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Research article

Aligning freight services and vehicle sales: Strategic cooperation for new energy commercial vehicle adoption

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Appendix

Proof of Table 3. (1) In Model T, $\frac{\partial \pi_{M(p_t)}}{\partial p_t} = a + n\lambda - 2ap_t$, and $\frac{\partial \pi^2{}_{M(p_t)}}{\partial (p_t)^2} = -2a < 0$. Thus, let $\frac{\partial \pi_{M(p_t)}}{\partial p_t} = 0$, we get $p_{t(\lambda)} = \frac{a+n\lambda}{2a}$. Replacing $p_{t(\lambda)} = \frac{a+n\lambda}{2a}$ into platform's profit function, then the second-order and first-order derivative of λ are $\frac{\partial \pi^2{}_{OFP(\lambda)}}{\partial (\lambda)^2} = n(-2+\beta) < 0$, and $\frac{\partial \pi_{OFP(\lambda)}}{\partial \lambda} = \frac{1}{2}n(2-\beta)(f-2\lambda) - \frac{a\beta}{2}$, respectively. Tus, let $\frac{\partial \pi_{OFP(\lambda)}}{\partial \lambda} = 0$, we obtain $\lambda^{T*} = \frac{1}{2}(f - \frac{a\beta}{n(2-\beta)})$. Replacing λ^{T*} into $p_{t(\lambda)}$, we can obtain $p_t^{T*} = \frac{1}{4}(2 + \frac{fn}{a} - \frac{\beta}{2-\beta})$, $d^{T*} = \frac{1}{4}(fn + a(3 - \frac{2}{2-\beta}))$, $\pi_M^{T*} = \frac{(fn(-2+\beta)+a(-4+3\beta))^2}{16a(-2+\beta)^2}$, $\pi_{OFP}^{T*} = 2a(\frac{1}{-2+\beta} + \frac{2fn}{-fn(-2+\beta)+a\beta})$. To satisfy the constraints in model T that

$\lambda^{T*} > 0,\ 0 < v_1 < 1,\ 0 < v_2 < 1,\ 0 < d_t < 1,\ p_t > 0$, we obtain Region 1: $-\frac{a\beta}{-2n+n\beta} = \underline{f}^T < f < \overline{f}^T = -\frac{3a\beta}{-2n+n\beta}$.

(2) The derivation of Model A is similar to Model T. To satisfy constraints that $\lambda^{A*} > 0,\ 0 < v_1 < 1,\ 0 < v_2 < 1,\ 0 < d_t < 1,\ p_t > 0$, we obtain the Region 2: $\max\{0, \frac{-2+\beta+2\theta}{2\theta}\} < a < 1$ and $\frac{\beta(-1+\theta-a\theta)}{n(-2+\beta+2(1-a)\theta)} = \underline{f}^A < f < \overline{f}^A =$ $\min\{\frac{\beta(6-3\beta+(1-a)(n-3(4-\beta))\theta-6(1-a)^2\theta^2)}{n(-2+\beta-2(-1+a)\theta)^2}, \frac{-4+\beta-(1-a)(-8+n+\beta)\theta-4(-1+a)^2\theta^2}{n(-2+\beta-2(-1+a)\theta)}\}$.

(3) In Model P, we obtain the first-order derivative of $\pi_{M(p_t)}$ with respect to p_t and w_o are $\frac{\partial\pi_{M(p_t)}}{\partial p_t} = \frac{-1+\tau-\beta m_o+2(1+(-1+\beta)\tau)p_t-2\beta w_o}{-1+\tau}$, $\frac{\partial\pi_{M(w_o)}}{\partial w_o} = \frac{n\lambda+\frac{\beta(m_o-2\tau p_t+2w_o)}{-1+\tau}}{\tau}$. The Hessian matrix is given by $H=\begin{pmatrix} \frac{2(1-(1-\beta)\tau)}{-1+\tau} & \frac{2\beta}{1-\tau} \\ \frac{2\beta}{1-\tau} & \frac{-2\beta}{(1-\tau)\tau} \end{pmatrix}$, and the sequential principal minors of Hessian matrix H is $\frac{2(1-(1-\beta)\tau)}{-1+\tau} < 0$, while $|H| = \frac{4\beta}{\tau-\tau^2} > 0$, which deduces the Hessian matrix H is negative definite. Hence, if there exists a maximum, this maximum uniquely solves the first-order optimality conditions (FOCs): $\frac{\partial\pi_{M(p_t)}}{\partial p_t} = 0$ and $\frac{\partial\pi_{M(w_o)}}{\partial w_o} = 0$. Solving the above equations simultaneously, we have $p_{t(\lambda)} = \frac{1}{2}(1 + n\lambda)$ and $w_{o(m_o,\lambda)} = \frac{\beta\tau+n\lambda(1+(-1+\beta)\tau)-\beta m_o}{2\beta}$. Then, substituting $p_{t(\lambda)}$ and $w_{o(m_o,\lambda)}$ into $\pi_{OFP(m_o,\lambda)}$, and $\frac{\partial^2\pi_{OFP(m_o,\lambda)}}{\partial(m_o)^2} = \frac{\beta}{(-1+\tau)\tau} < 0$, which implies $\pi_{OFP(m_o,\lambda)}$ is concave on m_o. Thus, according to $\frac{\partial\pi_{OFP(m_o,\lambda)}}{\partial m_o} = 0$, we obtain $m_{o(\lambda)} = \frac{(f\beta-(n+\beta)\lambda)(-1+\tau)}{2\beta}$. Then, $\frac{\partial^2\pi_{OFP(\lambda)}}{\partial(\lambda)^2} = \frac{(n-\beta)^2-(n^2+2n(3-2\beta)\beta+\beta^2)\tau}{4\beta\tau}$. To ensure $\frac{\partial^2\pi_{OFP(\lambda)}}{\partial(\lambda)^2} < 0,\ (n-\beta)^2-(n^2+2n(3-2\beta)\beta+\beta^2)\tau < 0$ should be satisified. In this context, we can obtain $\lambda^{P*} = \frac{\beta(2\beta\tau+f(\beta-\beta\tau+n(-1-3\tau+2\beta\tau)))}{(n-\beta)^2-(n^2+2n(3-2\beta)\beta+\beta^2)\tau}$, replacing λ^{P*} into response functions above, we can obtain the equilibrium results, as shown in Table 3. Among that, A=$\beta^4(1-\tau)(1-(1-\beta)\tau)-2n\beta^3(1-\tau)(2+6\tau-5\beta\tau-f\beta(1-(1-\beta)\tau))+n^2\beta^2(6+f^2\beta(1-\tau)(\beta+(4-\beta)(1-\beta)\tau)-2f\beta(1-\tau)(3+9\tau-8\beta\tau)+\tau(20-23\beta+(38-\beta(73-36\beta))\tau))+n^4((1-\tau)^2-2f\beta(1-\tau)(1+3\tau-2\beta\tau)+f^2\beta(\beta+(4+(2-3\beta)\beta)\tau-(1-\beta)(4-\beta(9-4\beta))\tau^2))+2n^3\beta(2(1-\tau)(-1-3(1-\beta)\tau)-f^2\beta(1-\tau)(\beta-(1-\beta)(4-3\beta)\tau)+f\beta(3+\tau(8-9\beta+(21-\beta(31-12\beta))\tau)))$. To satisfy the constraints that $\lambda^{P*} > 0,\ 0 < v_1 < 1,\ 0 < v_3 < v_2 < 1,\ 0 < d_t + d_o < 1,\ p_t > 0,\ p_o > 0$, we obtain Region 3: $\frac{\beta}{4-2\beta} < n \le \beta$, $\max\left\{\frac{n-2\beta}{-3n-2\beta+2n\beta}, \frac{n-\beta}{-3n+2n\beta}\right\} < \tau < 1$, $f <$

$\max\left\{\frac{n^2-2n\beta+\beta^2-n^2\tau-3n\beta\tau+2n\beta^2\tau}{2n^2-2n\beta-n^2\beta+n\beta^2+4n^2\tau-6n^2\beta\tau+2n^2\beta^2\tau}, \frac{-n^2+2n\beta-\beta^2+n^2\tau+9n\beta\tau+2\beta^2\tau-6n\beta^2\tau}{2n^2-2n\beta-n^2\beta+n\beta^2+4n^2\tau-6n^2\beta\tau+2n^2\beta^2\tau}\right\}$, and $\frac{-2\beta\tau}{-n+\beta-3n\tau-\beta\tau+2n\beta\tau} = \underline{f}^P < f < \frac{-n^2+3n\beta-2\beta^2+n^2\tau+9n\beta\tau+2\beta^2\tau-6n\beta^2\tau}{4n^2-2n^2\beta+4n^2\tau-6n^2\beta\tau+2n^2\beta^2\tau}$; or $\beta < n < 1$, $\max\left\{\frac{n^2-2n\beta+\beta^2}{n^2+6n\beta+\beta^2-4n\beta^2}, \frac{n-2\beta}{-3n-2\beta+2n\beta}\right\} < \tau < 1$, and $\underline{f}^P < f < \frac{-n^2+3n\beta-2\beta^2+n^2\tau+9n\beta\tau+2\beta^2\tau-6n\beta^2\tau}{4n^2-2n^2\beta+4n^2\tau-6n^2\beta\tau+2n^2\beta^2\tau}$, $\overline{f}^P = \min\{\frac{-n^2+3n\beta-2\beta^2+n^2\tau+9n\beta\tau+2\beta^2\tau-6n\beta^2\tau}{4n^2-2n^2\beta+4n^2\tau-6n^2\beta\tau+2n^2\beta^2\tau}, \max\{\frac{n^2-2n\beta+\beta^2-n^2\tau-3n\beta\tau+2n\beta^2\tau}{2n^2-2n\beta-n^2\beta+n\beta^2+4n^2\tau-6n^2\beta\tau+2n^2\beta^2\tau}, \frac{-n^2+2n\beta-\beta^2+n^2\tau+9n\beta\tau+2\beta^2\tau-6n\beta^2\tau}{2n^2-2n\beta-n^2\beta+n\beta^2+4n^2\tau-6n^2\beta\tau+2n^2\beta^2\tau}\}\}$, $\underline{\tau}^P = \max\{\frac{n-2\beta}{-3n-2\beta+2n\beta}, \frac{n-\beta}{-3n+2n\beta}, \frac{n^2-2n\beta+\beta^2}{n^2+6n\beta+\beta^2-4n\beta^2}\}$.

(4) Similarly, in B=$(\beta^4(1-\tau)(1-(1-8\beta)\tau)-2n\beta^3(1-\tau)(2+14\tau-4\beta\tau+f\beta(-1-15\tau+8\beta\tau))+n^4((1-\tau)^2-2f\beta(1-\tau)(1+7\tau-4\beta\tau)+f^2\beta(\beta+2(16-9\beta)\tau-(32-\beta(81-16(4-\beta)\beta))\tau^2))+2n^3\beta(2(1-\tau)(-1-7\tau+6\beta\tau)-f^2\beta(1-\tau)(\beta-(32-3\beta(13-4\beta))\tau)+f\beta(3+5\tau(2+23\tau)-4\beta\tau(3+37\tau-12\beta\tau)))+n^2\beta^2(6-2f\beta(1-\tau)(3+21\tau-16\beta\tau)+f^2\beta(1-\tau)(\beta+(32+\beta(-33+8\beta))\tau)+2\tau(26+99\tau-4\beta(5+43\tau-18\beta\tau))))$. To satisfy the constraints, we obtain Region 4: $\frac{\beta}{4-2\beta} < n \le \beta$, $\max\{\frac{-n+3\beta}{7n+3\beta-4n\beta}, \frac{-n+\beta}{7n+3\beta-4n\beta}\} < \tau < 1$, $f < \max\{\frac{(n^2+4n(5-3\beta)\beta+3\beta^2)\tau-(n-\beta)^2}{n(2-\beta)(n-\beta-(\beta-n(5-4\beta))\tau)}, \frac{(n-\beta)^2-(n^2+4n(2-\beta)\beta+\beta^2)\tau}{n(2-\beta)(n-\beta-(\beta-n(5-4\beta))\tau)}\}$, and $\frac{4\beta\tau}{n+\beta(-1+\tau)+n(7-4\beta)\tau} = \underline{f}^M < f < \frac{n^2(-1+\tau)+3\beta^2(-1+\tau)+4n\beta(1+(5-3\beta)\tau)}{n(-2+\beta)(\beta(-1+\tau)+n(-3+(-5+4\beta)\tau))}$; or $\beta < n < 1$, $\max\{\frac{n-3\beta}{-7n-3\beta+4n\beta}, \frac{(n-\beta)^2}{n^2+2n(7-4\beta)\beta+\beta^2}\} < \tau < 1$, and $\underline{f}^M < f < \frac{n^2(1-\tau)+3\beta^2(1-\tau)-4n\beta(1+(5-3\beta)\tau)}{n(2-\beta)(\beta(-1+\tau)+n(-3-(5-4\beta)\tau))}$, $\overline{f}^M = \min\{\frac{n^2(-1+\tau)+3\beta^2(-1+\tau)+4n\beta(1+(5-3\beta)\tau)}{n(-2+\beta)(\beta(-1+\tau)+n(-3+(-5+4\beta)\tau))}, \max\{\frac{-(n-\beta)^2+(n^2+4n(5-3\beta)\beta+3\beta^2)\tau}{n(-2+\beta)(-n+\beta+(\beta+n(-5+4\beta))\tau)}, \frac{(n-\beta)^2+(-n^2+4n(-2+\beta)\beta+\beta^2)\tau}{n(-2+\beta)(-n+\beta+(\beta+n(-5+4\beta))\tau)}\}\}$, $\underline{\tau}^M = \max\{\frac{n-3\beta}{-7n-3\beta+4n\beta}, \frac{n-\beta}{-7n+\beta+4n\beta}, \frac{(n-\beta)^2}{n^2+2n(7-4\beta)\beta+\beta^2}\}$.

Proof of Corollary 1. (1) $\frac{\partial\lambda^{T*}}{\partial f} = \frac{1}{2} > 0$; $\frac{\partial p_t^{T*}}{\partial f} = \frac{n}{4a} > 0$; $\frac{\partial d_t^{T*}}{\partial f} = \frac{n}{4} > 0$; $\frac{\partial\pi_M^{T*}}{\partial f} = \frac{1}{8}n(3+\frac{fn}{a}+\frac{2}{-2+\beta}) > 0$; $\frac{\partial\pi_{OFP}^{T*}}{\partial f} = \frac{fn(2-\beta)+a\beta}{4} > 0$; (2) $\frac{\partial\lambda^{T*}}{\partial n} = -\frac{a\beta}{2n^2(-2+\beta)} > 0$; $\frac{\partial p_t^{T*}}{\partial n} = \frac{f}{4a} > 0$; $\frac{\partial d_t^{T*}}{\partial n} = \frac{f}{4} > 0$; $\frac{\partial\pi_M^{T*}}{\partial n} = \frac{1}{8}n(3+\frac{fn}{a}+\frac{2}{-2+\beta}) > 0$; $\frac{\partial\pi_{OFP}^{T*}}{\partial n} = \frac{-f^2n^2(-2+\beta)^2+a^2\beta^2}{8n^2(-2+\beta)} > 0$; (3) $\frac{\partial\lambda^{T*}}{\partial\beta} = -\frac{a}{n(-2+\beta)^2} < 0$; $\frac{\partial p_t^{T*}}{\partial\beta} = -\frac{1}{2(-2+\beta)^2} < 0$; $\frac{\partial d_t^{T*}}{\partial\beta} = -\frac{a}{2(-2+\beta)^2} < 0$; $\frac{\partial\pi_M^{T*}}{\partial\beta} = \frac{a(4-3\beta)-fn(-2+\beta)}{4(-2+\beta)^3} < 0$; $\frac{\partial\pi_{OFP}^{T*}}{\partial\beta} = -\frac{(a-fn)^2-\frac{4a^2}{(-2+\beta)^2}}{8n} > 0$.

Proof of Corollary 2. (1) $\frac{\partial\lambda^{A*}}{\partial f} = \frac{(1-(1-a)\theta)(2-\beta-2(1-a)\theta)}{4-2\beta-(1-a)(8-n-2\beta)\theta+4(1-a)^2\theta^2} > 0$; $\frac{\partial p_t^{A*}}{\partial f} = \frac{-n(-2+\beta-2(-1+a)\theta)}{8-4\beta(1-(1-a)\theta)-2(1-a)\theta(8-n-4\theta+4a\theta)} > 0$; $\frac{\partial d_t^{A*}}{\partial f} = \frac{n(2-\beta-2(1-a)\theta)(1-2(1-a)\theta)}{8-4\beta(1-(1-a)\theta)-2(1-a)\theta(8-n-4\theta+4a\theta)}$, Since the denominator is positive, we need to determine the sign of the numerator $n(2-\beta-2(1-a)\theta)(1-2(1-a)\theta)$, thus when $-\frac{1}{2} < b <$

0 , that is, $a > \frac{-1+2\theta}{2\theta}$, $\frac{\partial d_t^{A*}}{\partial f} > 0$. $\frac{\partial \pi_M^{A*}}{\partial f} = \frac{n(-2+\beta+2(1-a)\theta)(1-(1-a)\theta)(-4-2fn+3\beta+fn\beta+(1-a)(8-n+2fn-3\beta)\theta-4(1-a)^2\theta^2)}{2(-4+2\beta+(-1+a)(-8+n+2\beta)\theta-4(-1+a)^2\theta^2)^2} > 0$; $\frac{\partial \pi_{OFP}^{A*}}{\partial f} = \frac{-fn(-2+\beta-2(-1+a)\theta)^2+\beta(-2+\beta+(-1+a)(-4+n+\beta)\theta-2(-1+a)^2\theta^2)}{2(-4+2\beta(1+(-1+a)\theta)+(-1+a)\theta(-8+n+4\theta-4a\theta))} > 0.$

(2) i) $\frac{\partial \lambda^{A*}}{\partial n} = \frac{(1-(1-a)\theta)(2(-1+a)fn^2\theta(1-(1-a)\theta)-2(\beta-(1-a)\beta\theta)^2+\beta(4-(1-a)\theta(12-2n-fn^2-2(1-a)(6-n)\theta+4(1-a)^2\theta^2)))}{n^2(2(-2+\beta)+(-1+a)(-8+n+2\beta)\theta-4(-1+a)^2\theta^2)^2}$, it is obvious that the denominator of $\frac{\partial \lambda^{A*}}{\partial n}$ is positive. Next, let $g_{1(f)} = (1-(1-a)\theta)(2(-1+a)fn^2\theta(1-(1-a)\theta)-2(\beta-(1-a)\beta\theta)^2+\beta(4-(1-a)\theta(12-2n-fn^2-2(1-a)(6-n)\theta+4(1-a)^2\theta^2)))$, $g_{1(f)}$ is a linear function of f, where the coefficient of the first-order term is $(1-(1-a)\theta)\Big(\big((1-a)n^2\beta\theta+2(-1+a)n^2\theta(1+(-1+a)\theta)\big)\Big) < 0$, and the constant term is $(4\beta-12(1-a)\beta\theta+2(1-a)n\beta\theta+2(1-a)^2(6-n)\beta\theta^2-4(1-a)^3\beta\theta^3-2(\beta-(1-a)\beta\theta)^2) > 0$. Considering the range of values of f , $\frac{-\beta+\beta\theta-a\beta\theta}{-2n+n\beta+2n\theta-2an\theta} < f < \min\{\frac{-4+\beta+(-1+a)(-8+n+\beta)\theta-4(-1+a)^2\theta^2}{n(-2+\beta-2(-1+a)\theta)}, \frac{\beta(6-3\beta-(-1+a)(n+3(-4+\beta))\theta+6(-1+a)^2\theta^2)}{n(-2+\beta-2(-1+a)\theta)^2}\}$, let $b = (1-a)\theta$, $0 < b < \frac{2-\beta}{2}$, we obtain $g_{1(f=\frac{-\beta+\beta\theta-a\beta\theta}{-2n+n\beta+2n\theta-2an\theta})} = \beta(1-b)^2\big(4-2\beta(1-b)-b(8-n-4b)\big) > 0$, then $g_{1(f=\overline{f}^A)} =$

$$\begin{cases} (1-b)(\beta-(n+\beta)b)(4-2\beta-(8-n-2\beta)b+4b) & \overline{f}^A = \frac{\beta+(8-n-\beta)b-4b^2-4}{n(2-\beta-2b)} \\ \frac{\beta(1-b)(2(-2+\beta)^2-b(16+n-4\beta)(2-\beta)+(48-n^2+n(4-\beta)-2(12-\beta)\beta)b^2-2(16+n-4\beta)b^3+8b^4)}{2-\beta-2b} & \overline{f}^A = \frac{\beta(6-3\beta+(n-3(4-\beta))b+6b^2)}{n(2-\beta-2b)^2} \end{cases}$$, if $n > \frac{\beta^2}{-2+\beta}$ and $b > max\{\frac{4+n-\beta-\sqrt{8n+n^2-2n\beta+\beta^2}}{4}, \frac{\beta}{n+\beta}\}$, $g_{1(f=\overline{f^A})} > 0$, then, if $f < \frac{2\beta(1-b)(-2+\beta-(-4+n+\beta)b-2b^2)}{-n^2b(2-\beta-2b)}$, $g_{1(f)} < 0$.

$\frac{\partial p_t^{A*}}{\partial n} = \frac{(1-(1-a)\theta)((1-a)\beta\theta+2f(-2+\beta+2\theta-2a\theta)^2)}{2(2(-2+\beta)+(-1+a)(-8+n+2\beta)\theta-4(-1+a)^2\theta^2)^2} > 0$. $\frac{\partial d_t^{A*}}{\partial n} = \frac{(1-(1-a)\theta)(1-2(1-a)\theta)((1-a)\beta\theta-2f(2-\beta-2\theta+2a\theta)^2)}{2(-4+2\beta+(-1+a)(-8+n+2\beta)\theta-4(-1+a)^2\theta^2)^2}$, and $\frac{(1+(-1+a)\theta)(-(-1+a)\beta\theta+2f(-2+\beta+2\theta-2a\theta)^2)}{2(-4+2\beta+(-1+a)(-8+n+2\beta)\theta-4(-1+a)^2\theta^2)^2} > 0$, thus, if $a > \frac{-1+2\theta}{2\theta}$, $\frac{\partial d_t^{A*}}{\partial n} < 0$.

$\frac{\partial \pi_M^{A*}}{\partial n} = \frac{(1-(1-a)\theta)^2(4+2fn-3\beta-fn\beta-(1-a)(8-(1-2f)n-3\beta)\theta+4(1-a)^2\theta^2)((1-a)\beta\theta+2f(\beta+2\theta-2a\theta-2)^2)}{-2(-4+2\beta(1+(-1+a)\theta)+(-1+a)\theta(-8+n+4\theta-4a\theta))^3} > 0$;

$\frac{\partial \pi_{OFP}^{A*}}{\partial n} = \frac{(1-(1-a)\theta)\left(fn(2-\beta-2\theta+2a\theta)-\beta(1-\theta+a\theta)\right)}{2n^2(-4+2\beta+(-1+a)(-8+n+2\beta)\theta-4(-1+a)^2\theta^2)^2} > 0.$

Following a similar analytical procedure, the proofs of the subsequent corollaries are omitted for brevity, only the threshold conditions are presented.

Corollary 3. $f_{1P} = \min\{-\frac{\beta(n+\beta)\tau}{n(n-\beta)(-1+(-1+\beta)\tau)}, \underline{f}^P\}$, $\beta_{1M} = \frac{1+(5+n)\tau-\sqrt{1+10(1-n)\tau+(25-(2-n)n)\tau^2}}{6\tau}$, or $f_{1M} = \min\{\frac{-n\beta\tau-3\beta^2\tau}{-n^2+n\beta-n^2\tau+5n\beta\tau+n^2\beta\tau-3n\beta^2\tau}, \underline{f}^M\}$, $\beta_1 \in \{\beta_{1M}, 1\}$, $f_1 \in \{f_{1M}, f_{1P}\}$.

Corollary 5. $\tau_{1M} = \min\{1, \frac{n^2-2n\beta+\beta^2}{n^2+6n\beta+\beta^2-4n\beta^2}\}$, $\tau_1 \epsilon \{\underline{\tau}^P, \tau_{1M}\}$; $\tau_{2P} = \max\{0, \frac{-(n-\beta)(3n+\beta)}{2n\beta+\beta^2+n^2(5-4\beta)}\}$, $\tau_2 \epsilon \{\tau_{2P}, \underline{\tau}\}$;

$f_2 \epsilon \{\frac{n^2(-1+\tau)-2n\beta(-1+\tau)+\beta^2(-1+(-7+4\beta)\tau)}{(2-\beta)(2n\beta(1-\tau)+\beta^2(1-\tau)-n^2(3+(5-4\beta)\tau))}, \frac{n^2(-1+\tau)-2n\beta(-1+\tau)+\beta^2(-1+(-15+8\beta)\tau)}{(2-\beta)(2n\beta(1-\tau)+\beta^2(1-\tau)-n^2(3+(13-8\beta)\tau))}\}$.

ii) $\tau_{3P} = \frac{2n^3(-1+\beta)+n^2(-4+\beta)\beta+2n(3-2\beta)\beta^2+\beta^4}{2n^3(1-\beta)+n^2(4-\beta)\beta+\beta^4-2n\beta^2(5-2(3-\beta)\beta)}$, $f_{3P} = \frac{\beta(-2n^3(-1+\tau)+\beta^3(-1+\tau)-n^2\beta(5+3\tau)+4n\beta^2(1-(-3+\beta)\tau))}{n(2n^3(1-\beta)(1-\tau)+n^2(4-\beta)\beta(1-\tau)-\beta^4(1-\tau)-2n\beta^2(3-2\beta+(5-2(3-\beta)\beta)\tau))}$, $f_{3M} = \frac{\beta(-2n^3(-1+\tau)+\beta^3(-1+\tau)-n^2\beta(5+11\tau)+4n\beta^2(1+(7-2\beta)\tau))}{n(2n^3(1-\beta)(1-\tau)+n^2(4-\beta)\beta(1-\tau)-\beta^4(1-\tau)-2n\beta^2(3-2\beta+(13-2\beta(7-2\beta))\tau))}$, $\tau_3 \epsilon \{\tau_{3P}, \tau_{3M}\}$, $f_3 \epsilon \{f_{3P}, f_{3M}\}$.

Proof of Proposition 1. (1) $d^{T*} - d^{N*} = \frac{1}{4}\left(fn + \frac{a\beta}{-2+\beta}\right)$. Given the constraint $f > -\frac{a\beta}{-2n+n\beta}$, it follows that $d^{T*} > d^{N*}$. Similarly, it can be verified that $d^{i*} > d^{N*}, \{i = T, A, P, M\}$.

(2) $\frac{\pi_M^{T*}}{\pi_M^{N*}} = \frac{\left(fn(-2+\beta)+a(-4+3\beta)\right)^2}{4a^2(-2+\beta)^2}$. To determine the magnitude relationship, we compare the term $\frac{fn(-2+\beta)+a(-4+3\beta)}{2a(-2+\beta)}$. Based on the constraint conditions, it is readily seen that $\pi_M^{T*} > \pi_M^{N*}$. Similarly, it can be verified that $\pi_M^{i*} > \pi_M^{N*}$, $\{i = T, A, P, M\}$.

Proof of Proposition 2. (1) i) Let $h_1 = \lambda^{A*} - \lambda^{T*} = \frac{(1+(-1+a)\theta)(fn(-2+\beta)+\beta+(-1+a)(-2fn+\beta)\theta)}{n(-4+2\beta(1+(-1+a)\theta)+(-1+a)\theta(-8+n+4\theta-4a\theta))} - \frac{1}{2}(f + \frac{a\beta}{n(-2+\beta)})$, then $\frac{\partial h_1}{\partial f} = \frac{-(-1+a)n\theta}{-8+4\beta(1+(-1+a)\theta)+2(-1+a)\theta(-8+n+4\theta-4a\theta)} < 0$, thus h_1 is a decreasing function of f, and f should satisfy $f > \frac{-\beta+\beta\theta-a\beta\theta}{-2n+n\beta+2n\theta-2an\theta}$, $h_{1(f=\frac{-\beta+\beta\theta-a\beta\theta}{-2n+n\beta+2n\theta-2an\theta})} = \frac{(-1+a)\beta(2+\beta(-1+\theta)+2(-1+a)\theta)}{2n(-2+\beta)(-2+\beta-2(-1+a)\theta)} < 0$. Since the function value of h_1 at the left endpoint of the domain is negative, then $h_1 < 0$, $\lambda^{A*} < \lambda^{T*}$.

ii) Let $h_2 = \lambda^{P*} - \lambda^{T*} = \frac{\beta(2\beta\tau+f(\beta-\beta\tau+n(-1-3\tau+2\beta\tau)))}{(n-\beta)^2-(n^2+2n(3-2\beta)\beta+\beta^2)\tau} - \frac{f}{2} + \frac{a\beta}{2n(2-\beta)}$, h_2 is a linear function of f, and the constant term is $\beta\left(\frac{a}{4n-2n\beta} + \frac{2\beta\tau}{(n-\beta)^2-(n^2+2n(3-2\beta)\beta+\beta^2)\tau}\right) < 0$, When $n < \beta$, the coefficient

$\frac{(n-\beta)(n+\beta)(-1+\tau)}{2(n-\beta)^2-2(n^2+2n(3-2\beta)\beta+\beta^2)\tau} < 0$, and vice versa. Therefore, when $n > \beta$ and $f > f_{4P} = \frac{\beta(-a(n-\beta)^2+4n(-2+\beta)\beta\tau+a(n^2+2n(3-2\beta)\beta+\beta^2)\tau)}{n(-2+\beta)(-n^2+\beta^2)(-1+\tau)}$, $\lambda^{P*} > \lambda^{T*}$.

Similarly, there is f_{4M}, if $f > f_{4M} = \frac{\beta(a(n-\beta)^2+8n(2-\beta)\beta\tau-a(n^2+2n(7-4\beta)\beta+\beta^2)\tau)}{n(2-\beta)(n^2-\beta^2)(1-\tau)}$, $\lambda^{M*} > \lambda^{T*}$. Thus, if $n > \beta$ and $f > f_4$, $\lambda^{i*} > \lambda^{T*}$; otherwise, $\lambda^{i*} < \lambda^{T*}$ $\{i = P, M\}$, where $f_4 \epsilon \{f_{4P}, f_{4M}\}$.

(2) i) $d^{A*} - d^{T*} = \frac{n(2-\beta)\theta(4+n-2\beta-4(1-a)\theta)f-16+2(10-3\beta)\beta+4(-1+a)(-8+n)\theta+\beta(a(32-3n-6\beta)+2(-18+n+5\beta))\theta+4(-1+a)((-2+\beta)^2+a(-4+3\beta))\theta^2}{4(-2+\beta)(-4+2\beta(1+(-1+a)\theta)+(-1+a)\theta(-8+n+4\theta-4a\theta))}$, the denominator is positive. The numerator is a linear function of f, $n(2-\beta)\theta(4+n-2\beta-4(1-a)\theta) > 0$, and the constant term $-16+2(10-3\beta)\beta+4(1-a)(8-n)\theta+\beta\big(a(32-3n-6\beta)-2(18-n-5\beta)\big)\theta-4(1-a)\big((2-\beta)^2-a(4-\beta)\big)\theta^2 < 0$. Thus, $d^{A*} > d^{T*}$ if $f < f_5 = \min\{\overline{f}^A, \frac{-2(2-\beta)(4-3\beta)+\big(a(8-n-2\beta)(-4+3\beta)-2(2-\beta)(-8+n+5\beta)\big)\theta-4(1-a)((-2+\beta)^2+a(-4+3\beta))\theta^2}{n(-2+\beta)\theta(4+n-2\beta+4(-1+a)\theta)}\}$.

ii) $d^{P*} - d^{T*} = \frac{h_4}{4(\beta-2)(n^2-2n\beta+\beta^2-n^2\tau-6n\beta\tau-\beta^2\tau+4n\beta^2\tau)}$, $h_4 = n(2-\beta)(n+\beta)(n+(3-2\beta)\beta)(1-\tau)f+\beta^2\big(2(2-\beta)(1-\beta)+a(4-3\beta)\big)(1-\tau)+n^2\big(4-2\beta+a(4-3\beta)\big)(1-\tau)-2n\beta(a(4-3\beta)(1+3\tau-2\beta\tau)+(2-\beta)(2-\beta+6\tau-5\beta\tau))$, is a linear function about f, and coefficient $n(2-\beta)(n+\beta)(n+(3-2\beta)\beta)(1-\tau) > 0$. Due to the constraint $f > \frac{2\beta\tau}{n-\beta+3n\tau+\beta\tau-2n\beta\tau}$, $h_{4(f=\frac{2\beta\tau}{n-\beta+3n\tau+\beta\tau-2n\beta\tau})} = \frac{n^2(-1+\tau)+\beta^2(-1+\tau)+2n\beta(1+(3-2\beta)\tau))(-\beta(2(-2+\beta)(-1+\beta)+a(4-3\beta))(1-\tau)-2n(2-\beta)(1+3(1-\beta)\tau)+an(4-3\beta)(1+(3-2\beta)\tau)}{\beta-\beta\tau-n(1+(3-2\beta)\tau)} > 0$, thus, $d^{P*} > d^{T*}$. Similarly, $d^{M*} - d^{T*} = \frac{fn^3(2-\beta)(1-\tau)+\beta^2\big(4-4a-10\beta+3a\beta+4\beta^2\big)(-1+\tau)+n^2\big(4-2\beta-4f(-2+\beta)^2\beta+a(-4+3\beta)\big)(-1+\tau)+n\beta(-2a(-4+3\beta)(-1-7\tau+4\beta\tau)-(-2+\beta)(4+f\beta(-7+4\beta)(-1+\tau)+28\tau-4\beta(1+5\tau)))}{4(-2+\beta)(n^2-2n\beta+\beta^2-n^2\tau-14n\beta\tau-\beta^2\tau+8n\beta^2\tau)} > 0.$

Proof of Proposition 3. (1) i) $\frac{\pi_M^{A*}}{\pi_M^{T*}} - 1 = \frac{h_5}{(fn(-2+\beta)+a(-4+3\beta))^2(-4+2\beta+(-1+a)(-8+n+2\beta)\theta-4(-1+a)^2\theta^2)^2}$, $h_5 = A_1 f^2 + B_1 f + C_1$, and $A_1 = (-1+a)n^2(-2+\beta)^2(4(-2+\beta)^2-4(-2+\beta)(-8+n-a(-6+\beta)+2\beta)\theta-(-1+a)(96+n^2+4n(-4+\beta)+16a(-3+\beta)+4(-12+\beta)\beta)\theta^2+8(-1+a)^2(-8+2a+n+2\beta)\theta^3-16(-1+a)^3\theta^4) < 0$; $B_1 = 2(-1+a)an(-2+\beta)\theta\big(-(-1+a)n^2(-4+3\beta)\theta-8n(-1+\beta)(1+(-1+a)\theta)(-2+\beta+2\theta-2a\theta)+8\beta(1+(-1+a)\theta)^2(-2+\beta+2\theta-2a\theta)\big) < 0$; $C_1 = a(-2(4-3\beta)^2(4+a-2\beta)(-2+\beta)-(-1+a)(-4+3\beta)(a(-8+n+2\beta)(-4+3\beta)-4(-2+\beta)^2(-20+2n+9\beta))\theta+4(-1+a)^2(a(4-3\beta)^2+(-2+\beta)^2(160+n^2+12n(-2+\beta)+9\beta(-16+3\beta)))\theta^2+4(-1+a)^3(-2+\beta)^2(160+n^2+6n(-4+\beta)-96\beta+9\beta^2)\theta^3-32(-1+a)^4(-2+\beta)^2(-10+n+3\beta)\theta^4+64(-1+a)^5(-2+\beta)^2\theta^5) > 0$. Thus, if $f < f_{10}$ and $h_{5(fmin)} < 0$, then $\pi_M^{A*} > \pi_M^{T*}$.

$\pi_M^{A*} > \pi_M^{T*}$ if $f < f_6 = \frac{(2\sqrt{(a-(1-a)a\theta)(-8+10\beta-3\beta^2-(2-\beta)(-8+n+3\beta)\theta+a(-8+6\beta)\theta-4(-1+a)(-2+\beta)\theta^2)(-4+2\beta(1+(-1+a)\theta)-(1-a)\theta(-8+n+4\theta-4a\theta))+a\theta((1-a)n^2(-4+3\beta)\theta+8n(1-\beta)(1-(1-a)\theta)(-2+\beta+2\theta-2a\theta)+8\beta(1-(1-a)\theta)^2(-2+\beta+2\theta-2a\theta)))}{n(2-\beta)(4(2-\beta)^2-4(2-\beta)(8-n-a(6-\beta)-2\beta)\theta-(1-a)(96+n^2-4n(4-\beta)-16a(3-\beta)-4(12-\beta)\beta)\theta^2+8(1-a)^2(8-2a-n-2\beta)\theta^3+16(1-a)^3\theta^4)}$,

$\theta < \theta_1 = \frac{2-3\beta+\beta^2}{2-2\beta+\beta^2}$ and $a > a_1$, where a_1 is the first root of $(-2+\beta)^2\beta^2(-1+\theta)^2 - (-2+\beta)(6\beta^2(7-5\theta)+\beta^3(-9+\theta)-16\beta(-4+\theta)(-1+\theta)+32(-1+\theta)^2)(-1+\theta)a - 4\theta(-(4-3\beta)^2(-2+\beta)+(-64+\beta(96+\beta(-47+7\beta)))\theta+8(-2+\beta)^2\theta^2)a^2+4\theta^2(-(4-3\beta)^2+4(-2+\beta)^2\theta)a^3=0$.

ii) $\frac{\pi_{OFP}^{A*}}{\pi_{OFP}^{T*}} - 1 = \frac{h_6}{(fn(-2+\beta)-a\beta)^2(-4+2\beta(1+(-1+a)\theta)+(-1+a)\theta(-8+n+4\theta-4a\theta))}$, and $\big(-4+2\beta(1+(-1+a)\theta)+(-1+a)\theta(-8+n+4\theta-4a\theta)\big) < 0$, $h_6 = A_2f^2+B_2f+C_2$, $A_2 = (-1+a)n^2(-2+\beta)\theta(-(-2+\beta)(n+2\beta)+4(-1+a)\beta\theta) > 0$, $B_2 = 2(-1+a)n(-2+\beta)\beta\big(-4+2\beta(1+(-1+a)\theta)+\theta(8-8a-2n+an-4(-1+a)^2\theta)\big) < 0$, $C_2 = -2(-1+a^2)(-2+\beta)\beta^2 - (-1+a)(2(4+a^2(-4+\beta)-2\beta)\beta^2+n(-16+\beta(16+(-4+a^2)\beta)))\theta+2(-1+a)^2(-2(-8+n)n+(-12+n)n\beta+2(-1+a^2+n)\beta^2+\beta^3)\theta^2-8(-1+a)^3n(-2+\beta)\theta^3 < 0$. In view of the maximum value of f is $-\frac{3a\beta}{-2n+n\beta}$, then substituting f_{max} into h_6, and we find $h_{6(f_{max})} < 0$, thus, $\frac{\pi_{OFP}^{A*}}{\pi_{OFP}^{T*}} < 1$ always holds.

(2) In Model P, $\frac{\pi_M^{P*}}{\pi_M^{T*}} - 1 = \frac{h_7}{(fn(-2+\beta)+a(-4+3\beta))^2(n^2(-1+\tau)+\beta^2(-1+\tau)+2n\beta(1+(3-2\beta)\tau))^2}$, $h_7 = A_3a^2+B_3a+C_3$, where $A_3 = a^2(4-3\beta)^2(n^2(-1+\tau)+\beta^2(-1+\tau)+2n\beta(1+(3-2\beta)\tau))^2 < 0$, $C_3 = -f^2n^2(2-\beta)^2\big(n^2(-1+\tau)+\beta^2(-1+\tau)+2n\beta(1+(3-2\beta)\tau)\big)^2 < 0$, according to $a < 1$, we next discuss the value of $h_{7(a=1)} = (-1+\tau)((n-\beta)^4\beta(-8+5\beta)-(n-\beta)^2\beta(n^2(-8+5\beta)+\beta^2(8+\beta(-11+4\beta))-2n\beta(24+\beta(-35+12\beta)))\tau - f^2n^2(-2+\beta)^2(n+\beta)(-(n-3\beta)(n-\beta)^2+(n^3+n^2(11-8\beta)\beta+\beta^2(16+\beta(-19+4\beta))+n\beta(16+\beta(-25+12\beta)))\tau)+2fn(n-\beta)(2-\beta)(n^3(4-3\beta)(1-\tau)-n\beta^2(4+\beta+(12-\beta(17-4\beta))\tau)+\beta^3(4-\beta+(12-\beta(15-4\beta))\tau)-n^2\beta(4(1+7\tau)-\beta(5+43\tau-16\beta\tau)))) > 0$, thus, there must have a threshold regarding a, when $a < a_{2P}$, $h_7 < 0$, combine the definition domain, $a_{2P} = max\{\underline{a}, the\ first\ root\ of -\big(fn(2-\beta)+a(4-3\beta)\big)^2\big(n^2(1-\tau)+\beta^2(1-\tau)-2n\beta(1+(3-2\beta)\tau)\big)^2+4a(2-\beta)^2(\beta^4(1-\tau)(1-(1-\beta)\tau)+2n\beta^3(-1+\tau)(2+6\tau-5\beta\tau+f\beta(-1+(-1+\beta)\tau))+n^2\beta^2(6-f^2\beta(-1+\tau)(\beta+(4-\beta)(1-\beta)\tau)-2f\beta(1-\tau)(3+9\tau-8\beta\tau)+\tau(20-23\beta+(38-\beta(73-36\beta))\tau))+n^4((1-\tau)^2-2f\beta(1-\tau)(1+3\tau-2\beta\tau)+f^2\beta(\beta+(4+(2-3\beta)\beta)\tau+(-1+\beta)(4+\beta(-9+4\beta))\tau^2))-2n^3\beta(2(1-\tau)(1+3(1-\beta)\tau)-$

$f^2\beta(1-\tau)(\beta+(1-\beta)(4-3\beta)\tau)+f\beta(3+\tau(8-9\beta+(21-\beta(31-12\beta))\tau))))=0\}$. When $a<a_{2P}$, $\frac{\pi_M^{P*}}{\pi_M^{T*}}<1$.

Similarly, in Model M, when $a<a_{2M}$, $\frac{\pi_M^{M*}}{\pi_M^{T*}}<1$, $a_{2M}=max\{\underline{a}, the\ first\ root\ of -(fn(-2+\beta)+a(-4+3\beta))^2(n^2(-1+\tau)+\beta^2(-1+\tau)+2n\beta(1+(7-4\beta)\tau))^2+4a(-2+\beta)^2(-\beta^4(-1+\tau)(1+(-1+8\beta)\tau)+2n\beta^3(-1+\tau)(2+14\tau-4\beta\tau+f\beta(-1-15\tau+8\beta\tau))+n^4((-1+\tau)^2-2f\beta(-1+\tau)(-1-7\tau+4\beta\tau)+f^2\beta(\beta+2(16-9\beta)\tau+(-32+\beta(81+16(-4+\beta)\beta))\tau^2))+2n^3\beta(-2(-1+\tau)(-1-7\tau+6\beta\tau)-f^2\beta(-1+\tau)(-\beta+(32+3\beta(-13+4\beta))\tau)+f\beta(3+5\tau(2+23\tau)+4\beta\tau(-3-37\tau+12\beta\tau)))+n^2\beta^2(6-2f\beta(-1+\tau)(-3-21\tau+16\beta\tau)-f^2\beta(-1+\tau)(\beta+(32+\beta(-33+8\beta))\tau)+2\tau(26+99\tau+4\beta(-5-43\tau+18\beta\tau))))=0\}$, $a_2\epsilon\{a_{2P},a_{2M}\}$.

The proofs of the subsequent propositions follow similar analysis and are thus omitted; only the threshold conditions are presented.

Proposition 5. $f_{7P}=\frac{\beta(n+\sqrt{2}\sqrt{\frac{(2(-2+\beta)+(1-a)(8-n-2\beta)\theta-4(1-a)^2\theta^2)(n^2(2-\beta)^2+\beta^4+2n(2-\beta)\beta(2-\beta-2\theta+2a\theta))}{-2+\beta}})}{-n(-2+\beta)(n+2\beta)-4(-1+a)n\beta\theta}$, $\tau_{4P}=-\frac{N}{M}$, $f_7\epsilon\{f_{7P},f_{7M}\}$, $\tau_4\in\{\tau_{4P},\tau_{4M}\}$, $M=(n-\beta)^2\beta^2+2(-1+a)\big(n^3(-2+\beta)+\beta^4-n^2(-4+\beta)\beta(-7+6\beta)-n\beta^2(2+\beta+4\beta^2)\big)\theta+(1-a)^2\Big(n^4-8n^3(1-\beta)^2+\beta^4-8n(\beta^2+\beta^4)-2n^2\beta\big(56-\beta(47-8\beta)\big)\Big)\theta^2+4(1-a)^3n(n^2+2n(7-4\beta)\beta+\beta^2)\theta^3-2fn\beta((n-\beta)(2-\beta)(n+\beta)+(1-a)\big(n\beta^2-(4-\beta)\beta^2+n^2(4+6\beta-4\beta^2)\big)\theta-2(1-a)^2(n-\beta)(n+\beta)\theta^2)+f^2n^2((2-\beta)^2(n+\beta)^2-2(1-a)(2-\beta)(2\beta^2+n^2(2-\beta)(1+2\beta)-2n(2-\beta)\beta(1+2\beta))\theta+4(1-a)^2(n^2+\beta^2+2n\beta(1+(3-2\beta)\beta))\theta^2)$, $N=2fn\beta(n-\beta)((2-\beta)(n+\beta)-(1-a)(2n(2-\beta)+(4-\beta)\beta)\theta-2(1-a)^2(n+\beta)\theta^2)+f^2n^2(-(2-\beta)^2(n+\beta)^2+2(1-a)(2-\beta)(n^2(2-\beta)+2n(2-\beta)\beta+2\beta^2)\theta-4(1-a)^2(n^2+2n(1-\beta)\beta+\beta^2)\theta^2)+(n-\beta)^2(2(1-a)n\beta\theta(1-(1-a)\theta)-(\beta-(1-a)\beta\theta)^2-(1-a)n\theta(4-(1-a)\theta(8-n-4(1-a)\theta))$,

Proposition 9. $\tau_5=\frac{(n-\beta)^2(n^2(31-14\beta)\beta+7\beta^3-4n^3(3-2\beta)+2n((19-12\beta)\beta^2+\sqrt{16n^4(2-\beta)^2-8n^3(2-\beta)\beta^2+32(2-\beta)^2\beta^4+8n(2-\beta)\beta^3(4+\beta)+n^2\beta^2(64-(32-\beta)\beta)}))}{24n(3-2\beta)\beta^4+7\beta^5-4n^5(7-4\beta)-n^4\beta(137-4(49-16\beta)\beta)+2n^2\beta^3(77-94\beta+32\beta^2)+4n^3\beta^2(47-2\beta(29-8\beta))}$;

$f_8=\frac{(n-\beta)^5(4n+7\beta)-2(n-\beta)^3(4n^3+n^2(47-30\beta)\beta+2n(19-12\beta)\beta^2+7\beta^3)\tau+(n-\beta)(4n^5+87n^4\beta+4(103-15n)n^3\beta^2+2(93-284n)n^2\beta^3+4n(18+n(-55+48n))\beta^4+(7+16n(-3+4n))\beta^5)\tau^2}{n((n-\beta)^4(7(2-\beta)\beta+n(14-3\beta))-2(n-\beta)^2(7(2-\beta)\beta^3+n^3(14-3\beta(5-2\beta))+n^2\beta(82-\beta(61-6\beta))+n\beta^2(82-\beta(85-24\beta)))\tau+(7(2-\beta)\beta^5+2n^3\beta^2(3-2\beta)(58-41\beta)+n^5(14-27\beta+12\beta^2)+n\beta^4(150-167\beta-48\beta^2)+n^4\beta(150-\beta(323-(244-64\beta)\beta))+2n^2\beta^3(174-267\beta+2(75-16\beta)\beta^2))\tau^2)}$.

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