knowing that $\frac{\partial^2 \pi_{m,1}}{\partial^2 w} = -\beta < 0$, Since $\beta > 0$, it can be concluded that the manufacturer's profit function is concave in terms of w, and there exists a unique optimal value for w. Setting

$$\frac{\partial \pi_{m,1}}{\partial w} = \frac{a + \tau\delta - 2\beta w + \beta h + c_2\beta - c_2\beta\rho\delta}{2} = 0, \tag{A3}$$

gives the wholesale price corresponding to the maximum profit:

$$w^* = \frac{a + \tau\delta + c_2\beta - c_2\beta\rho\delta + h\beta}{2\beta}. \tag{A4}$$

Substitute the retailer's and manufacturer's best responses into the technology vendor's profit function in equation (3) in the main text, and calculate the partial derivative of equation (3) with respect to the technology fee. Knowing that $\frac{\partial^2 \pi_{t,1}}{\partial^2 h} = -\frac{\mu\beta}{2} < 0$, it can be concluded that the technology vendor's reduced profit function is strictly concave in h, and there exists an optimal value for h. Setting

$$\frac{\partial \pi_{t,1}}{\partial h} = \frac{a - c_2\beta + \tau\delta - 2\mu h\beta + c_2\beta\rho\delta + c_1\beta\left[1 + r_0\left(1 + k_1\theta_{1,1} - k_2\theta_2\right)\right]}{4} = 0, \tag{A5}$$

gives the technology fee corresponding to the maximum profit:

$$h^* = \frac{a + \tau\delta - c_2\beta + c_2\beta\rho\delta + c_1\beta\left[1 + r_0\left(1 + k_1\theta_{1,1} - k_2\theta_2\right)\right]}{2\mu\beta}. \tag{A6}$$

Since each stage yields a unique best response, the backward-induction procedure is well defined. Under the admissible parameter region in which $p^* \geq 0, w^* \geq 0, h^* \geq 0,$ and the resulting market demand satisfies $q^* = a - \beta p^* + \tau\delta > 0,$ the IPR pricing subgame admits a unique subgame-perfect equilibrium.

A note on the technology-fee discount. In the original formulation, a proportional technology-fee discount was introduced to capture the idea that the technology vendor may offer a concession when the manufacturer provides financing support. However, because the unit technology fee is chosen endogenously by the technology vendor, this proportional discount is equivalent to a rescaling of the decision variable. Let $z = A(\mu)h$ denote the effective technology fee paid by the manufacturer, where $A(\mu) > 0$ is the proportional adjustment factor. Maximizing the technology vendor's profit with respect to h is equivalent to maximizing it with respect to z. As a result, the optimal nominal technology fee h^* changes with $A(\mu)$, but the effective fee $A(\mu)h^*$, equilibrium quantity, retail price, wholesale price, and equilibrium profits do not. Therefore, μ has no independent equilibrium effect in the present model and is not used in the numerical analysis.

Appendix B. The proof of proposition 2

Taking the partial derivative of the retail price with respect to equation (11) in the main text, and knowing that $\frac{\partial^2 \pi_{r,2}}{\partial^2 p} = -2\beta < 0$, it can be concluded that the retailer's profit function is concave in terms of p, and there exists a unique optimal value for p. Setting

$$\frac{\partial \pi_{r,2}}{\partial p} = a - 2\beta p + \tau\delta + \beta w = 0, \tag{B1}$$

gives the retail price corresponding to the maximum profit:

$$p^* = \frac{a + \tau\delta + w\beta}{2\beta}. \tag{B2}$$

Substitute the retailer's best response into the manufacturer's profit function in equation (10) in the main text, and calculate the partial derivative of equation (10) with respect to the wholesale price. Knowing that $\frac{\partial^2 \pi_{m,2}}{\partial^2 w} = -\beta < 0$, it can be concluded that the manufacturer's profit function is concave in terms of w, and there exists a unique optimal value for w. Setting

$$\frac{\partial \pi_{m,2}}{\partial w} = \frac{a + \tau\delta - 2\beta w + \beta h + c_2\beta - c_2\beta\rho\delta - c_1\beta r_2}{2} = 0, \tag{B3}$$

gives the wholesale price corresponding to the maximum profit:

$$w^* = \frac{a + \tau\delta + \beta h + c_2\beta - c_2\beta\rho\delta - c_1\beta r_2}{2\beta}. \tag{B4}$$

Substitute the retailer's and manufacturer's best responses into the technology vendor's profit function in equation (9) in the main text, and calculate the partial derivative of equation (9) with respect to the technology fee. Knowing that $\frac{\partial^2 \pi_{t,2}}{\partial^2 h} = -\frac{\beta}{2} < 0$, it can be concluded that the technology vendor's reduced profit function is strictly concave in h, and there exists a unique optimal value for h. Setting

$$\frac{\partial \pi_{t,2}}{\partial h} = \frac{a + \tau\delta - 2\beta h - c_2\beta + c_2\beta\rho\delta + c_1\beta + 2c_1\beta r_2}{4} = 0, \tag{B5}$$

gives the technology fee corresponding to the maximum profit:

$$h = \frac{a + \tau\delta - c_2\beta + c_2\beta\rho\delta + c_1\beta + 2c_1\beta r_2}{2\beta}. \tag{B6}$$

Since each stage yields a unique best response, the backward-induction procedure is well defined. Under the admissible parameter region in which $p^* \geq 0, w^* \geq 0, h^* \geq 0,$ and the resulting market demand satisfies $q^* = a - \beta p^* + \tau\delta > 0,$ the IDF pricing subgame admits a unique subgame-perfect equilibrium.

Appendix C. The proof of proposition 3

Taking the partial derivative of the retail price with respect to equation (17) in the main text, and knowing that $\frac{\partial^2 \pi_{r,3}}{\partial^2 p} = -2\beta < 0$, it can be concluded that t the retailer's profit function is strictly concave in p, and there exists a unique optimal value for p. Setting

$$\frac{\partial \pi_{r,3}}{\partial p} = a - 2\beta p + \tau\delta + \beta w = 0, \tag{C1}$$

gives the retail price corresponding to the maximum profit:

$$p^* = \frac{a + \tau\delta + w\beta}{2\beta}. \tag{C2}$$

Substitute the retailer's best response into the manufacturer's profit function in equation (16) in the main text, and calculate the partial derivative of equation (16) with respect to the wholesale price.

Knowing that $\frac{\partial^2 \pi_{m,3}}{\partial^2 w} = -\beta < 0$, it can be concluded that the manufacturer's reduced profit function is strictly concave in w, and there exists a unique optimal value for w. Setting

$$\frac{\partial \pi_{m,3}}{\partial w} = \frac{a + \tau\delta - 2\beta w + \beta h(1-\eta) + c_2\beta - c_2\beta\rho\delta + c_1\beta\eta}{2} = 0, \tag{C3}$$

gives the wholesale price corresponding to the maximum profit:

$$w^* = \frac{a + \tau\delta + \beta h(1-\eta) + c_2\beta - c_2\beta\rho\delta + c_1\beta\eta}{2\beta}. \tag{C4}$$

Substitute the retailer's and manufacturer's best responses into the technology vendor's profit function in equation (15) in the main text, and calculate the partial derivative of equation (15) with respect to the technology fee. Knowing that $\frac{\partial^2 \pi_{t,3}}{\partial^2 h} = -\frac{\beta(1-\eta)^2}{2} < 0$, it can be concluded that the technology vendor's reduced profit function is strictly concave in h for $0 \le \eta \le \bar{\eta} < 1$, and there exists a unique optimal value for h. Setting

$$\frac{\partial \pi_{t,3}}{\partial h} = \frac{(1-\eta)\left[a + \tau\delta + (1-2\eta)c_1\beta + 2(\eta-1)\beta h - c_2\beta + c_2\beta\rho\delta\right]}{4} = 0, \tag{C5}$$

gives the technology fee corresponding to the maximum profit:

$$h^* = \frac{a + \tau\delta - c_2\beta + c_2\beta\rho\delta + c_1\beta(1-2\eta)}{2\beta(1-\eta)}. \tag{C6}$$

The strict inequality $\frac{\partial^2 \pi_{t,3}}{\partial h^2} = -\frac{\beta(1-\eta)^2}{2} < 0$ holds under the admissible parameter restriction $\beta > 0$ and $0 \le \eta \le \bar{\eta} < 1$. Therefore, for any admissible equity share η, the technology vendor's reduced profit function is strictly concave in (h), and the first-order condition yields a unique optimal technology fee h^*.

The boundary case $\eta = 1$ is excluded from the IEF model. Indeed, $\lim_{\eta \to 1} \frac{\partial^2 \pi_{t,3}}{\partial h^2} = 0,$ which implies that the technology vendor's reduced profit function loses strict concavity at this boundary. Moreover, the denominator of the interior solution $h^* = \frac{a + \tau\delta - c_2\beta + c_2\beta\rho\delta + c_1\beta(1-2\eta)}{2\beta(1-\eta)}$ contains $1-\eta$, and therefore becomes zero when $\eta = 1$. Hence, the interior equilibrium derived above is not defined at $\eta = 1$. Economically, $\eta = 1$ corresponds to full acquisition of the technology vendor by the manufacturer, rather than partial internal equity financing. In that case, the technology vendor no longer retains residual ownership or an independent Stackelberg-leader role. Therefore, this boundary case is outside the scope of the IEF model.

Combining the three stages, the retailer's profit function is strictly concave in p, the manufacturer's reduced profit function is strictly concave in w, and the technology vendor's reduced profit function is strictly concave in h for $0 \le \eta \le \bar{\eta} < 1$. Hence, each stage yields a unique best response. Since the

backward-induction procedure involves only these unique best responses, it is well defined. Under the admissible parameter region in which $p^* \geq 0, w^* \geq 0, h^* \geq 0,$ *and* $q^* = a - \beta p^* + \tau\delta > 0,$, the IEF pricing subgame admits a unique subgame-perfect equilibrium.

Appendix D. The proof of proposition 4

Taking the partial derivative of the retail price with respect to equation (23) in the main text, and knowing that $\frac{\partial^2 \pi_{r,4}}{\partial^2 p} = -2\beta < 0$, it can be concluded that the retailer's profit function is concave in terms of p, and there exists an optimal value for p. Setting

$$\frac{\partial \pi_{r,4}}{\partial p} = a - 2\beta p + \tau\delta + \beta w = 0, \tag{D1}$$

gives the retail price corresponding to the maximum profit:

$$p = \frac{a + \tau\delta + w\beta}{2\beta}. \tag{D2}$$

Substitute equation (20) into equation (22) in the main text, and calculate the partial derivative of equation (22) with respect to the wholesale price. Knowing that $\frac{\partial^2 \pi_{m,4}}{\partial^2 w} = -\beta < 0$, it can be concluded that the manufacturer's profit function is concave in terms of w, and there exists an optimal value for w. Setting

$$\frac{\partial \pi^*_{m,4}}{\partial w} = \frac{a + \tau\delta + c_2\beta - c_2\beta\rho\delta + (1 - \phi + \mu\phi - \varphi\eta)h\beta}{2} + \frac{\left[\eta\varphi + r_2(\phi + \varphi - 1)\right]c_1\beta - \beta w}{2}, \tag{D3}$$

gives the wholesale price corresponding to the maximum profit:

$$w = \frac{a + \tau\delta + c_2\beta - c_2\beta\rho\delta + (1 - \phi + \mu\phi - \varphi\eta)h\beta + \left[\eta\varphi + r_2(\phi + \varphi - 1)\right]c_1\beta}{2\beta}. \tag{D4}$$

Substitute equation (20) and (22) into equation (21) in the main text, and calculate the partial derivative of equation (21) with respect to the technology fee. Knowing that $\frac{\partial^2 \pi_{t,4}}{\partial^2 h} = -\frac{\beta(1 - \phi + \mu\phi - \varphi\eta)^2}{2} < 0$, it can be concluded that the technology vendor's reduced profit function is strictly concave in h, and there exists an optimal value for h. Setting

$$\frac{\partial \pi^*_{t,4}}{\partial h} = \frac{(1 - \phi + \mu\phi - \eta\varphi)\begin{bmatrix} a + c_1\beta + \tau\delta + c_1\beta(2r_2 - 2\eta\varphi + \phi r_1 - 2\phi r_2 - 2\varphi r_2) \\ -c_2\beta + c_2\beta\rho\delta - (1 - \phi + \mu\phi - \eta\varphi)2h\beta \end{bmatrix}}{4}, \tag{D5}$$

gives the technology fee corresponding to the maximum profit:

$$h = \frac{a + \tau\delta + c_1\beta - c_2\beta + c_2\beta\rho\delta + c_1\beta(2r_2 - 2\eta\varphi + \phi r_1 - 2\phi r_2 - 2\varphi r_2)}{2\beta(1 - \phi + \mu\phi - \eta\varphi)}. \tag{D6}$$

Appendix E. The proof of proposition 5

1. According to

$$\frac{\partial \pi_{m,1}^{*}}{\partial \delta}=\frac{\left(\tau+c_2\beta\rho\right)\left[a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\left(1+r_1\right)\right]}{16\beta}, \tag{E1}$$

$$\frac{\partial \pi_{r,1}^{*}}{\partial \delta}=\frac{\left(\tau+c_2\beta\rho\right)\left[a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\left(1+r_1\right)\right]}{32\beta}, \tag{E2}$$

So, when

$$a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\left(1+r_1\right)>0, \tag{E3}$$

$\pi_{m,1}^{*}$ and $\pi_{r,1}^{*}$ increase as δ increases.

According to

$$\frac{\partial \pi_{t,1}^{*}}{\partial \delta}=\frac{\left(\tau+c_2\beta\rho\right)\left[a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\left(1+r_1\right)\right]}{8\beta}-n\delta\left(1+r_1\right), \tag{E4}$$

So, when

$$\left(\tau+c_2\beta\rho\right)\left[a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\left(1+r_1\right)\right]-8\beta n\delta\left(1+r_1\right)>0, \tag{E5}$$

$\pi_{t,1}^{*}$ increases as δ increases.

2. Due to $\frac{\partial \pi_{t,1}^{*}}{\partial n}=-\frac{\left(1+r_1\right)\delta^2}{2}<0$, so, $\pi_{t,1}^{*}$ decreases as n increases.

3. Let

$$\Omega_1=a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\left(1+r_1\right), \tag{E6}$$

$$\Omega_2=\left(\tau+c_2\beta\rho\right)\left[a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\left(1+r_1\right)\right]-8\beta n\delta\left(1+r_1\right). \tag{E7}$$

Appendix F. The proof of proposition 6

1. According to

$$\frac{\partial \pi_{m,2}^{*}}{\partial \delta}=\frac{\left(\tau+c_2\beta\rho\right)\left(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\right)}{16\beta}+n\delta r_2, \tag{F1}$$

So, when

$$\left(\tau+c_2\beta\rho\right)\left(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\right)+16\beta n\delta r_2>0, \tag{F2}$$

$\pi_{m,2}^{*}$ increases as δ increases.

According to

$$\frac{\partial \pi_{r,2}^{*}}{\partial \delta}=\frac{\left(\tau+c_2\beta\rho\right)\left(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\right)}{32\beta}, \tag{F3}$$

So, when

$$a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta>0, \tag{F4}$$

$\pi_{r,2}^*$ increases as δ increases.

According to

$$\frac{\partial\pi_{t,2}^*}{\partial\delta}=\frac{(\tau+c_2\beta\rho)(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta)}{8\beta}-n\delta(1+r_2), \tag{F5}$$

So, when

$$(\tau+c_2\beta\rho)(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta)-8\beta n\delta(1+r_2)>0, \tag{F6}$$

$\pi_{t,2}^*$ increases as δ increases.

2. Due to $\frac{\partial\pi_{m,2}^*}{\partial n}=\frac{r_2\delta^2}{2}>0$, so, $\pi_{m,2}^*$ increases as n increases. Due to $\frac{\partial\pi_{t,2}^*}{\partial n}=-\frac{(1+r_2)\delta^2}{2}<0$, so, $\pi_{t,2}^*$ decreases as n increases.

3. Let

$$\Omega_3=(\tau+c_2\beta\rho)(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta)+16\beta n\delta r_2, \tag{F7}$$

$$\Omega_4=a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta, \tag{F8}$$

$$\Omega_5=(\tau+c_2\beta\rho)(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta)-8\beta n\delta(1+r_2). \tag{F9}$$

Appendix G. The proof of proposition 7

1. According to

$$\frac{\partial\pi_{m,3}^*}{\partial\delta}=\frac{(\tau+c_2\beta\rho)(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta)}{16\beta}-n\delta\eta, \tag{G1}$$

So, when

$$(\tau+c_2\beta\rho)(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta)-16\beta n\delta\eta>0, \tag{G2}$$

$\pi_{m,3}^*$ increases as δ increases.

According to

$$\frac{\partial\pi_{r,3}^*}{\partial\delta}=\frac{(\tau+c_2\beta\rho)(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta)}{32\beta}, \tag{G3}$$

So, when

$$a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta>0, \tag{G4}$$

$\pi_{r,3}^*$ increases as δ increases.

According to

$$\frac{\partial \pi_{t,3}^{*}}{\partial \delta}=\frac{\left(\tau+c_2\beta\rho\right)\left(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\right)}{8\beta}-n\delta\left(1-\eta\right), \tag{G5}$$

So, when

$$\left(\tau+c_2\beta\rho\right)\left(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\right)-8\beta n\delta\left(1-\eta\right)>0, \tag{G6}$$

$\pi_{t,3}^{*}$ increases as δ increases.

2. Due to $\frac{\partial \pi_{m,3}^{*}}{\partial n}=-\frac{\eta\delta^2}{2}<0$, so, $\pi_{m,3}^{*}$ increases as n increases. Due to $\frac{\partial \pi_{t,3}^{*}}{\partial n}=-\frac{\left(1-\eta\right)\delta^2}{2}<0$, so, $\pi_{t,3}^{*}$ decreases as n increases.

3. Let

$$\Omega_6=\left(\tau+c_2\beta\rho\right)\left(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\right)-16\beta n\delta\eta, \tag{G7}$$

$$\Omega_7=a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta, \tag{G8}$$

$$\Omega_8=\left(\tau+c_2\beta\rho\right)\left(a+\tau\delta-c_2\beta+c_2\beta\rho\delta-c_1\beta\right)-8\beta n\delta\left(1-\eta\right). \tag{G9}$$

Appendix H. The proof of proposition 8

1. According to

$$\begin{aligned}\frac{\partial \pi_{m,4}^{*}}{\partial \delta}=&\frac{\left(\tau+c_2\beta\rho\right)\left(a+\tau\delta-c_1\beta-c_2\beta-c_1\beta\phi r_1+c_2\beta\rho\delta\right)}{16\beta}\\&-n\delta\left[\eta\varphi-\left(1-\phi-\varphi\right)r_2\right],\end{aligned} \tag{H1}$$

So, when

$$\begin{aligned}&\left(\tau+c_2\beta\rho\right)\left(a+\tau\delta-c_1\beta-c_2\beta-c_1\beta\phi r_1+c_2\beta\rho\delta\right)\\&-16\beta n\delta\left[\eta\varphi-\left(1-\phi-\varphi\right)r_2\right]>0,\end{aligned} \tag{H2}$$

$\pi_{m,4}^{*}$ increases as δ increases.

According to

$$\frac{\partial \pi_{r,4}^{*}}{\partial \delta}=\frac{\left(\tau+c_2\beta\rho\right)\left(a+\tau\delta-c_1\beta-c_2\beta-c_1\beta\phi r_1+c_2\beta\rho\delta\right)}{32\beta}, \tag{H3}$$

So, when

$$\left(\tau+c_2\beta\rho\right)\left(a+\tau\delta-c_1\beta-c_2\beta-c_1\beta\phi r_1+c_2\beta\rho\delta\right)>0, \tag{H4}$$

$\pi_{r,4}^{*}$ increases as δ increases.

According to

$$\frac{\partial \pi_{t,4}^{*}}{\partial \delta}=\frac{\left(\tau+c_2\beta\rho\right)\left(a+\tau\delta-c_1\beta-c_2\beta-c_1\beta\phi r_1+c_2\beta\rho\delta\right)}{8\beta}-n\delta\left[1+\left(1-\phi-\varphi\right)r_2+\phi r_1-\eta\varphi\right], \tag{H5}$$

So, when

$$(\tau + c_2\beta\rho)(a + \tau\delta - c_1\beta - c_2\beta - c_1\beta\phi r_1 + c_2\beta\rho\delta) - 8\beta n\delta\left[1 + (1-\phi-\varphi)r_2 + \phi r_1 - \eta\varphi\right] > 0, \quad \text{(H6)}$$

$\pi_{t,4}^*$ increases as δ increases.

2. Due to

$$\frac{\partial \pi_{m,4}^*}{\partial n} = \frac{\delta^2\left[(1-\phi-\varphi)r_2 - \eta\varphi\right]}{2}, \quad \text{(H7)}$$

so, when

$$(1-\phi-\varphi)r_2 - \eta\varphi > 0, \quad \text{(H8)}$$

$\pi_{m,4}^*$ increases as n increases.

Due to

$$\frac{\partial \pi_{t,4}^*}{\partial n} = -\frac{\delta^2\left[1 + \phi r_1 + (1-\phi-\varphi)r_2 - \eta\varphi\right]}{2} < 0, \quad \text{(H9)}$$

so, $\pi_{t,4}^*$ decreases as n increases.

3. Due to

$$\frac{\partial \pi_{m,4}^*}{\partial \phi} = mr_2 - \frac{n\delta^2 r_2}{2} - \frac{c_1\beta^2 r_1\left[a + \tau\delta - c_1\beta(1+\phi r_1) - c_2\beta + c_2\beta\delta\rho\right]}{16}, \quad \text{(H10)}$$

So, when

$$16mr_2 - 8n\delta^2 r_2 - c_1\beta^2 r_1\left[a + \tau\delta - c_1\beta(1+\phi r_1) - c_2\beta + c_2\beta\delta\rho\right] > 0, \quad \text{(H11)}$$

$\pi_{m,4}^*$ increases as ϕ increases.

Due to

$$\frac{\partial \pi_{r,4}^*}{\partial \phi} = -\frac{c_1\beta^2 r_1\left[a + \tau\delta - c_2\beta + c_2\beta\rho\delta - c_1\beta(1+\phi r_1)\right]}{32}, \quad \text{(H12)}$$

So, when

$$-c_1\beta^2 r_1\left[a + \tau\delta - c_2\beta + c_2\beta\rho\delta - c_1\beta(1+\phi r_1)\right] > 0, \quad \text{(H13)}$$

$\pi_{r,4}^*$ increases as ϕ increases.

Due to

$$\frac{\partial \pi_{m,4}^*}{\partial \phi} = m(r_1 - r_2) - \frac{n\delta^2(r_1 - r_2)}{2} - \frac{c_1\beta^2 r_1\left[a + \tau\delta - c_1\beta(1+\phi r_1) - c_2\beta + c_2\beta\delta\rho\right]}{8}, \quad \text{(H14)}$$

So, when

$$(8m - 4n\delta^2)(r_1 - r_2) - c_1\beta^2 r_1\left[a + \tau\delta - c_1\beta(1+\phi r_1) - c_2\beta + c_2\beta\delta\rho\right] > 0, \quad \text{(H15)}$$

$\pi_{t,4}^*$ increases as ϕ increases.

4. Let

$$\begin{aligned}\Omega_9 &= \left(\tau + c_2\beta\rho\right)\left(a + \tau\delta - c_1\beta - c_2\beta - c_1\beta\phi r_1 + c_2\beta\rho\delta\right)\\ &\quad -16\beta n\delta\left[\eta\varphi - \left(1-\phi-\varphi\right)r_2\right],\end{aligned} \tag{H16}$$

$$\Omega_{10} = \left(\tau + c_2\beta\rho\right)\left(a + \tau\delta - c_1\beta - c_2\beta - c_1\beta\phi r_1 + c_2\beta\rho\delta\right), \tag{H17}$$

$$\begin{aligned}\Omega_{11} &= \left(\tau + c_2\beta\rho\right)\left(a + \tau\delta - c_1\beta - c_2\beta - c_1\beta\phi r_1 + c_2\beta\rho\delta\right)\\ &\quad -8\beta n\delta\left[1 + \left(1-\phi-\varphi\right)r_2 + \phi r_1 - \eta\varphi\right],\end{aligned} \tag{H18}$$

$$\Omega_{12} = \left(1-\phi-\varphi\right)r_2 - \eta\varphi, \tag{H19}$$

$$\Omega_{13} = 16mr_2 - 8n\delta^2 r_2 - c_1\beta^2 r_1\left[a + \tau\delta - c_1\beta\left(1+\phi r_1\right) - c_2\beta + c_2\beta\delta\rho\right], \tag{H20}$$

$$\Omega_{14} = -c_1\beta^2 r_1\left[a + \tau\delta - c_2\beta + c_2\beta\rho\delta - c_1\beta\left(1+\phi r_1\right)\right], \tag{H21}$$

$$\Omega_{15} = \left(8m - 4n\delta^2\right)\left(r_1 - r_2\right) - c_1\beta^2 r_1\left[a + \tau\delta - c_1\beta\left(1+\phi r_1\right) - c_2\beta + c_2\beta\delta\rho\right]. \tag{H22}$$

Appendix I. The proof of Figure 5.

$$\varphi_1 = 4.28\times10^{-3}\phi^2 - 0.99\phi - 1.02\times10^{-14}, \tag{I1}$$

$$\varphi_2 = 4.28\times10^{-3}\phi^2 - 0.99\phi + 0.99, \tag{I2}$$

$$\varphi_3 = 0.02\phi^2 - 10.02\phi + 4.72\times10^{-14}, \tag{I3}$$

$$\varphi_4 = 0.02\phi^2 - 10.02\phi + 1, \tag{I4}$$

$$\varphi_5 = -0.06\phi^2 + 31.98\phi - 1.3\times10^{-13}, \tag{I5}$$

$$\varphi_6 = -0.06\phi^2 + 31.98\phi + 1, \tag{I6}$$

$$\varphi_7 = 58.26 - 3.19\times10^{-3}(0.82\phi - 134)^2 - 1.02\phi, \tag{I7}$$

$$\varphi_8 = 57.26 - 3.19\times10^{-3}(0.82\phi - 134)^2 - 1.02\phi, \tag{I8}$$

$$\varphi_9 = 325.04 - 0.02(0.82\phi - 141.5)^2 - 2.34\phi, \tag{I9}$$

$$\varphi_{10} = 326.04 - 0.02(0.82\phi - 141.5)^2 - 2.34\phi, \tag{I10}$$

$$\varphi_{11} = 3.8\phi + 0.04(0.82\phi - 143)^2 - 911.9, \tag{I11}$$

$$\varphi_{12} = 3.8\phi + 0.04(0.82\phi - 143)^2 - 912.9. \tag{I12}$$

Appendix J. The proof of Figure 8.

$$\varphi_{13} = 4.06\times10^{-3}\phi^2 - 1.03\phi, \tag{J1}$$

$$\varphi_{14} = 4.11\times10^{-3}\phi^2 - 1.03\phi + 1, \tag{J2}$$

$$\varphi_{15} = 1.68\phi^2 - 875.15\phi + 2.91\times10^{-12}, \tag{J3}$$

$$\varphi_{16} = 1.68\phi^2 - 875.15\phi + 1, \tag{J4}$$

$$\varphi_{17} = -1.68\phi^2 - 877.39\phi + 1.09\times10^{-12}, \tag{J5}$$

$$\varphi_{18} = -1.68\phi^2 - 877.39\phi + 1, \tag{J6}$$

$$\varphi_{19} = 65.31 - 3.06\times10^{-3}(0.82\phi - 146)^2 - 1.01\phi, \tag{J7}$$

$$\varphi_{20} = 66.31 - 3.06\times10^{-3}(0.82\phi - 146)^2 - 1.01\phi, \tag{J8}$$

$$\varphi_{21} = 26644.99 - 1.25(0.82\phi - 146)^2 - 126.7\phi, \tag{J9}$$

$$\varphi_{22} = 26645.99 - 1.25(0.82\phi - 146)^2 - 126.7\phi, \tag{J10}$$

$$\varphi_{23} = 125.3\phi + 1.25(0.82\phi - 146)^2 - 26643.99, \tag{J11}$$

$$\varphi_{24} = 125.3\phi + 1.25(0.82\phi - 146)^2 - 26644.99. \tag{J12}$$

Appendix K.

We set $\frac{\sigma_\varepsilon}{a_0} = 0, 0.05, 0.10, 0.20, 0.30$ and conduct 3,000 simulations under $\delta = 0.2$.

Table 12. Feasible IPR share under alternative demand volatility.

Demand volatility	Mean $\bar{\phi}$	Median $\bar{\phi}$	95% interval	$P(\bar{\phi} < 0.5)$
0	0.7391	0.72	[0.4600, 1.0000]	6.20%
0.05	0.7378	0.72	[0.4600, 1.0000]	6.30%
0.10	0.7345	0.72	[0.4400, 1.0000]	7.10%
0.20	0.7210	0.70	[0.4000, 1.0000]	11.00%
0.30	0.7014	0.68	[0.3400, 1.0000]	16.83%

Appendix L.

Table 13. Monte Carlo convergence.

Simulations	**Median** δ^*	**Median** $n^*(1)$	**Median** $\bar{\phi}(0.2)$	$(P(\bar{\phi} < 0.5)$	**IPR–IEF intersection probability**
1,000	0.7759	421.11	0.725	8.50%	55.60%
3,000	0.7742	419.07	0.700	7.00%	53.20%
5,000	0.7747	418.98	0.725	6.76%	54.20%
10,000	0.7748	419.71	0.725	6.93%	54.57%

The estimates remain stable between 3,000 and 10,000 simulations. The main 5,000-sample analysis therefore provides numerically reliable results.



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