



Research article

Climate uncertainty and asset pricing decisions: A machine learning analysis

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Appendix A: Model results

A.1. Optimal decision

Dynamic programming in continuous time is solved by stochastic optimal control theory. The problem of welfare maximization for the social planner is:

$$J(A_t, Z_t, T_t, E_t) = \max_{L_{C,t}, L_{B,t}, L_{G,t}} \min_{\Lambda_{A,t}, \Lambda_{T,t}} E_t \left[\int_t^\infty \exp(-\rho s) \left\{ \rho \frac{C_s^{1-\eta}}{1-\eta} + \varrho_A \frac{1}{2} \Lambda'_{A,s} \Lambda_{A,s} + \varrho_T \frac{1}{2} \Lambda'_{T,s} \Lambda_{T,s} \right\} ds \right],$$

subject to technological production conditions given by

$$\begin{aligned} C_t &= \exp(-Z_t) Y_t = \exp(-Z_t) A_t (L_{C,t})^{1-\alpha-\nu} (B_t^\omega + G_t^\omega)^{\nu/\omega}, \\ B_t &= \bar{A}_B (L_{B,t})^{\alpha_B}, \\ G_t &= \bar{A}_G (L_{G,t})^{\alpha_G}, \\ L_t &= L_{C,t} + L_{B,t} + L_{G,t} = 1, \end{aligned}$$

and the stochastic processes in productivity, carbon stocks, temperature, and climate damage

$$dT_t = [\lambda B_t + \sigma'_T \Lambda_{T,t}] dt + \sigma'_T dW_t,$$

$$\begin{aligned}
dZ_t &= [\nabla D](T_t) [\lambda B_t + \sigma'_T \Lambda_{T,t}] dt + \frac{|\sigma_T|^2}{2} [\nabla^2 D](T_t) dt + \\
&\quad \sigma'_T [\nabla D](T_t) dW_t, \\
dE_t &= [\psi B_t - \phi E_t + \sigma'_T \Lambda_{T,t}] dt + \sigma'_T dW_t, \\
\frac{dA_t}{A_t} &= [\mu_A + \sigma'_A \Lambda_{A,t}] dt + \sigma'_A dW_t.
\end{aligned}$$

The HJB equation can be obtained by solving the social planner problem using stochastic optimal control theory:

$$\begin{aligned}
\rho J &= \frac{\rho}{1-\eta} \exp(-Z_t(1-\eta)) A_t^{1-\eta} \left(1 - B_t^{1/\alpha_B} A_B^{-1/\alpha_B} - G_t^{1/\alpha_G} A_G^{-1/\alpha_G}\right)^{(1-\alpha-\nu)(1-\eta)} (B_t^\omega + G_t^\omega)^{\frac{\nu(1-\eta)}{\omega}} \\
&\quad + \frac{1}{2} \varrho_A \Lambda'_{A,t} \Lambda_{A,t} + \frac{1}{2} \varrho_T \Lambda'_{T,t} \Lambda_{T,t} + (\mu_A A_t + \sigma_A \Lambda_{A,t} A_t) J_A + \frac{1}{2} \sigma_A^2 A_t^2 J_{AA} \\
&\quad + (\lambda B_t + \sigma_T \Lambda_{T,t}) J_T + \frac{1}{2} \sigma_T^2 J_{TT} + (\psi B_t - \phi E_t + \sigma_T \Lambda_{T,t}) J_E + \frac{1}{2} \sigma_T^2 J_{EE} \\
&\quad + [\nabla D](\lambda B_t + \sigma_T \Lambda_{T,t}) J_Z + \frac{1}{2} \sigma_T^2 [\nabla^2 D] J_Z + \frac{1}{2} \sigma_T^2 [\nabla D]^2 J_{ZZ},
\end{aligned}$$

where the value function and the controls rely on the state variables, and the expression for the function's state dependence is left out. Minimizing for the worst-case model distortion results in the following conditions:

$$\begin{aligned}
\Lambda_{A,t}^* &= -\frac{1}{\varrho_A} \sigma_A A_t J_A, \\
\Lambda_{T,t}^* &= -\frac{1}{\varrho_T} \sigma_T [J_T + J_E + [\nabla D](T_t) J_Z].
\end{aligned}$$

The worst-case distortion can be substituted into the equation to obtain:

$$\begin{aligned}
\rho J &= \frac{\rho}{1-\eta} \exp(-Z_t(1-\eta)) A_t^{1-\eta} \left(1 - B_t^{1/\alpha_B} A_B^{-1/\alpha_B} - G_t^{1/\alpha_G} A_G^{-1/\alpha_G}\right)^{(1-\alpha-\nu)(1-\eta)} (B_t^\omega + G_t^\omega)^{\frac{\nu(1-\eta)}{\omega}} \\
&\quad - \phi E_t J_E + \mu_A A_t J_A + \frac{1}{2} \sigma_A^2 A_t^2 J_{AA} - \frac{1}{2\varrho_A} \sigma_A^2 A_t^2 J_A^2 - \frac{1}{2\varrho_T} \sigma_T^2 [J_T + J_E + [\nabla D] J_Z]^2 \\
&\quad + [\lambda J_T + [\nabla D] \lambda J_Z + \psi J_E] B_t + \frac{1}{2} \sigma_T^2 (J_{TT} + J_{EE} + [\nabla D]^2 J_{ZZ} + [\nabla^2 D] J_Z).
\end{aligned}$$

The first-order conditions for optimal B_t and G_t decisions are given below.

$$\begin{aligned}
0 &= \rho \exp(-Z_t(1-\eta)) A_t^{1-\eta} L_{C,t}^{(1-\alpha-\nu)(1-\eta)} (B_t^\omega + G_t^\omega)^{\frac{\nu(1-\eta)}{\omega}} \left\{ -\frac{(1-\alpha-\nu)}{\alpha_B A_B^{1/\alpha_B} L_{C,t}} B_t^{1/\alpha_B-1} + \frac{\nu}{(B_t^\omega + G_t^\omega)} B_t^{\omega-1} \right\} \\
&\quad + \lambda J_T + \lambda [\nabla D] J_Z + \psi J_E, \\
0 &= \rho \exp(-Z_t(1-\eta)) A_t^{1-\eta} L_{C,t}^{(1-\alpha-\nu)(1-\eta)} (B_t^\omega + G_t^\omega)^{\frac{\nu(1-\eta)}{\omega}} \left\{ -\frac{(1-\alpha-\nu)}{\alpha_G A_G^{1/\alpha_G} L_{C,t}} G_t^{1/\alpha_G-1} + \frac{\nu}{(B_t^\omega + G_t^\omega)} G_t^{\omega-1} \right\}.
\end{aligned}$$

A.2. Carbon tax

The decentralized mechanism of price generation in a decentralized market economy using a social planner framework is examined in this paper. The income from production in the polluting sector must be taxed in order to align the social planner's result with the competitive equilibrium result in a decentralized economy. It is first necessary to determine the significant effects of the decentralization mechanism in order to determine this ideal tax. Consequently, the first-order conditions of the firm's profit-maximization problem are directly calculated to determine prices. The equation for the final-product firm's profit-maximization problem is as follows:

$$\max_{L_{C,t}, B_t, G_t} \int_0^\infty \frac{\pi_t}{\pi_0} \left\{ A_t L_{C,t}^{1-\alpha-\nu} (B_t^\omega + G_t^\omega)^{\nu/\omega} - w_t L_{C,t} - P_{B,t} B_t - P_{G,t} G_t \right\} dt.$$

According to a derivation of the first-order condition, the following expression can be used to determine the wage w_t , the spot price of polluting inputs $P_{B,t}$, and the spot price of clean inputs $P_{G,t}$:

$$w_t = (1 - \alpha - \nu) \frac{Y_t}{L_{C,t}}, P_{B,t} = \nu \frac{Y_t}{B_t} \left(\frac{B_t^\omega}{B_t^\omega + G_t^\omega} \right), P_{G,t} = \nu \frac{Y_t}{G_t} \left(\frac{G_t^\omega}{B_t^\omega + G_t^\omega} \right).$$

After that, the decentralization framework's profit maximization problem for polluting input and clean input firms can be expressed as follows:

$$\max_{L_{B,t}} \int_0^\infty \frac{\pi_t}{\pi_0} \left\{ (1 - \tau) P_{B,t} A_B L_{B,t}^{\alpha_B} - w_t L_{B,t} \right\} dt, \quad \max_{L_{G,t}} \int_0^\infty \frac{\pi_t}{\pi_0} \left\{ P_{G,t} A_G L_{G,t}^{\alpha_G} - w_t L_{G,t} \right\} dt.$$

The input prices of clean and polluting input firms are derived from the first-order condition using the following expression:

$$P_{G,t} = \frac{w_t G_t^{\frac{1-\alpha_G}{\alpha_G}}}{\alpha_G \bar{A}_G^{1/\alpha_G}}, \quad P_{B,t} = \frac{1}{1 - \tau} \frac{w_t B_t^{\frac{1-\alpha_B}{\alpha_B}}}{\alpha_B \bar{A}_B^{1/\alpha_B}}.$$

The optimal choice of B_t in the decentralized framework is determined by the two expressions for $P_{B,t}$ in the joint decentralized firm problem, which satisfies the following expression:

$$0 = \nu \frac{B_t^{\omega-1}}{B_t^\omega + G_t^\omega} - \frac{(1 - \alpha - \nu) B_t^{\frac{1-\alpha_B}{\alpha_B}}}{\alpha_B A_B^{1/\alpha_B}} \frac{1}{L_{C,t} (1 - \tau)}.$$

Given the social planner problem's previous derivation, the socially optimal selection of B_t satisfies the following expression:

$$0 = \rho \exp(-Z_t(1 - \eta)) A_t^{1-\eta} L_{C,t}^{(1-\alpha-\nu)(1-\eta)} (B_t^\omega + G_t^\omega)^{\frac{\nu(1-\eta)}{\omega}} \left\{ \nu \frac{B_t^{\omega-1}}{(B_t^\omega + G_t^\omega)} - \frac{(1 - \alpha - \nu) B_t^{\frac{1-\alpha_B}{\alpha_B}}}{\alpha_B A_B^{1/\alpha_B}} \frac{1}{L_{C,t}} \right\} + \lambda J_T + \lambda [\nabla D] J_Z + \psi J_E.$$

The tax expression that follows can be obtained by solving τ for these expressions:

$$\tau = -(\lambda J_T + \lambda [\nabla D] J_Z + \psi J_E) / \left\{ \rho \exp(-Z_t(1 - \eta)) A_t^{1-\eta} L_{C,t}^{(1-\alpha-\nu)(1-\eta)} (B_t^\omega + G_t^\omega)^{\frac{\nu(1-\eta)}{\omega}-1} B_t^{\omega-1} \right\}.$$

A.3. SDF, risk-free rate, risk, and uncertainty prices

Under the CRRA utility, the stochastic discount factor (SDF) is given by the following equation:

$$\pi_t = \exp(-\rho t) \rho C_t^{-\eta} \zeta_{A,t} \zeta_{T,t},$$

where the adjustment terms $\zeta_{A,t}$ and $\zeta_{T,t}$ reflect the main distinctions from the conventional preference framework:

$$\begin{aligned} \zeta_{i,t} &= \exp\left(-\frac{1}{2} \int_0^t (\Lambda_{i,\tau}^*)' (\Lambda_{i,\tau}^*) d\tau + \int_0^t (\Lambda_{i,\tau}^*)' dW_\tau\right), \\ d\zeta_{i,t} &= \zeta_{i,t} (\Lambda_{i,t}^*)' dW_t. \end{aligned}$$

Taking the differential of the SDF's logarithmic transformation leads to the following expression:

$$d \ln(\pi_t) = -\rho dt - \eta d \ln(C_t) - \frac{1}{2} \left(|\Lambda_{A,t}^*|^2 + |\Lambda_{T,t}^*|^2 \right) + (\Lambda_{A,t}^* + \Lambda_{T,t}^*) dW_t.$$

Subsequently, we apply Ito's Lemma to expand the stochastic differential $d\pi_t/\pi_t$ and derive the expression for $C_t^{-\eta}$ through the following procedure:

$$\begin{aligned} \frac{d\pi_t}{\pi_t} &= -\rho dt - (\mu_A + \sigma_A \Lambda_{A,t}^*) dt - \eta (\lambda B_t + \sigma_T \Lambda_{T,t}^*) \frac{C_T}{C} dt - \frac{1}{2} \eta |\sigma_T|^2 \left(\frac{C_{TT}}{C} - (\eta + 1) \left(\frac{C_T}{C} \right)^2 \right) dt \\ &\quad - [\nabla D](T_t) (\lambda B_t + \sigma_T \Lambda_{T,t}^*) dt - (\psi B_t - \phi E_t + \sigma_T \Lambda_{T,t}^*) dt + \frac{1}{2} \sigma_T^2 dt + \frac{1}{2} \sigma_T^2 [[\nabla D](T_t)]^2 dt + \frac{1}{2} \sigma_A^2 dt \\ &\quad - \frac{1}{2} \sigma_T^2 [\nabla^2 D](T_t) dt + (\Lambda_{A,t}^* + \Lambda_{T,t}^*) dW_t - \eta \frac{C_T}{C} \sigma_T dW_t - (\sigma_T + \sigma_T [\nabla D](T_t) + \sigma_A) dW_t. \end{aligned}$$

The SDF requires systematic adjustment to incorporate climate model uncertainty, necessitating a probabilistic framework that accounts for structural differences across climate model ensembles:

$$\begin{aligned} \Lambda_{A,t}^* &= -\frac{1}{\varrho_A} \sigma_A A_t J_A, \\ \Lambda_{T,t}^* &= -\frac{1}{\varrho_T} \sigma_T [J_T + J_E + [\nabla D](T_t) J_Z]. \end{aligned}$$

From the above derivation, we obtain an explicit expression for the risk-free rate:

$$\begin{aligned} r_{f,t} &= \rho + \left(\mu_A - \frac{1}{\varrho_A} J_A |\sigma_A|^2 A_t \right) + \eta \left(\lambda B_t - \frac{1}{\varrho_T} [J_T + J_E + [\nabla D](T_t) J_Z] |\sigma_T|^2 \right) \frac{C_T}{C} \\ &\quad + \frac{1}{2} \eta |\sigma_T|^2 \left(\frac{C_{TT}}{C} - (\eta + 1) \left(\frac{C_T}{C} \right)^2 \right) + [\nabla D](T_t) \left(\lambda B_t - \frac{1}{\varrho_T} [J_T + J_E + [\nabla D](T_t) J_Z] |\sigma_T|^2 \right) \\ &\quad + \frac{1}{2} |\sigma_T|^2 [\nabla^2 D](T_t) + \left(\psi B_t - \phi E_t - \frac{1}{\varrho_T} [J_T + J_E + [\nabla D](T_t) J_Z] |\sigma_T|^2 \right) \\ &\quad - \frac{1}{2} |\sigma_T|^2 - \frac{1}{2} [[\nabla D](T_t)]^2 |\sigma_T|^2 - \frac{1}{2} |\sigma_A|^2 (A_t)^2, \end{aligned}$$

and derive the following expression for the risk price:

$$\begin{aligned}\sigma_{\pi,T} &= \sigma_T \left[\eta \frac{C_T}{C} + \frac{1}{\varrho_T} J_T \right], \\ \sigma_{\pi,Z} &= \sigma_T [\nabla D](T_t) \left[\frac{1}{\varrho_T} J_Z + 1 \right], \\ \sigma_{\pi,E} &= \sigma_T \left[\frac{1}{\varrho_T} J_E + 1 \right], \\ \sigma_{\pi,A} &= \sigma_A A_t \left[\frac{1}{\varrho_A} J_A + 1 \right].\end{aligned}$$

Appendix B: Calibration

B.1. Production parameters

For the calibration of production-side parameters (A_C, A_B, A_G), we employ the baseline model specification that excludes climate effects. In this framework, the social planner's optimization problem is characterized by the following Hamilton-Jacobi-Bellman (HJB) equation:

$$0 = -\rho J(A_t) + \rho \frac{C_t^{1-\eta}}{1-\eta} + \mu_A A_t J_A + \frac{1}{2} \sigma_A^2 A_t^2 J_{AA}.$$

The first-order optimality condition is given by:

$$\begin{aligned}B_t &= \left[\frac{1}{\left\{ \frac{(1-\alpha-\nu)}{\nu \tilde{A}_B^{1/\tilde{\alpha}}} \left(1 + \left(\frac{A_B}{A_G} \right)^{1/\tilde{\alpha}} \right)^{\frac{G\omega}{\tilde{\alpha}\omega-1}} \right\} + \left(\frac{1}{A_B} \right)^{1/\tilde{\alpha}} + \left(\frac{A_B}{A_G} \right)^{\frac{1}{\tilde{\alpha}\omega-1}} \left(\frac{1}{A_G} \right)^{1/\tilde{\alpha}}} \right]^{\tilde{\alpha}}, \\ G_t &= \left(\frac{A_B}{A_G} \right)^{\frac{\tilde{\alpha}}{\tilde{\alpha}\omega-1}} B_t.\end{aligned}$$

We adopt a combined empirical-theoretical calibration strategy to identify the productivity parameters A_B and A_G . First, we rely on empirical estimates of carbon emissions B_t and the share of renewable energy in total energy consumption. These empirical targets are matched to the model-implied expressions for B_t and G_t . We set the quarterly emissions target at $B_t = 4.0$. Data from the International Energy Agency (IEA) indicate that renewable energy accounts for approximately 10%–20% of global energy use. Consistent with these figures and the model's quarterly structure, we calibrate the renewable energy target at $G_t = 0.56$. Solving the resulting system of equilibrium conditions yields calibrated parameter values of $A_B = 100.43$ and $A_G = 53.40$.

To calibrate the parameter A_C , we use global economic output data obtained from the World Bank. According to the World Bank, annual global GDP is approximately USD 80 trillion. Converting annual GDP into quarterly terms by simple averaging yields an initial quarterly GDP of 20 trillion. Substituting these values into the production function yields the calibrated value of A_C :

$$A_C = \left(L_C^{-(1-\alpha-\nu)} \right) \left(Y * (B^\omega + G^\omega)^{-\frac{\nu}{\omega}} \right).$$

B.2. Preference parameters for recursive preferences

Within the recursive preference framework, calibration centers on two key parameters: the coefficient of relative risk aversion and the elasticity of intertemporal substitution (EIS). We set $\theta = 1$, consistent with empirical estimates of the EIS reported in the literature and with the findings of [1], who documents values close to unity. We then identify the coefficient of relative risk aversion using the SDF. In the benchmark model without climate factors, the HJB equation takes the form

$$\frac{d\pi}{\pi} = h_J dt - \xi \mu_A dt + \frac{1}{2} \sigma_A^2 \xi (1 + \xi) dt - \xi \sigma_A dW_t.$$

Using quarterly empirical estimates of the market price of risk, $MPR_{data} = 0.21$, and output volatility, $\sigma_{A,data} = 0.01$, the implied coefficient of relative risk aversion is

$$\xi = \frac{0.21}{0.01} = 21.$$

In the benchmark model without climate factors, substituting the value function $J = \frac{\xi_0 A^{1-\xi}}{1-\xi}$ into the HJB equation yields

$$r_f = \rho + \mu_A - \sigma_A^2 \xi.$$

Substituting the calibrated parameters ξ , μ_A , σ_A , and ρ gives

$$r_f = 0.00675,$$

which corresponds to an annualized rate of approximately 2.7%.

Appendix C: Results of parameter sensitivity

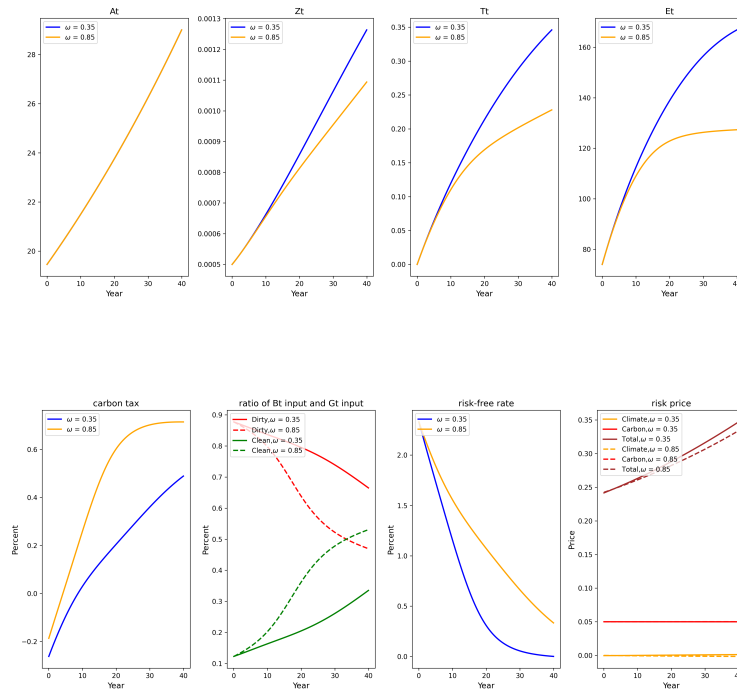


Figure 6. Effects of ω .

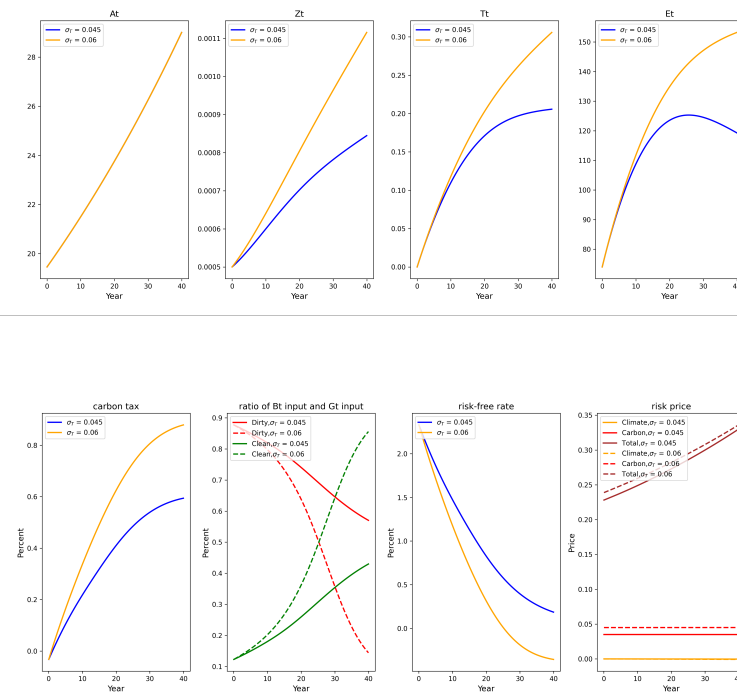


Figure 7. Effects of σ_T .

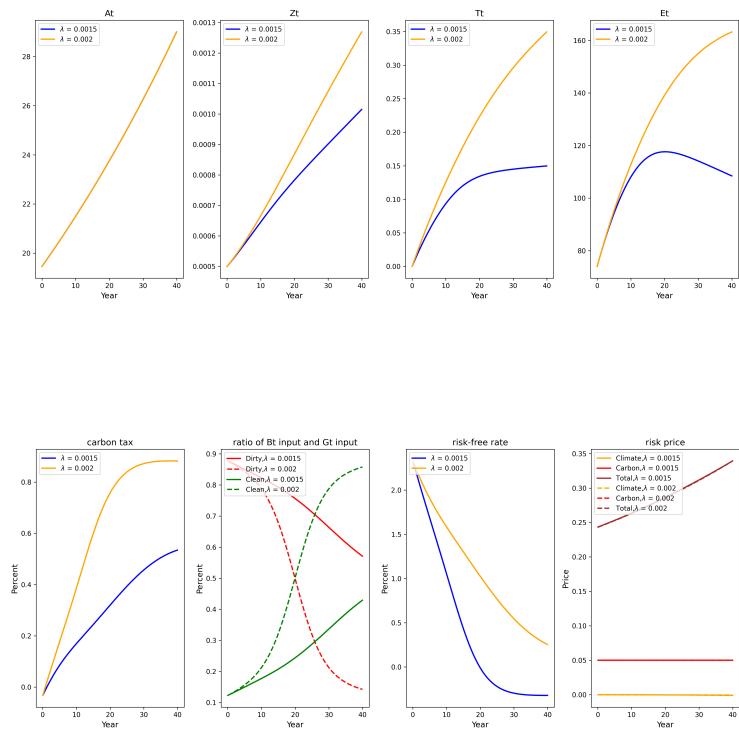


Figure 8. Effects of λ .

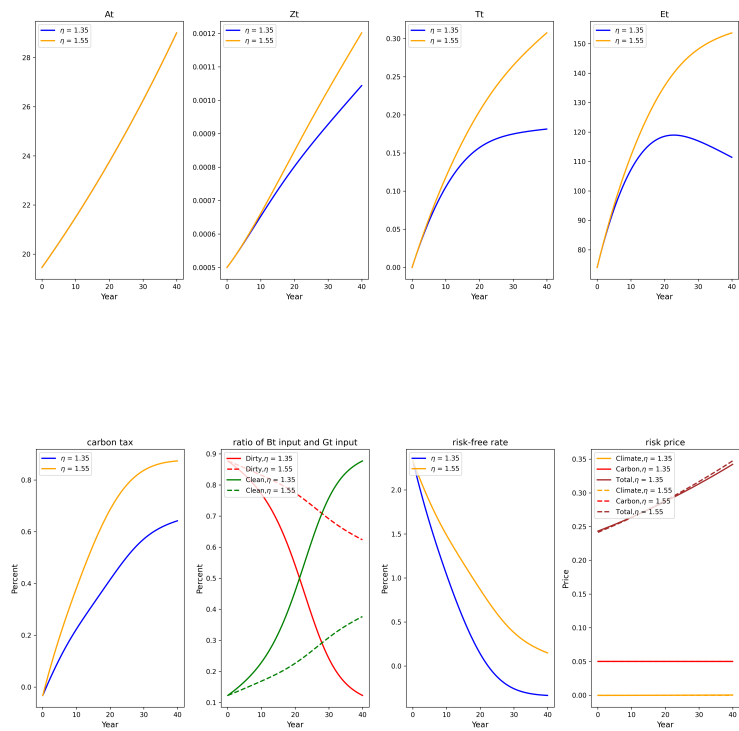


Figure 9. Effects of η .

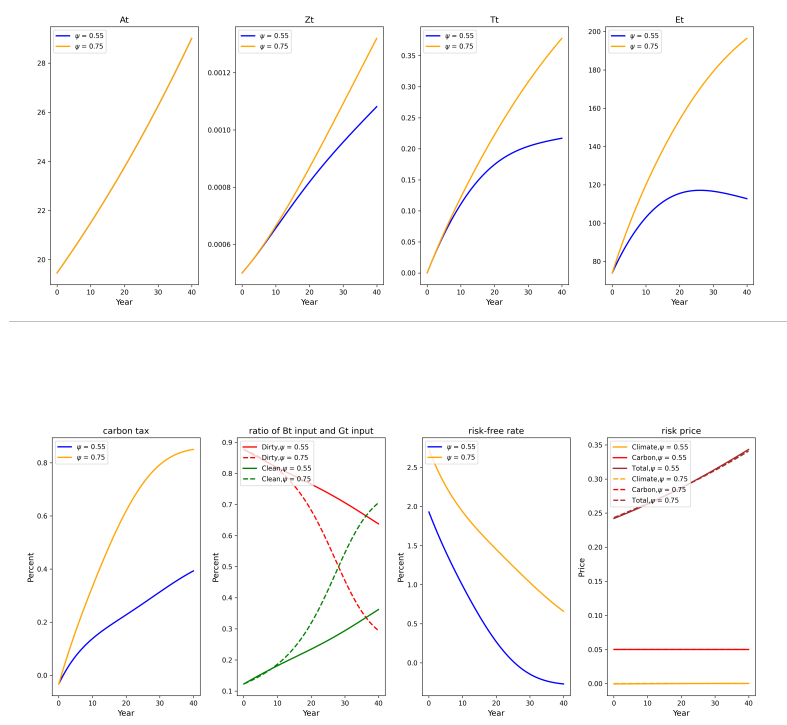


Figure 10. Effects of ψ .

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1. A. Vissing-Jørgensen, Limited asset market participation and the elasticity of intertemporal substitution, *J. Polit. Econ.*, **110** (2002), 825–853. <https://doi.org/10.3386/w8896>



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