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Research article

Strategy selection of logistics services and live-streaming formats in the fresh agri-food supply chain

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Appendix A. Proofs

Proof of Lemma 4.1 and Theorem 4.2. According to Eq (3.1), the Hessian matrix of $\pi_S(p_s, k)$ can be divided.

$$H_1 = \begin{pmatrix} -\frac{2\beta}{\theta} & \frac{1}{\theta} \\ \frac{1}{\theta} & -\lambda \end{pmatrix} \tag{A.1}$$

Thus, $\pi_S(p_s, k)$ is a concave function with p_s and k only when $\lambda > \frac{1}{2\beta\theta}$. Let $\frac{\partial \pi_S}{\partial p_s} = 0$ and $\frac{\partial \pi_S}{\partial k} = 0$. We can get the best response function of the supplier as summarized in Eqs (A.2) and (A.3).

$$p_s(p_l, e) = \frac{(1-e)rp_c(\beta\theta\lambda - 1) + c_m(\beta\theta\lambda - 1) + \theta^2\lambda + \beta\theta\lambda p_l - p_l}{2\beta\theta\lambda - 1}, \tag{A.2}$$

$$k(p_l, e) = \frac{\theta - \beta(1-e)rp_c - \beta c_m - \beta p_l}{2\beta\theta\lambda - 1}. \tag{A.3}$$

By taking the first-order derivative of Eqs (A.2) and (A.3) with respect to p_l and e, we have $\frac{\partial p_s(p_l,e)}{\partial e} = -\frac{rp_c(\beta\theta\lambda-1)}{2\beta\theta\lambda-1}$, $\frac{\partial p_s(p_l,e)}{\partial p_l} = \frac{\beta\theta\lambda-1}{2\beta\theta\lambda-1}$ $\frac{\partial k(p_l,e)}{\partial e} = \frac{\beta rp_c}{2\beta\theta\lambda-1}$, and $\frac{\partial k(p_l,e)}{\partial p_l} = -\frac{\beta}{2\beta\theta\lambda-1}$.

As $\lambda > \frac{1}{2\beta\theta}$, it is easy to verify that (1) $\frac{\partial k(p_l,e)}{\partial e} > 0$ and $\frac{\partial k(p_l,e)}{\partial p_l} < 0$; (2) $\frac{\partial p_s(p_l,e)}{\partial e} < 0$ when $\theta > \frac{1}{\beta\lambda}$, $\frac{\partial p_s(p_l,e)}{\partial e} > 0$ when $\theta < \frac{1}{\beta\lambda}$; $\frac{\partial p_s(p_l,e)}{\partial p_l} > 0$ when $\theta > \frac{1}{\beta\lambda}$, $\frac{\partial p_s(p_l,e)}{\partial p_l} < 0$ when $\theta < \frac{1}{\beta\lambda}$.

Substituting Eqs (A.2) and (A.3) into Eq (3.2), the Hessian matrix of $\pi_L(p_l, e)$ can be derived.

$$H_2 = \begin{pmatrix} -\frac{2\beta^2\lambda}{2\beta\theta\lambda-1} & \frac{\beta^2\lambda rp_c}{2\beta\theta\lambda-1} \\ \frac{\beta^2\lambda rp_c}{2\beta\theta\lambda-1} & -\gamma \end{pmatrix} \tag{A.4}$$

Thus, $\pi_L(p_l, e)$ is a concave function with p_l and e only when $\gamma > \frac{\beta^2\lambda r^2 s^2}{2(2\beta\theta\lambda-1)}$. Let $\frac{\partial \pi_L}{\partial p_l} = 0$ and $\frac{\partial \pi_L}{\partial e} = 0$. We can get the optimal solutions p_l^* e^*. Substituting p_l^* and e^* into Eqs (A.2) and (A.3), we can get p_s^* and k^*. □

Proof of Proposition 4.3 and Corollary 4.4. By computing the first-order derivative of the equilibrium solution under Scenario S with respect to p_c, we have $\frac{\partial p_l^{S*}}{\partial p_c} = \frac{\gamma r(2\beta\theta\lambda-1)\big(2\beta\lambda r p_c(\theta-\beta c_l-\beta c_m)-\beta^2\lambda r^2 p_c^2-2\gamma(2\beta\theta\lambda-1)\big)}{\big(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\big)^2}$, $\frac{\partial e^{S*}}{\partial p_c} = \frac{\beta\lambda r\big(\beta^2\lambda r^2 p_c^2(\theta-\beta c_l-\beta c_m)-2\gamma(2\beta\theta\lambda-1)(2\beta r p_c-(\theta-\beta c_l-\beta c_m))\big)}{\big(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\big)^2}$, $\frac{\partial p_s^{S*}}{\partial p_c} = \frac{\gamma r(1-\beta\theta\lambda)\big(2\beta\lambda r p_c(\theta-\beta c_l-\beta c_m)-\beta^2\lambda r^2 p_c^2-2\gamma(2\beta\theta\lambda-1)\big)}{\big(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\big)^2}$, $\frac{\partial k^{S*}}{\partial p_c} = \frac{\beta\gamma r\big(2\beta\lambda r p_c(\theta-\beta c_l-\beta c_m)-\beta^2\lambda r^2 p_c^2-2\gamma(2\beta\theta\lambda-1)\big)}{\big(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\big)^2}$, $\frac{\partial D^{S*}}{\partial p_c} = \frac{\beta^2\gamma\lambda r\big(2\beta\lambda r p_c(\theta-\beta c_l-\beta c_m)-\beta^2\lambda r^2 p_c^2-2\gamma(2\beta\theta\lambda-1)\big)}{\big(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\big)^2}$, $\frac{\partial \pi_L^{S*}}{\partial p_c} = \frac{\beta\gamma\lambda r(\theta-\beta c_l-\beta c_m-\beta r p_c)(\beta\lambda r p_c(\theta-\beta c_l-\beta c_m)-2\gamma(2\beta\theta\lambda-1))}{\big(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\big)^2}$, $\frac{\partial \pi_S^{S*}}{\partial p_c} = \frac{\beta\gamma^2\lambda r(2\beta\theta\lambda-1)(\theta-\beta c_l-\beta c_m-\beta r p_c)\big(2\beta\lambda r p_c(\theta-\beta c_l-\beta c_m)-\beta^2\lambda r^2 p_c^2-2\gamma(2\beta\theta\lambda-1)\big)}{\big(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\big)^3}$.

(1) As $2\beta\theta\lambda - 1 > 0$, when $\gamma > \gamma_1$, we have $\frac{\partial p_l^{S*}}{\partial p_c} < 0$, $\frac{\partial p_s^{S*}}{\partial p_c} < 0$, $\frac{\partial k^{S*}}{\partial p_c} < 0$, $\frac{\partial D^{S*}}{\partial p_c} < 0$, and $\frac{\partial \pi_S^{S*}}{\partial p_c} < 0$; when $\gamma < \gamma_1$, we have $\frac{\partial p_l^{S*}}{\partial p_c} > 0$, $\frac{\partial p_s^{S*}}{\partial p_c} > 0$, $\frac{\partial k^{S*}}{\partial p_c} > 0$, $\frac{\partial D^{S*}}{\partial p_c} > 0$, and $\frac{\partial \pi_S^{S*}}{\partial p_c} > 0$.

(2) When $r > r_1$ and $\gamma > \gamma_2$, or when $r < r_1$ and $\gamma < \gamma_2$, we have $\frac{\partial e^{S*}}{\partial p_c} < 0$; otherwise, we have $\frac{\partial e^{S*}}{\partial p_c} > 0$.

(3) When $\gamma > \gamma_3$, we have $\frac{\partial \pi_L^{S*}}{\partial p_c} < 0$; when $\gamma < \gamma_3$, we have $\frac{\partial \pi_L^{S*}}{\partial p_c} > 0$. □

Proof of Lemma 4.5 and Theorem 4.6. According to Eq (3.4), we can get the second-order condition $\frac{\partial^2 \pi_K}{\partial k^2} = -\lambda$. Thus, π_K is a concave function with k. If we let $\frac{\partial \pi_K}{\partial k} = 0$, then we can get the best response function of the KOL as summarized in Eq (A.5).

$$k(p_s) = \frac{\xi p_s}{\theta\lambda} \tag{A.5}$$

Substituting Eq (A.5) into Eq (3.3), we can get the second-order condition $\frac{\partial^2 \pi_S}{\partial (p_s)^2} = -\frac{2(1-\xi)(\beta\theta\lambda-\xi)}{\theta^2\lambda}$. Thus, $\pi_S(p_s)$ is a concave function with p_s only when $\lambda > \frac{\xi}{\beta\theta}$. If we let $\frac{\partial \pi_S}{\partial p_s} = 0$, then we can get the best response function of the supplier as summarized in Eq (A.6).

$$p_s(p_l, e) = \frac{-(e-1)rp_c(\beta\theta\lambda-\xi) + c_m(\beta\theta\lambda-\xi) - f\theta\lambda\xi\phi + f\theta\lambda\phi - \theta^2\lambda\xi + \theta^2\lambda + \beta\theta\lambda p_l - \xi p_l}{2(\xi-1)(\xi-\beta\theta\lambda)} \tag{A.6}$$

Substituting Eq (A.6) into Eq (A.5), we have

$$k(p_l, e) = \frac{\xi(-(e-1)rp_c(\beta\theta\lambda-\xi) + c_m(\beta\theta\lambda-\xi) - f\theta\lambda\xi\phi + f\theta\lambda\phi - \theta^2\lambda\xi + \theta^2\lambda + \beta\theta\lambda p_l - \xi p_l)}{2\theta\lambda(\xi-1)(\xi-\beta\theta\lambda)} \tag{A.7}$$

By taking the first-order derivative of Eqs (A.6) and (A.7) with respect to p_l and e, we have $\frac{\partial p_s(p_l,e)}{\partial e} = -\frac{rp_c}{2(1-\xi)} < 0$, $\frac{\partial p_s(p_l,e)}{\partial p_l} = \frac{1}{2(1-\xi)} > 0$, $\frac{\partial k(p_l,e)}{\partial e} = -\frac{\xi r p_c}{2\theta\lambda(1-\xi)} < 0$, and $\frac{\partial k(p_l,e)}{\partial p_l} = \frac{\xi}{2\theta\lambda(1-\xi)} > 0$.

Substituting Eqs (A.6) and (A.7) into Eq (3.2), the Hessian matrix of $\pi_L(p_l, e)$ can be derived.

$$H_3 = \begin{pmatrix} -\frac{\beta\theta\lambda-\xi}{\theta^2\lambda(1-\xi)} & \frac{rp_c(\beta\theta\lambda-\xi)}{2\theta^2\lambda(1-\xi)} \\ \frac{rp_c(\beta\theta\lambda-\xi)}{2\theta^2\lambda(1-\xi)} & -\gamma \end{pmatrix} \tag{A.8}$$

Thus, $\pi_L(p_l, e)$ is a concave function with p_l and e only when $\gamma > \frac{r^2 p_c^2(\beta\theta\lambda-\xi)}{4\theta^2\lambda(1-\xi)}$. If we let $\frac{\partial \pi_L}{\partial p_l} = 0$ and $\frac{\partial \pi_L}{\partial e} = 0$, we can get the optimal solutions p_l^* and e^*. Substituting p_l^* and e^* into Eqs (A.6) and (A.7), we can get p_s^* and k^*. □

Proof of Proposition 4.7 and Corollary 4.8. By computing the first-order derivative of the equilibrium solution under Scenario KA with respect to f, we have $\frac{\partial p_l^{KA*}}{\partial f} = \frac{2\gamma\theta^3\lambda^2(1-\xi)^2\phi}{(\beta\theta\lambda-\xi)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)} > 0$, $\frac{\partial e^{KA*}}{\partial f} = \frac{\theta\lambda(1-\xi)r\phi p_c}{4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)} > 0$, $\frac{\partial p_s^{KA*}}{\partial f} = \frac{\theta\lambda\phi\left(3\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}{(\beta\theta\lambda-\xi)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}$, $\frac{\partial k^{KA*}}{\partial f} = \frac{\xi\phi\left(3\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}{(\beta\theta\lambda-\xi)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}$, $\frac{\partial D^{KA*}}{\partial f} = \frac{\gamma\theta\lambda(1-\xi)\phi}{4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)} > 0$, $\frac{\partial \pi_L^{KA*}}{\partial f} = \frac{\gamma\theta\lambda(1-\xi)\phi(\theta\lambda(1-\xi)(f\phi+\theta)-(\beta\theta\lambda-\xi)(c_l+c_m+rp_c))}{(\beta\theta\lambda-\xi)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}$, $\frac{\partial \pi_S^{KA*}}{\partial f} = \frac{2\gamma^2\theta^3\lambda^2(1-\xi)^2\phi(\theta\lambda(1-\xi)(f\phi+\theta)-(\beta\theta\lambda-\xi)(c_l+c_m+rp_c))}{(\beta\theta\lambda-\xi)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)^2} - T$, $\frac{\partial \pi_K^{KA*}}{\partial f} = \frac{\Gamma_1}{(\beta\theta\lambda-\xi)^2\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)^2}$, $\Gamma_1 = 3\gamma^2\theta^4\lambda^3\xi(1-\xi)^2\phi(f\phi+\theta)(2\beta\theta\lambda-5\xi) + T(\beta\theta\lambda-\xi)^2\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)^2 - \gamma\theta^2\lambda^2\xi(1-\xi)\phi(\beta\theta\lambda-\xi)\left(2r^2p_c^2(f\phi+\theta)(\beta\theta\lambda-4\xi)+\gamma\theta(2\beta\theta\lambda+\xi)(c_l+rp_c)\right) + \lambda\xi r^2\phi p_c^2(\beta\theta\lambda-\xi)^2\left(\beta\gamma\theta^2\lambda(c_l+rp_c)-\xi r^2p_c^2(f\phi+\theta)\right) - \gamma\theta^2\lambda^2\xi\phi c_m(\beta\theta\lambda-\xi)\left(\gamma\theta(1-\xi)(2\beta\theta\lambda+\xi)-\beta r^2p_c^2(\beta\theta\lambda-\xi)\right)$.

(1) As $(\beta\theta\lambda-\xi)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right) > 0$, we have $\frac{\partial p_l^{KA*}}{\partial f} > 0$, $\frac{\partial e^{KA*}}{\partial f} > 0$, $\frac{\partial D^{KA*}}{\partial f} > 0$.

(2) As $\beta\theta\lambda-\xi > 0$, when $r < r_2$, we have $\frac{\partial p_s^{KA*}}{\partial f} > 0$ and $\frac{\partial k^{KA*}}{\partial f} > 0$; when $r > r_2$, we have $\frac{\partial p_s^{KA*}}{\partial f} < 0$ and $\frac{\partial k^{KA*}}{\partial f} < 0$.

(3) When $r < r_3$, we have $\frac{\partial \pi_L^{KA*}}{\partial f} > 0$, $\frac{\partial \pi_S^{KA*}}{\partial f} > 0$; when $r > r_3$, we have $\frac{\partial \pi_L^{KA*}}{\partial f} < 0$, $\frac{\partial \pi_S^{KA*}}{\partial f} < 0$.

(4) When $T > T_1$, we have $\frac{\partial \pi_K^{KA*}}{\partial f} > 0$; when $T < T_1$, we have $\frac{\partial \pi_K^{KA*}}{\partial f} < 0$. □

Proof of Proposition 4.9 and Corollary 4.10. By computing the first-order derivative of the equilibrium solution under Scenario KA with respect to p_c, we have $\frac{\partial p_l^{KA*}}{\partial p_c} = \frac{2\gamma\theta^2\lambda(1-\xi)r\left(2\theta\lambda(1-\xi)rp_c(f\phi+\theta)-4\gamma\theta^2\lambda(1-\xi)-rp_c(\beta\theta\lambda-\xi)(2c_l+2c_m+rp_c)\right)}{\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)^2}$, $\frac{\partial e^{KA*}}{\partial p_c} = \frac{r\left(4\gamma\theta^2\lambda(1-\xi)(\theta\lambda(1-\xi)(f\phi+\theta)-(\beta\theta\lambda-\xi)(c_l+c_m+2rp_c))+\theta\lambda(1-\xi)r^2p_c^2(f\phi+\theta)(\beta\theta\lambda-\xi)-r^2p_c^2(c_l+c_m)(\beta\theta\lambda-\xi)^2\right)}{\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)^2}$, $\frac{\partial p_s^{KA*}}{\partial p_c} = \frac{\gamma\theta^2\lambda r\left(-2\theta\lambda(1-\xi)rp_c(f\phi+\theta)+4\gamma\theta^2\lambda(1-\xi)+rp_c(\beta\theta\lambda-\xi)(2c_l+2c_m+rp_c)\right)}{\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)^2}$, $\frac{\partial k^{KA*}}{\partial p_c} = \frac{\gamma\theta\xi r\left(-2\theta\lambda(1-\xi)rp_c(f\phi+\theta)+4\gamma\theta^2\lambda(1-\xi)+rp_c(\beta\theta\lambda-\xi)(2c_l+2c_m+rp_c)\right)}{\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)^2}$, $\frac{\partial D^{KA*}}{\partial p_c} = \frac{\gamma r(\beta\theta\lambda-\xi)\left(2\theta\lambda(1-\xi)rp_c(f\phi+\theta)-4\gamma\theta^2\lambda(1-\xi)-rp_c(\beta\theta\lambda-\xi)(2c_l+2c_m+rp_c)\right)}{\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)^2}$, $\frac{\partial \pi_L^{KA*}}{\partial p_c} = \frac{\gamma r((\beta\theta\lambda-\xi)(c_l+c_m+rp_c)-\theta\lambda(1-\xi)(f\phi+\theta))(\theta\lambda(1-\xi)(4\gamma\theta-rp_c(f\phi+\theta))+rp_c(c_l+c_m)(\beta\theta\lambda-\xi))}{\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)^2}$, $\frac{\partial \pi_S^{KA*}}{\partial p_c} = \frac{2\gamma^2\theta^2\lambda(1-\xi)r((\beta\theta\lambda-\xi)(c_l+c_m+rp_c)-\theta\lambda(1-\xi)(f\phi+\theta))(2\theta\lambda(1-\xi)(2\gamma\theta-rp_c(f\phi+\theta))+rp_c(\beta\theta\lambda-\xi)(2c_l+2c_m+rp_c))}{\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)^3}$, $\frac{\partial \pi_K^{KA*}}{\partial p_c} = \frac{\gamma\theta^2\lambda\xi r\left(4\gamma\theta^2\lambda(1-\xi)-(2\theta\lambda(1-\xi)rp_c(f\phi+\theta)-rp_c(\beta\theta\lambda-\xi)(2c_l+2c_m+rp_c))\right)}{(\beta\theta\lambda-\xi)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)^3}$
$\times\left(\beta\lambda r^2p_c^2(f\phi+\theta)(\beta\theta\lambda-\xi)-\gamma\left((\beta\theta\lambda-\xi)(2\beta\theta\lambda-\xi)(c_l+c_m+rp_c)+\theta\lambda(1-\xi)(f\phi+\theta)(2\beta\theta\lambda+\xi)\right)\right)$.

(1) When $\gamma > \gamma_4$, we have $\frac{\partial p_l^{KA*}}{\partial p_c} < 0$, $\frac{\partial p_s^{KA*}}{\partial p_c} > 0$, $\frac{\partial k^{KA*}}{\partial p_c} > 0$, and $\frac{\partial D^{KA*}}{\partial p_c} < 0$; when $\gamma < \gamma_4$, we have $\frac{\partial p_l^{KA*}}{\partial p_c} > 0$, $\frac{\partial p_s^{KA*}}{\partial p_c} < 0$, $\frac{\partial k^{KA*}}{\partial p_c} < 0$, and $\frac{\partial D^{KA*}}{\partial p_c} > 0$.

(2) When $r > r_4$ and $\gamma > \gamma_5$, or when $r < r_4$ and $\gamma < \gamma_5$, we have $\frac{\partial e^{KA*}}{\partial p_c} < 0$; otherwise, we have $\frac{\partial e^{KA*}}{\partial p_c} > 0$.

(3) When $r < r_3$ and $\gamma < \gamma_6$, or when $r > r_3$, we have $\frac{\partial \pi_L^{KA*}}{\partial p_c} > 0$, $\frac{\partial \pi_S^{KA*}}{\partial p_c} > 0$; when $r < r_3$ and $\gamma > \gamma_6$, we have $\frac{\partial \pi_L^{KA*}}{\partial p_c} < 0$, $\frac{\partial \pi_S^{KA*}}{\partial p_c} < 0$.

(4) When $\gamma_6 < \gamma < \gamma_7$, we have $\frac{\partial \pi_K^{KA*}}{\partial p_c} > 0$; when $\gamma > \gamma_7$ or $\gamma < \gamma_6$, we have $\frac{\partial \pi_K^{KA*}}{\partial p_c} < 0$. □

Proof of Lemma 4.11 and Theorem 4.12. According to Eq (3.6), the Hessian matrix of $\pi_K(x,k)$ can be divided.

$$H_4 = \begin{pmatrix} -\frac{2\beta}{\theta} & \frac{1}{\theta} \\ \frac{1}{\theta} & -\lambda \end{pmatrix} \tag{A.9}$$

Thus $\pi_K(x,k)$ is a concave function with x and k only when $\lambda > \frac{1}{2\beta\theta}$. Let $\frac{\partial \pi_K}{\partial x} = 0$ and $\frac{\partial \pi_K}{\partial k} = 0$, we can get the best response function of the KOL as summarized in Eqs (A.10) and (A.11).

$$x(w,e,p_l) = \frac{-(e-1)rp_c(\beta\theta\lambda - 1) + \theta\lambda(f\phi + \theta + \beta(-w)) + p_l(\beta\theta\lambda - 1)}{2\beta\theta\lambda - 1} \tag{A.10}$$

$$k(w,e,p_l) = \frac{\beta(e-1)rp_c + f\phi + \theta - \beta p_l + \beta(-w)}{2\beta\theta\lambda - 1} \tag{A.11}$$

Substituting Eqs (A.10) and (A.11) into Eq (3.5), we can get the second-order consition $\frac{\partial^2 \pi_S}{\partial w^2} = -\frac{2\beta^2\lambda}{2\beta\theta\lambda - 1} < 0$. Thus, $\pi_S(w)$ is a concave function with w. If we let $\frac{\partial \pi_S}{\partial w} = 0$, we can get the best response function of the supplier as summarized in Eq (A.12).

$$w(e,p_l) = \frac{\beta(e-1)rp_c + \beta c_m + f\phi + \theta - \beta p_l}{2\beta} \tag{A.12}$$

Substituting Eq (A.12) into Eqs (A.10) and (A.11), we have

$$x(e,p_l) = \frac{-(3\beta\theta\lambda - 2)\,(erp_c - rp_c - p_l) - \beta\theta\lambda c_m + f\theta\lambda\phi + \theta^2\lambda}{2(2\beta\theta\lambda - 1)} \tag{A.13}$$

$$k(e,p_l) = \frac{\beta erp_c - \beta c_m - \beta rp_c + f\phi + \theta - \beta p_l}{2(2\beta\theta\lambda - 1)} \tag{A.14}$$

By taking the first-order derivative of Eqs (A.12) to (A.14) with respect to p_l and e, we have $\frac{\partial x(e,p_l)}{\partial e} = -\frac{rp_c(3\beta\theta\lambda-2)}{2(2\beta\theta\lambda-1)}$, $\frac{\partial x(e,p_l)}{\partial p_l} = \frac{3\beta\theta\lambda-2}{2(2\beta\theta\lambda-1)}$, $\frac{\partial k(e,p_l)}{\partial e} = \frac{\beta rp_c}{2(2\beta\theta\lambda-1)}$, $\frac{\partial k(e,p_l)}{\partial p_l} = -\frac{\beta}{2(2\beta\theta\lambda-1)}$, $\frac{\partial w(e,p_l)}{\partial e} - \frac{rp_c}{2}$, $\frac{\partial w(e,p_l)}{\partial p_l} = -\frac{1}{2}$, $\frac{\partial p_s(e,p_l)}{\partial e} = -\frac{rp_c(\beta\theta\lambda-1)}{2(2\beta\theta\lambda-1)}$, and $\frac{\partial p_s(e,p_l)}{\partial p_l} - \frac{\beta\theta\lambda-1}{2(2\beta\theta\lambda-1)}$.

As $\lambda > \frac{1}{2\beta\theta}$, (1) $\frac{\partial k(p_l,e)}{\partial e} > 0$; $\frac{\partial k(p_l,e)}{\partial p_l} < 0$; (2) $\frac{\partial x(p_l,e)}{\partial e} < 0$ when $\theta > \frac{2}{3\beta\lambda}$, $\frac{\partial x(p_l,e)}{\partial e} > 0$ when $\theta < \frac{2}{3\beta\lambda}$; $\frac{\partial x(p_l,e)}{\partial p_l} > 0$ when $\theta > \frac{2}{3\beta\lambda}$, $\frac{\partial x(p_l,e)}{\partial p_l} < 0$ when $\theta < \frac{2}{3\beta\lambda}$; and (3) $\frac{\partial w(p_l,e)}{\partial e} > 0$; $\frac{\partial w(p_l,e)}{\partial p_l} < 0$. (4) $\frac{\partial p_s(p_l,e)}{\partial e} < 0$ when $\theta > \frac{1}{\beta\lambda}$, $\frac{\partial p_s(p_l,e)}{\partial e} > 0$ when $\theta < \frac{1}{\beta\lambda}$; $\frac{\partial p_s(p_l,e)}{\partial p_l} > 0$ when $\theta > \frac{1}{\beta\lambda}$, $\frac{\partial p_s(p_l,e)}{\partial p_l} < 0$ when $\theta < \frac{1}{\beta\lambda}$.

Substituting Eqs (A.12) to (A.14) into Eq (3.2), the Hessian matrix of $\pi_L(p_l,e)$ can be derived.

$$H_5 = \begin{pmatrix} -\frac{\beta^2\lambda}{2\beta\theta\lambda-1} & \frac{\beta^2\lambda rp_c}{2(2\beta\theta\lambda-1)} \\ \frac{\beta^2\lambda rp_c}{2(2\beta\theta\lambda-1)} & -\gamma \end{pmatrix} \tag{A.15}$$

Thus, $\pi_L(p_l,e)$ is a concave function with p_l and e only when $\gamma > \frac{\beta^2\lambda r^2 p_c^2}{4(2\beta\theta\lambda-1)}$. Let $\frac{\partial \pi_L}{\partial p_l} = 0$ and $\frac{\partial \pi_L}{\partial e} = 0$, we can get the optimal solutions p_l^* and e^*. Substituting p_l^* and e^* into Eqs (A.12) to (A.14), we can get w^*, x^* and k^*. □

Proof of Proposition 4.13 and Corollary 4.14. By computing the first-order derivative of the equilibrium solution under Scenario KR with respect to f, we have $\frac{\partial p_l^{KR*}}{\partial f} = \frac{2\gamma\phi(2\beta\theta\lambda-1)}{\beta\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)} >$

0, $\frac{\partial e^{KR*}}{\partial f} = \frac{\beta\lambda r\phi p_c}{4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2} > 0$, $\frac{\partial w^{KR*}}{\partial f} = \frac{\gamma\phi(2\beta\theta\lambda-1)}{\beta\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)} > 0$, $\frac{\partial x^{KR*}}{\partial f} = \frac{\phi\left(\gamma(5\beta\theta\lambda-2)-\beta^2\lambda r^2 p_c^2\right)}{\beta\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$, $\frac{\partial p_s^{KR*}}{\partial f} = \frac{\phi\left(\gamma(7\beta\theta\lambda-3)-\beta^2\lambda r^2 p_c^2\right)}{\beta\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$, $\frac{\partial k^{KR*}}{\partial f} = \frac{\gamma\phi}{4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2} > 0$, $\frac{\partial D^{KR*}}{\partial f} = \frac{\beta\gamma\lambda\phi}{4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2} > 0$, $\frac{\partial \pi_L^{KR*}}{\partial f} = \frac{\gamma\lambda\phi(\theta-\beta(c_l+c_m+rp_c)+f\phi)}{4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2} > 0$, $\frac{\partial \pi_S^{KR*}}{\partial f} = \frac{2\gamma^2\lambda\phi(2\beta\theta\lambda-1)(\theta-\beta(c_l+c_m+rp_c)+f\phi)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2} > 0$, $\frac{\partial \pi_K^{KR*}}{\partial f} = \frac{\gamma^2\lambda\phi(2\beta\theta\lambda-1)(\theta-\beta(c_l+c_m+rp_c)+f\phi)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2} > 0$.

(1) As $4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2 > 0$ and $\theta-\beta(c_l+c_m+rp_c)+f\phi > 0$, we have $\frac{\partial p_l^{KR*}}{\partial f} > 0$, $\frac{\partial e^{KR*}}{\partial f} > 0$, $\frac{\partial w^{KR*}}{\partial f} > 0$, $\frac{\partial k^{KR*}}{\partial f} > 0$, $\frac{\partial D^{KR*}}{\partial f} > 0$, $\frac{\partial \pi_L^{KR*}}{\partial f} > 0$, $\frac{\partial \pi_S^{KR*}}{\partial f} > 0$, $\frac{\partial \pi_K^{KR*}}{\partial f} > 0$.

(2) As $5\beta\theta\lambda-2 > 0$, when $r < r_5$, we have $\frac{\partial x^{KR*}}{\partial f} > 0$; when $r > r_5$, we have $\frac{\partial x^{KR*}}{\partial f} < 0$.

(3) As $7\beta\theta\lambda-3 > 0$, when $r < r_6$, we have $\frac{\partial p_s^{KR*}}{\partial f} > 0$; when $r > r_6$, we have $\frac{\partial p_s^{KR*}}{\partial f} < 0$. □

Proof of Proposition 4.15 and Corollary 4.16. By computing the first-order derivative of the equilibrium solution under Scenario KR with respect to p_c, we have $\frac{\partial p_l^{KR*}}{\partial p_c} = \frac{2\gamma r(2\beta\theta\lambda-1)\left(2\beta\lambda rp_c(\theta-\beta(c_l+c_m)+f\phi)-\beta^2\lambda r^2 p_c^2-4\gamma(2\beta\theta\lambda-1)\right)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2}$, $\frac{\partial e^{KR*}}{\partial p_c} = \frac{\beta\lambda r\left(\beta^2\lambda r^2 p_c^2(\theta-\beta(c_l+c_m)+f\phi)+4\gamma(2\beta\theta\lambda-1)(-\beta(c_l+c_m+2rp_c)+f\phi+\theta)\right)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2}$, $\frac{\partial w^{KR*}}{\partial p_c} = \frac{\gamma r(2\beta\theta\lambda-1)\left(2\beta\lambda rp_c(\theta-\beta(c_l+c_m)+f\phi)-\beta^2\lambda r^2 p_c^2-4\gamma(2\beta\theta\lambda-1)\right)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2}$, $\frac{\partial x^{KR*}}{\partial p_c} = \frac{\gamma r(2-3\beta\theta\lambda)\left(2\beta\lambda rp_c(\theta-\beta(c_l+c_m)+f\phi)-\beta^2\lambda r^2 p_c^2-4\gamma(2\beta\theta\lambda-1)\right)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2}$, $\frac{\partial p_s^{KR*}}{\partial p_c} = \frac{\gamma r(1-\beta\theta\lambda)\left(2\beta\lambda rp_c(\theta-\beta(c_l+c_m)+f\phi)-\beta^2\lambda r^2 p_c^2-4\gamma(2\beta\theta\lambda-1)\right)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2}$, $\frac{\partial k^{KR*}}{\partial p_c} = \frac{\beta\gamma r\left(2\beta\lambda rp_c(\theta-\beta(c_l+c_m)+f\phi)-\beta^2\lambda r^2 p_c^2-4\gamma(2\beta\theta\lambda-1)\right)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2}$, $\frac{\partial D^{KR*}}{\partial p_c} = \frac{\beta^2\gamma\lambda r\left(2\beta\lambda rp_c(\theta-\beta(c_l+c_m)+f\phi)-\beta^2\lambda r^2 p_c^2-4\gamma(2\beta\theta\lambda-1)\right)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2}$, $\frac{\partial \pi_L^{KR*}}{\partial p_c} = \frac{\beta\gamma\lambda r(\theta-\beta(c_l+c_m+rp_c)+f\phi)(\beta\lambda rp_c(\theta-\beta(c_l+c_m)+f\phi)-4\gamma(2\beta\theta\lambda-1))}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2}$, $\frac{\partial \pi_S^{KR*}}{\partial p_c} = \frac{2\beta\gamma^2\lambda r(2\beta\theta\lambda-1)(\theta-\beta(c_l+c_m+rp_c)+f\phi)\left(2\beta\lambda rp_c(\theta-\beta(c_l+c_m)+f\phi)-\beta^2\lambda r^2 p_c^2-4\gamma(2\beta\theta\lambda-1)\right)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^3}$, $\frac{\partial \pi_K^{KR*}}{\partial p_c} = \frac{\beta\gamma^2\lambda r(2\beta\theta\lambda-1)(\theta-\beta(c_l+c_m+rp_c)+f\phi)\left(2\beta\lambda rp_c(\theta-\beta(c_l+c_m)+f\phi)-\beta^2\lambda r^2 p_c^2-4\gamma(2\beta\theta\lambda-1)\right)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^3}$.

(1) When $\gamma < \gamma_8$, we have $\frac{\partial p_l^{KR*}}{\partial p_c} > 0$, $\frac{\partial w^{KR*}}{\partial p_c} > 0$, $\frac{\partial k^{KR*}}{\partial p_c} > 0$, $\frac{\partial D^{KR*}}{\partial p_c} > 0$, $\frac{\partial \pi_L^{KR*}}{\partial p_c} > 0$, $\frac{\partial \pi_S^{KR*}}{\partial p_c} > 0$, and $\frac{\partial \pi_K^{KR*}}{\partial p_c} > 0$; when $\gamma > \gamma_8$, we have $\frac{\partial p_l^{KR*}}{\partial p_c} < 0$, $\frac{\partial w^{KR*}}{\partial p_c} < 0$, $\frac{\partial k^{KR*}}{\partial p_c} < 0$, $\frac{\partial D^{KR*}}{\partial p_c} < 0$, $\frac{\partial \pi_L^{KR*}}{\partial p_c} < 0$, $\frac{\partial \pi_S^{KR*}}{\partial p_c} < 0$, and $\frac{\partial \pi_K^{KR*}}{\partial p_c} < 0$.

(2) When $r > r_7$ and $\gamma > \gamma_9$, or when $r < r_7$ and $\gamma < \gamma_9$, we have $\frac{\partial e^{KR*}}{\partial p_c} < 0$; otherwise, we have $\frac{\partial e^{KR*}}{\partial p_c} > 0$.

(3) When $\lambda < \frac{2}{3\beta\theta}$ and $\gamma < \gamma_8$, or when $\lambda > \frac{2}{3\beta\theta}$ and $\gamma > \gamma_8$, we have $\frac{\partial x^{KR*}}{\partial p_c} >$; otherwise, we have $\frac{\partial x^{KR*}}{\partial p_c} < 0$.

(4) When $\lambda < \frac{1}{\beta\theta}$ and $\gamma < \gamma_8$, or when $\lambda > \frac{1}{\beta\theta}$ and $\gamma > \gamma_8$, $\frac{\partial p_s^{KR*}}{\partial p_c} > 0$. otherwise, $\frac{\partial p_s^{KR*}}{\partial p_c} < 0$. □

Proof of Proposition 5.1. As $p_l^{S*} - p_l^{KA*} = -\frac{2\beta\gamma f\theta^3\lambda^2(1-\xi)^2\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}{\beta(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)} + \frac{\beta\gamma r^2 p_c^2(\beta\theta\lambda-\xi)\left(2\beta^2\theta^2\lambda^2\xi-2\beta\theta\lambda\xi-\beta\theta\lambda+\xi\right)(c_l+c_m+rp_c)}{\beta(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)} + \frac{4\gamma^2\theta^3\lambda(1-\xi)\xi(\beta\theta\lambda-1)(2\beta\theta\lambda-1)+\gamma\theta r^2p_c^2\left(2\beta^3\theta^3\lambda^3(1-\xi)^2-(2\beta\theta\lambda-1)(\beta\theta\lambda-\xi)^2\right)}{\beta(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}$, $p_l^{S*} - p_l^{KR*} = \frac{\gamma(2\beta\theta\lambda-1)\left(\beta^2\lambda r^2 p_c^2(\theta-\beta(c_l+c_m+rp_c))-2f\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\right)}{\beta\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$, $p_l^{KA*} - p_l^{KR*} = \frac{2\gamma(\beta\theta\lambda-1)(f\phi+\theta)\left(r^2p_c^2\left(\beta^2\theta^2\lambda^2\left(1-\xi^2\right)+\xi(\beta\theta\lambda-1)(2\beta\theta\lambda-\xi)\right)-4\gamma\theta^2\lambda\xi(1-\xi)(2\beta\theta\lambda-1)\right)}{\beta(\beta\theta\lambda-\xi)\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)} - \frac{2\beta\gamma r^2p_c^2(\beta\theta\lambda-1)(\beta\theta\lambda-\xi)(\beta\theta\lambda\xi+\beta\theta\lambda-\xi)(c_l+c_m+rp_c)}{\beta(\beta\theta\lambda-\xi)\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}$, and it is easy to verify that $2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2 > 0$. There are thresholds f_{p_l1}, f_{p_l2}, and f_{p_l3} such that the following hold. (1) When $f > f_{p_l1}$ we have $p_l^{S*} < p_l^{KA*}$; otherwise, when $f < f_{p_l1}$, we have $p_l^{S*} > p_l^{KA*}$. (2) When $f > f_{p_l2}$ we have $p_l^{S*} < p_l^{KR*}$; otherwise, when $f < f_{p_l2}$ we have $p_l^{S*} > p_l^{KR*}$. (3) When $r > r_8$ and $f > f_{p_l3}$, or when $r < r_8$ and $f < f_{p_l3}$, we have $p_l^{KA*} > p_l^{KR*}$; otherwise, when $r > r_8$ and $f < f_{p_l3}$, or when $r < r_8$ and $f > f_{p_l3}$, we have $p_l^{KA*} < p_l^{KR*}$. □

Proof of Proposition 5.2. As $e^{S*} - e^{KA*} = \frac{2\gamma r p_c\left(2\beta^2\theta^2\lambda^2\xi - 2\beta\theta\lambda\xi - \beta\theta\lambda + \xi\right)(c_l+c_m+rp_c) + 2\gamma\theta^2\lambda(1-\xi) r p_c}{\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)} - \frac{f\theta\lambda(1-\xi) r\phi p_c\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)+\beta\theta\lambda\xi r^3 p_c^3(\beta\theta\lambda-1)}{\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}$, $e^{S*} - e^{KR*} = \frac{\beta\lambda r p_c\left(2\gamma(2\beta\theta\lambda-1)(\theta-\beta c_l-\beta c_m-\beta r p_c)-f\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\right)}{\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)}$, $e^{KA*} - e^{KR*} = \frac{rp_c(\beta\theta\lambda-1)\left(f\lambda\phi\left(4\gamma\theta(1-\xi)+\beta\xi r^2p_c^2\right)+4\gamma\theta^2\lambda(1-\xi)-4\gamma(\beta\theta\lambda\xi+\beta\theta\lambda-\xi)(c_l+c_m+rp_c)+\beta\theta\lambda\xi r^2p_c^2\right)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}$, there are thresholds f_{e1}, f_{e2}, and f_{e3} such that the following hold. (1) When $f < f_{e1}$, we have $e^{S*} > e^{KA*}$; otherwise, when $f > f_{e1}$, we have $e^{S*} < e^{KA*}$. (2) When $f < f_{e2}$, we have $e^{S*} > e^{KR*}$; otherwise, when $f > f_{e2}$, we have $e^{S*} < e^{KR*}$. (3) When $f < f_{e3}$, we have $e^{KA*} < e^{KR*}$; otherwise, when $f > f_{e3}$, we have $e^{KA*} > e^{KR*}$. □

Proof of Proposition 5.3. As $k^{S*} - k^{KA*} = \frac{(-\gamma)\theta r^2p_c^2(\beta\theta\lambda-\xi)^2-\xi(f\phi+\theta)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(3\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}{(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)} + \frac{4\gamma^2\theta^3\lambda(1-\xi)(\beta\theta\lambda-\xi)+\gamma(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)\left(\beta r^2p_c^2(\beta\theta\lambda\xi+\beta\theta\lambda-\xi)-2\gamma\theta(2\beta\theta\lambda-\xi)\right)}{(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}$, $k^{S*} - k^{KR*} = \frac{\gamma\left(2\gamma(2\beta\theta\lambda-1)(\theta-\beta c_l-\beta c_m-\beta rp_c)-f\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\right)}{\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)}$, there are thresholds f_{k1} and f_{k2} such that the following hold. (1) When $r < r_2$ and $f < f_{k1}$, or when $r > r_2$ and $f > f_{k1}$, we have $k^{S*} > k^{KA*}$; otherwise, when $r < r_2$ and $f > f_{k1}$, or when $r > r_2$ and $f < f_{k1}$, we have $k^{S*} < k^{KA*}$. (2) When $f < f_{k2}$, we have $k^{S*} > k^{KR*}$; otherwise, when $f > f_{k2}$, we have $k^{S*} < k^{KR*}$. □

Proof of Proposition 5.4. As $p_s^{S*} - p_s^{KA*} = -\frac{\beta\theta\lambda(f\phi+\theta)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(3\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}{\beta(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)} - \frac{\beta\gamma(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)\left(2\gamma\theta^2\lambda(2\beta\theta\lambda\xi-2\xi+1)-r^2p_c^2(\beta\theta\lambda\xi+\beta\theta\lambda-\xi)\right)}{\beta(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)} + \frac{\theta(\beta\theta\lambda-\xi)\left(3\beta\gamma\theta\lambda+\beta^2\lambda\left(-r^2\right)p_c^2-\gamma\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}{\beta(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}$, $p_s^{S*} - p_s^{KR*} = \frac{2\gamma^2(1-\beta\theta\lambda)(2\beta\theta\lambda-1)(\theta-\beta(c_l+c_m+rp_c))-f\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(\gamma(7\beta\theta\lambda-3)-\beta^2\lambda r^2p_c^2\right)}{\beta\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)}$, and it is easy to verify that $7\beta\gamma\theta\lambda + \beta^2\lambda\left(-r^2\right)p_c^2 - 3\gamma > 0$, there are thresholds f_{p_s1} and f_{p_s2} such that we have the following. (1) When $r < r_2$ and $f < f_{p_s1}$, or when $r > r_2$ and $f > f_{p_s1}$, we have $p_s^{S*} > p_s^{KA*}$; otherwise, when $r < r_2$ and $f > f_{p_s1}$, or when $r > r_2$ and $f < f_{p_s1}$, we have $p_s^{S*} < p_s^{KA*}$. (2) When $f < f_{p_s2}$ we have $p_s^{S*} > p_s^{KR*}$; otherwise, when $f > f_{p_s2}$, we have $p_s^{S*} < p_s^{KR*}$. □

Proof of Proposition 5.5. As $D^{S*} - D^{KR*} = \frac{\beta\gamma\lambda\left(2\gamma(2\beta\theta\lambda-1)(\theta-\beta(c_l+c_m+rp_c))-f\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\right)}{\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)}$, $D^{S*} - D^{KA*} = \frac{\gamma\left(-f\theta\lambda(1-\xi)\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)+2\gamma\theta^2\lambda(1-\xi)+2\gamma\left(2\beta^2\theta^2\lambda^2\xi-2\beta\theta\lambda\xi-\beta\theta\lambda+\xi\right)(c_l+c_m+rp_c)-\beta\theta\lambda\xi r^2p_c^2(\beta\theta\lambda-1)\right)}{\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}$, $D^{KA*} - D^{KR*} = \frac{\gamma(\beta\theta\lambda-1)\left(f\lambda\phi\left(4\gamma\theta(1-\xi)+\beta\xi r^2p_c^2\right)+4\gamma\theta^2\lambda(1-\xi)-4\gamma(\beta\theta\lambda\xi+\beta\theta\lambda-\xi)(c_l+c_m+rp_c)+\beta\theta\lambda\xi r^2p_c^2\right)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}$, there are thresholds f_{D1}, f_{D2}, and f_{D3} such that the following hold. (1) When $f < f_{D1}$, we have $D^{S*} > D^{KA*}$; otherwise, when $f > f_{D1}$, we have $D^{S*} < D^{KA*}$. (2) When $f < f_{D2}$, we have $D^{S*} > D^{KR*}$; otherwise, when $f > f_{D2}$, we have $D^{S*} < D^{KR*}$. (3) When $f < f_{D3}$, we have $D^{KA*} < D^{KR*}$; otherwise, when $f > f_{D3}$, we have $D^{KA*} > D^{KR*}$. □

Proof of Proposition 5.6. (1) As $\pi_L^{S*} - \pi_L^{KA*} = -\frac{\gamma\theta\lambda(1-\xi)\phi\left(2f\left(\theta^2\lambda(1-\xi)-(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)\right)+f^2\theta\lambda(1-\xi)\phi\right)}{2(\beta\theta\lambda-\xi)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)} + \frac{-\gamma\theta\lambda(c_l+c_m+rp_c)\left(2\gamma\theta(1-\xi)-\beta\xi r^2p_c^2(\beta\theta\lambda-1)\right)-\gamma^2\left(2\beta^2\theta^2\lambda^2\xi-2\beta\theta\lambda\xi-\beta\theta\lambda+\xi\right)(c_l+c_m+rp_c)^2}{\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)} + \frac{2\gamma^2\theta^4\lambda^2(1-\xi)(2\beta\theta\lambda\xi-3\xi+1)+\gamma\theta^2\lambda\xi r^2p_c^2(\beta\theta\lambda-1)(\beta\theta\lambda\xi-2\beta\theta\lambda+\xi)}{2(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}$, we let $\pi_L^{S*} - \pi_L^{KA*} = 0$ and we can get $f_{\pi_l1} = \frac{(\theta-\beta c_l-\beta c_m-\beta rp_c)\sqrt{\lambda(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}}{\theta\lambda(1-\xi)\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)} - \frac{\theta\lambda(1-\xi)\phi\left(\theta^2\lambda(1-\xi)-(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)\right)}{\theta^2\lambda^2(1-\xi)^2\phi^2}$, $f_{\pi_l2} = -\frac{(\theta-\beta c_l-\beta c_m-\beta rp_c)\sqrt{\lambda(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2p_c^2(\beta\theta\lambda-\xi)\right)}}{\theta\lambda(1-\xi)\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2p_c^2\right)} - \frac{\theta\lambda(1-\xi)\phi\left(\theta^2\lambda(1-\xi)-(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)\right)}{\theta^2\lambda^2(1-\xi)^2\phi^2}$. Thus, there are thresholds f_{π_l1}, f_{π_l2}, and $f > f_{\pi_l3}$ such that when $f < f_{\pi_l2}$ or $f > f_{\pi_l1}$, we have $\pi_L^{S*} < \pi_L^{KA*}$; otherwise, when $f_{\pi_l2} < f < f_{\pi_l1}$, we have $\pi_L^{S*} > \pi_L^{KA*}$.

(2) As $\pi_L^{S*} - \pi_L^{KR*} = \frac{\gamma\lambda\left(2\gamma(2\beta\theta\lambda-1)(\theta-\beta c_l-\beta c_m-\beta r p_c)^2-\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(2f\phi(\theta-\beta c_l-\beta c_m-\beta r p_c)+f^2\phi^2\right)\right)}{2\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$, we let $\pi_L^{S*} - \pi_L^{KR*} = 0$ and we can get $f_{\pi_l 3} = -\frac{(\theta-\beta(c_l+c_m+rp_c))\left(1-\frac{\sqrt{\beta^4\lambda^2 r^4 p_c^4+2\gamma(2\beta\theta\lambda-1)\left(4\gamma(2\beta\theta\lambda-1)-3\beta^2\lambda r^2 p_c^2\right)}}{2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2}\right)}{\phi}$, $f_{\pi_l 4} = -\frac{(\theta-\beta(c_l+c_m+rp_c))\left(\frac{\sqrt{\beta^4\lambda^2 r^4 p_c^4+2\gamma(2\beta\theta\lambda-1)\left(4\gamma(2\beta\theta\lambda-1)-3\beta^2\lambda r^2 p_c^2\right)}}{2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2}+1\right)}{\phi}$. As $f_{\pi_l 4} < 0$, thus when $f > f_{\pi_l 3}$, we have $\pi_L^{S*} < \pi_L^{KR*}$; otherwise, when $f < f_{\pi_l 3}$, we have $\pi_L^{S*} > \pi_L^{KR*}$. □

Proof of Proposition5.7. (1) As $\pi_S^{S*} - \pi_S^{KA*} = \frac{A_1 f^2 + A_2 f + A_3}{2(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2\left(4\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)^2}$, in which $A_1 = -2\gamma^2\theta^4\lambda^3(1-\xi)^3\phi^2\left(-4\beta\gamma\theta\lambda+\beta^2\lambda r^2 p_c^2+2\gamma\right)^2$, $A_2 = 2\left(-4\beta\gamma\theta\lambda+\beta^2\lambda r^2 p_c^2+2\gamma\right)^2\left(2\gamma^2\theta^3\lambda^2(1-\xi)^2\phi\left((\beta\theta\lambda-\xi)(c_l+c_m+rp_c)-\theta^2\lambda(1-\xi)\right)\right.$ $\left.+T(\beta\theta\lambda-\xi)\left(4\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)^2\right)$, $A_3 = 8\gamma^4\theta^6\lambda^3(1-\xi)^2(2\beta\theta\lambda-1)(2\beta\theta\lambda\xi-3\xi+1)+\gamma^2\lambda(\beta\theta\lambda-\xi)\left(2\beta^2\theta^2\lambda^2\xi-2\beta\theta\lambda\xi-\beta\theta\lambda+\xi\right)$ $(c_l+c_m+rp_c)^2\left(\beta^2 r^4 p_c^4(\beta\theta\lambda-\xi)-8\gamma^2\theta^2(1-\xi)(2\beta\theta\lambda-1)\right)+2\beta\gamma^2\theta\lambda r^4 p_c^4(\beta\theta\lambda-\xi)$ $\left(2\beta^3\theta^3\lambda^3\xi^2-4\beta^3\theta^3\lambda^3\xi+4\beta^2\theta^2\lambda^2\xi+\beta^2\theta^2\lambda^2-2\beta\theta\lambda\xi^2-2\beta\theta\lambda\xi+\xi^2\right)(c_l+c_m+rp_c)+16\gamma^3\theta^3\lambda^2(1-\xi)(2\beta\theta\lambda-1)(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)\left(\gamma\theta\xi-\gamma\theta+\beta^2\theta\lambda\xi r^2 p_c^2+\beta\xi\left(-r^2\right)p_c^2\right)+\gamma^2\theta^2\lambda r^4 p_c^4$ $\left(2\beta^4\theta^4\lambda^4\xi^3-6\beta^4\theta^4\lambda^4\xi^2+6\beta^4\theta^4\lambda^4\xi-6\beta^3\theta^3\lambda^3\xi-\beta^3\theta^3\lambda^3+6\beta^2\theta^2\lambda^2\xi^2+3\beta^2\theta^2\lambda^2\xi-2\beta\theta\lambda\xi^3-3\beta\theta\lambda\xi^2+\xi^3\right)+$ $8\gamma^3\theta^4\lambda^2(1-\xi)\xi r^2 p_c^2(\beta\theta\lambda-1)(2\beta\theta\lambda-1)(\beta\theta\lambda\xi-2\beta\theta\lambda+\xi)$, we let $\pi_S^{S*} - \pi_S^{KA*} = 0$ and we can get $f_{\pi_s 1} = \frac{T(\beta\theta\lambda-\xi)\left(4\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)^2}{2\gamma^2\theta^4\lambda^3(1-\xi)^3\phi^2} - \frac{\theta^2\lambda(1-\xi)-(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)}{\theta\lambda(1-\xi)\phi} + \frac{\sqrt{(\beta\theta\lambda-\xi)\Delta_1}\left(-4\gamma\theta^2\lambda\xi+4\gamma\theta^2\lambda-\beta\theta\lambda r^2 p_c^2+\xi r^2 p_c^2\right)}{\gamma^2\theta^4\lambda^3(1-\xi)^3\phi^2\left(4\beta\gamma\theta\lambda-\beta^2\lambda r^2 p_c^2-2\gamma\right)}$, $f_{\pi_s 2} = \frac{T(\beta\theta\lambda-\xi)\left(4\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)^2}{2\gamma^2\theta^4\lambda^3(1-\xi)^3\phi^2} - \frac{\theta^2\lambda(1-\xi)-(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)}{\theta\lambda(1-\xi)\phi} - \frac{\sqrt{(\beta\theta\lambda-\xi)\Delta_1}\left(-4\gamma\theta^2\lambda\xi+4\gamma\theta^2\lambda-\beta\theta\lambda r^2 p_c^2+\xi r^2 p_c^2\right)}{\gamma^2\theta^4\lambda^3(1-\xi)^3\phi^2\left(4\beta\gamma\theta\lambda-\beta^2\lambda r^2 p_c^2-2\gamma\right)}$, in which $\Delta_1 = 2\beta^2\gamma^3\theta^4\lambda^4(1-\xi)^3\phi\left(\gamma\phi(c_l+c_m+rp_c)^2+8\theta r^2 T p_c^2\right)(2\beta\theta\lambda-1)+4\gamma^2\theta^3\lambda^2(1-\xi)^2\phi$ $(c_l+c_m+rp_c)\left(T(\beta\theta\lambda-\xi)\left(-4\beta\gamma\theta\lambda+\beta^2\lambda r^2 p_c^2+2\gamma\right)^2-\beta\gamma^2\theta^2\lambda^2(1-\xi)\phi(2\beta\theta\lambda-1)\right)+(1-\xi)^3\ \phi\left(2\gamma^2\gamma^2\theta^6\lambda^4\phi(2\beta\theta\lambda-1)-4\beta^4\gamma^2\theta^5\lambda^5 r^4 T p_c^4-16\gamma^4\theta^5\lambda^3 T(2\beta\theta\lambda-1)^2\right)+T^2(\beta\theta\lambda-\xi)$ $\left(-4\beta\gamma\theta\lambda+\beta^2\lambda r^2 p_c^2+2\gamma\right)^2\left(-4\gamma\theta^2\lambda\xi+4\gamma\theta^2\lambda-\beta\theta\lambda r^2 p_c^2+\xi r^2 p_c^2\right)^2$. Since $A_1 < 0$, there are thresholds $f_{\pi_s 1}$ and $f_{\pi_s 2}$ such that when $f < f_{\pi_s 2}$ or $f > f_{\pi_s 1}$, we have $\pi_S^{S*} < \pi_S^{KA*}$; otherwise, when $f_{\pi_s 1} < f < f_{\pi_s 1}$, we have $\pi_S^{S*} > \pi_S^{KA*}$.

(2) As $\pi_S^{S*} - \pi_S^{KR*} = \frac{\gamma^2\lambda(2\beta\theta\lambda-1)(\theta-\beta(c_l+c_m+rp_c))^2\left(8\gamma^2(2\beta\theta\lambda-1)^2-\beta^4\lambda^2 r^4 p_c^4\right)}{2\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2} - \frac{\gamma^2\lambda\phi(2\beta\theta\lambda-1)\left(2f(\theta-\beta(c_l+c_m+rp_c))+f^2\phi\right)}{\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)^2}$, we let $\pi_S^{S*} - \pi_S^{KR*} = 0$ and we can get $f_{\pi_s 3} = \frac{(\theta-\beta(c_l+c_m+rp_c))\left(\sqrt{2}\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)-2\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\right)}{2\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$, $f_{\pi_s 4} = -\frac{(\theta-\beta(c_l+c_m+rp_c))\left(2\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)+\sqrt{2}\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\right)}{2\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$. Since $f_{\pi_s 4} < 0$, when $f > f_{\pi_s 3}$, we have $\pi_S^{S*} < \pi_S^{KR*}$; otherwise, when $f < f_{\pi_s 3}$, we have $\pi_S^{S*} > \pi_S^{KR*}$. □

Appendix B. Thresholds

All threshold values used in this paper are summarized in Table B1 and Table B2.

Table B1. Threshold values of γ and r.

Notation	Definition	Notation	Definition
γ_1	$\frac{2\beta\lambda r p_c(\theta-\beta c_l-\beta c_m)-\beta^2\lambda r^2 p_c^2}{2(2\beta\theta\lambda-1)}$	r_1	$\frac{\theta-\beta c_l-\beta c_m}{2\beta p_c}$
γ_2	$\frac{\beta^2\lambda r^2 p_c^2(\theta-\beta c_l-\beta c_m)}{2(2\beta\theta\lambda-1)(2\beta r p_c-(\theta-\beta c_l-\beta c_m))}$	r_2	$\frac{\theta\sqrt{3\gamma\lambda(1-\xi)}}{p_c\sqrt{\beta\theta\lambda-\xi}}$
γ_3	$\frac{\beta\lambda r p_c(\theta-\beta c_l-\beta c_m)}{2(2\beta\theta\lambda-1)}$	r_3	$\frac{\theta\lambda(1-\xi)(f\phi+\theta)-(c_l+c_m)(\beta\theta\lambda-\xi)}{p_c(\beta\theta\lambda-\xi)}$
γ_4	$\frac{2\theta\lambda(1-\xi)r p_c(f\phi+\theta)-r p_c(\beta\theta\lambda-\xi)(2c_l+2c_m+r p_c)}{4\theta^2\lambda(1-\xi)}$	r_4	$\frac{(c_l+c_m)(\beta\theta\lambda-\xi)-\theta\lambda^2(1-\xi)(f\phi+\theta)}{2p_c(\beta\theta\lambda-\xi)}$
γ_5	$\frac{r^2 p_c^2(c_l+c_m)(\beta\theta\lambda-\xi)^2-\theta\lambda(1-\xi)r^2 p_c^2(f\phi+\theta)(\beta\theta\lambda-\xi)}{4\theta^2\lambda(1-\xi)(\theta\lambda(1-\xi)(f\phi+\theta)-(\beta\theta\lambda-\xi)(c_l+c_m+2r p_c))}$	r_5	$\frac{\sqrt{\gamma(5\beta\theta\lambda-2)}}{\beta p_c\sqrt{\lambda}}$
γ_6	$\frac{\theta\lambda(1-\xi)r p_c(f\phi+\theta)-r p_c(c_l+c_m)(\beta\theta\lambda-\xi)}{4\theta^2\lambda(1-\xi)}$	r_6	$\frac{\sqrt{\gamma(7\beta\theta\lambda-3)}}{\beta p_c\sqrt{\lambda}}$
γ_7	$\frac{\beta\lambda r^2 p_c^2(f\phi+\theta)(\beta\theta\lambda-\xi)}{(\beta\theta\lambda-\xi)(2\beta\theta\lambda-\xi)(c_l+c_m+r p_c)+\theta\lambda(1-\xi)(f\phi+\theta)(2\beta\theta\lambda+\xi)}$	r_7	$\frac{\theta-\beta(c_l+c_m)+f\phi}{2\beta p_c}$
γ_8	$\frac{2\beta\lambda r p_c(\theta-\beta(c_l+c_m)+f\phi)-\beta^2\lambda r^2 p_c^2}{4(2\beta\theta\lambda-1)}$	r_8	$\frac{\sqrt{4\gamma\theta^2\lambda\xi(1-\xi)(2\beta\theta\lambda-1)}}{p_c\sqrt{\beta^2\theta^2\lambda^2\left(1-\xi^2\right)+\xi(\beta\theta\lambda-1)(2\beta\theta\lambda-\xi)}}$
γ_9	$\frac{\beta^2\lambda r^2 p_c^2(\theta-\beta(c_l+c_m)+f\phi)}{4(2\beta\theta\lambda-1)(\beta(c_l+c_m+2r p_c)-f\phi-\theta)}$		

Table B2. Threshold values about f.

Notation	Definition
$f_{p_l 1}$	$\frac{\beta r^2 p_c^2(\beta\theta\lambda-\xi)\left(2\beta^2\theta^2\lambda^2\xi-2\beta\theta\lambda\xi-\beta\theta\lambda+\xi\right)(c_l+c_m+rp_c)}{2\beta\theta^3\lambda^2(1-\xi)^2\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}+\frac{4\gamma\theta^3\lambda(1-\xi)\xi(\beta\theta\lambda-1)(2\beta\theta\lambda-1)+\theta r^2 p_c^2\left(2\beta^3\theta^3\lambda^3(1-\xi)^2-(2\beta\theta\lambda-1)(\beta\theta\lambda-\xi)^2\right)}{2\beta\theta^3\lambda^2(1-\xi)^2\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$
$f_{p_l 2}$	$\frac{\beta^2\lambda r^2 p_c^2(\theta-\beta(c_l+c_m+rp_c))}{2\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$
$f_{p_l 3}$	$\frac{\beta r^2 p_c^2(\beta\theta\lambda-\xi)(\beta\theta\lambda\xi+\beta\theta\lambda-\xi)(c_l+c_m+rp_c)}{\phi\left(r^2 p_c^2\left(\beta^2\theta^2\lambda^2\left(1-\xi^2\right)+\xi(\beta\theta\lambda-1)(2\beta\theta\lambda-\xi)\right)-4\gamma\theta^2\lambda\xi(1-\xi)(2\beta\theta\lambda-1)\right)}-\frac{\theta}{\phi}$
$f_{p_s 1}$	$-\frac{\beta\gamma(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)\left(2\gamma\theta^2\lambda(2\beta\theta\lambda\xi-2\xi+1)-r^2 p_c^2(\beta\theta\lambda\xi+\beta\theta\lambda-\xi)\right)}{\beta\theta\lambda\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(3\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)}-\frac{\beta\theta^2\lambda\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(3\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)}{\beta\theta\lambda\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(3\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)}$ $+\frac{\theta(\beta\theta\lambda-\xi)\left(3\beta\gamma\theta\lambda+\beta^2\lambda\left(-r^2\right)p_c^2-\gamma\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)}{\beta\theta\lambda\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(3\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)}$
$f_{p_s 2}$	$\frac{2\gamma^2(1-\beta\theta\lambda)(2\beta\theta\lambda-1)(\theta-\beta(c_l+c_m+rp_c))}{\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(7\beta\gamma\theta\lambda+\beta^2\lambda\left(-r^2\right)p_c^2-3\gamma\right)}$
f_{e1}	$\frac{2\gamma\theta^2\lambda(1-\xi)+2\gamma\left(2\beta^2\theta^2\lambda^2\xi-2\beta\theta\lambda\xi-\beta\theta\lambda+\xi\right)(c_l+c_m+rp_c)-\beta\theta\lambda\xi r^2 p_c^2(\beta\theta\lambda-1)}{\theta\lambda(1-\xi)\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$
f_{e2}	$\frac{2\gamma(2\beta\theta\lambda-1)(\theta-\beta c_l-\beta c_m-\beta rp_c)}{\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$
f_{e3}	$\frac{-4\gamma\theta^2\lambda(1-\xi)+4\gamma(\beta\theta\lambda\xi+\beta\theta\lambda-\xi)(c_l+c_m+rp_c)-\beta\theta\lambda\xi r^2 p_c^2}{\lambda\phi\left(4\gamma\theta(1-\xi)+\beta\xi r^2 p_c^2\right)}$
f_{k1}	$\frac{\left(4\gamma^2\theta^3\lambda(1-\xi)(\beta\theta\lambda-\xi)+\gamma(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)\left(\beta r^2 p_c^2(\beta\theta\lambda\xi+\beta\theta\lambda-\xi)-2\gamma\theta(2\beta\theta\lambda-\xi)\right)\right)-\gamma\theta r^2 p_c^2(\beta\theta\lambda-\xi)^2}{\xi\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(3\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)}-\frac{\theta}{\phi}$
f_{k2}	$\frac{2\gamma(2\beta\theta\lambda-1)(\theta-\beta c_l-\beta c_m-\beta rp_c)}{\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$
f_{D1}	$\frac{2\gamma\theta^2\lambda(1-\xi)+2\gamma\left(2\beta^2\theta^2\lambda^2\xi-2\beta\theta\lambda\xi-\beta\theta\lambda+\xi\right)(c_l+c_m+rp_c)-\beta\theta\lambda\xi r^2 p_c^2(\beta\theta\lambda-1)}{\theta\lambda(1-\xi)\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$
f_{D2}	$\frac{2\gamma(2\beta\theta\lambda-1)(\theta-\beta(c_l+c_m+rp_c))}{\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$
f_{D3}	$\frac{-4\gamma\theta^2\lambda(1-\xi)+4\gamma(\beta\theta\lambda\xi+\beta\theta\lambda-\xi)(c_l+c_m+rp_c)-\beta\theta\lambda\xi r^2 p_c^2}{\lambda\phi\left(4\gamma\theta(1-\xi)+\beta\xi r^2 p_c^2\right)}$
$f_{\pi_s 1}$	$\frac{T(\beta\theta\lambda-\xi)\left(4\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)^2}{2\gamma^2\theta^4\lambda^3(1-\xi)^3\phi^2}-\frac{\theta^2\lambda(1-\xi)-(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)}{\theta\lambda(1-\xi)\phi}+\frac{\sqrt{(\beta\theta\lambda-\xi)\Delta_1}\left(-4\gamma\theta^2\lambda\xi+4\gamma\theta^2\lambda-\beta\theta\lambda r^2 p_c^2+\xi r^2 p_c^2\right)}{\gamma^2\theta^4\lambda^3(1-\xi)^3\phi^2\left(4\beta\gamma\theta\lambda-\beta^2\lambda r^2 p_c^2-2\gamma\right)}$
$f_{\pi_s 2}$	$\frac{T(\beta\theta\lambda-\xi)\left(4\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)^2}{2\gamma^2\theta^4\lambda^3(1-\xi)^3\phi^2}-\frac{\theta^2\lambda(1-\xi)-(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)}{\theta\lambda(1-\xi)\phi}-\frac{\sqrt{(\beta\theta\lambda-\xi)\Delta_1}\left(-4\gamma\theta^2\lambda\xi+4\gamma\theta^2\lambda-\beta\theta\lambda r^2 p_c^2+\xi r^2 p_c^2\right)}{\gamma^2\theta^4\lambda^3(1-\xi)^3\phi^2\left(4\beta\gamma\theta\lambda-\beta^2\lambda r^2 p_c^2-2\gamma\right)}$
$f_{\pi_s 3}$	$\frac{(\theta-\beta(c_l+c_m+rp_c))\left(\sqrt{2}\left(4\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)-2\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\right)}{2\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}$
$f_{\pi_l 1}$	$\frac{(\theta-\beta c_l-\beta c_m-\beta rp_c)\sqrt{\lambda(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)}}{\theta\lambda(1-\xi)\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}-\frac{\theta\lambda(1-\xi)\phi\left(\theta^2\lambda(1-\xi)-(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)\right)}{\theta^2\lambda^2(1-\xi)^2\phi^2}$
$f_{\pi_l 2}$	$-\frac{(\theta-\beta c_l-\beta c_m-\beta rp_c)\sqrt{\lambda(\beta\theta\lambda-\xi)\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)\left(4\gamma\theta^2\lambda(1-\xi)-r^2 p_c^2(\beta\theta\lambda-\xi)\right)}}{\theta\lambda(1-\xi)\phi\left(2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2\right)}-\frac{\theta\lambda(1-\xi)\phi\left(\theta^2\lambda(1-\xi)-(\beta\theta\lambda-\xi)(c_l+c_m+rp_c)\right)}{\theta^2\lambda^2(1-\xi)^2\phi^2}$
$f_{\pi_l 3}$	$-\frac{(\theta-\beta(c_l+c_m+rp_c))}{\phi}\left(1-\frac{\sqrt{\beta^4\lambda^2 r^4 p_c^4+2\gamma(2\beta\theta\lambda-1)\left(4\gamma(2\beta\theta\lambda-1)-3\beta^2\lambda r^2 p_c^2\right)}}{2\gamma(2\beta\theta\lambda-1)-\beta^2\lambda r^2 p_c^2}\right)$

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