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Research article

Integrated scheduling of drone-based medical sample delivery and testing

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Appendix

For the cases $v^1 \le v^2$ and $v^1 > v^2$, the following lemmas can be derived about optimality conditions of the restricted temporal decision:

Lemma 1: If $v^1 \le v^2$ and $cir\_time \cdot v^1 \le Q$, the optimal solution of the restricted temporal decision satisfies:

$$\widehat{l_n} = L$$

$$\widehat{l_{n-1}} = L - cir\_time$$

The minimized objective function value is:

$$\widehat{ft_n} = L + c + cir\_time \cdot v^1 / v^2$$

Proof: When $v^1 \le v^2$, the testing speed exceeds the sampling speed, and the testing institution evidently experiences little backlog time. Under this condition, the objective function can be expressed as:

$$ft_n = l_n + c + q_n / v^2 + \max\{0, ft_{n-1} - at_n\}$$

Among them, the unknown terms comprise l_n, q_n, and $ft_{n-1} - at_n$. Considering different situations for these variables, the proof of Lemma 1 is partitioned into four distinct cases:

Case 1), where $ft_{n-1} \le at_n$ and $l_n = L$;
Case 2), where $ft_{n-1} \le at_n$ and $l_n > L$;
Case 3), where $ft_{n-1} > at_n$ and $l_n = L$;

Case 4), where $ft_{n-1} > at_n$ and $l_n > L$.

For Case 1), the testing institution exhibits no backlog time when receiving the last batch of samples. In addition, the time at which the last trip picks up samples at the collection point equals the time at which the collection point finishes its sample collection. Thus, it must be satisfied that:

$$L - l_{n-1} \geq cir\_time$$

The objective function can be rewritten as:

$$\min ft_n = L + c + q_n / v^2 + 0$$

Consider that the unknown terms comprise only q_n; the objective function is equivalent to:

$$\min q_n$$

Since

$$q_n = \left(l_n - l_{n-1}\right) \cdot v^1 = \left(L - l_{n-1}\right) \cdot v^1$$

$$L - l_{n-1} \geq cir\_time$$

$$q_n \geq cir\_time \cdot v^1$$

This ensures that

$$\min q_n = cir\_time \cdot v^1$$

Let

$$q_n^* = cir\_time \cdot v^1$$

Under these conditions, Case 1) achieves the optimal solution, which is denoted as

$$l_{n-1}^* = L - q_n / v^1 = L - cir\_time$$

$$ft_n^* = L + c + cir\_time \cdot v^1 / v^2$$

For Case 2), the testing institution also exhibits no backlog time when receiving the last batch of samples. However, the time at which the last trip picks up samples at the collection point exceeds the time at which the collection point finishes its sample collection. In the optimal solution of Case 2), this scenario will happen only when

$$L - l_{n-1} < cir\_time$$

In this case, the objective function can be rewritten as:

$$\min ft_n = l_n + c + q_n / v^2 + 0$$

Additionally, provided

$$q_n = \left(L - l_{n-1}\right) \cdot v^1$$

$$ft_n - ft_n^* = \left(l_n - L\right) + \left(q_n - q_n^*\right) / v^2 = \left(l_n - L\right) - \left(l_{n\text{-}1}^* \text{-} l_{n\text{-}1}\right) \cdot v^1 / v^2 > \left(l_n - L\right) - \left(l_{n\text{-}1}^* \text{-} l_{n\text{-}1}\right)$$

Since

$$L - l_{n-1}^* = cir\_time$$

And the time interval between each pair of consecutive trips satisfies

$$l_n - l_{n-1} \geq cir\_time$$

It can be obtained that

$$\left(l_n - L\right) - \left(l_{n-1}^* - l_{n-1}\right) \geq 0$$

$$ft_n - ft_n^* > 0$$

Consequently, any solution obtained in Case 2) is inferior to the optimal solution in Case 1). Figure 11 shows the comparison of timelines for Cases 1) and 2).

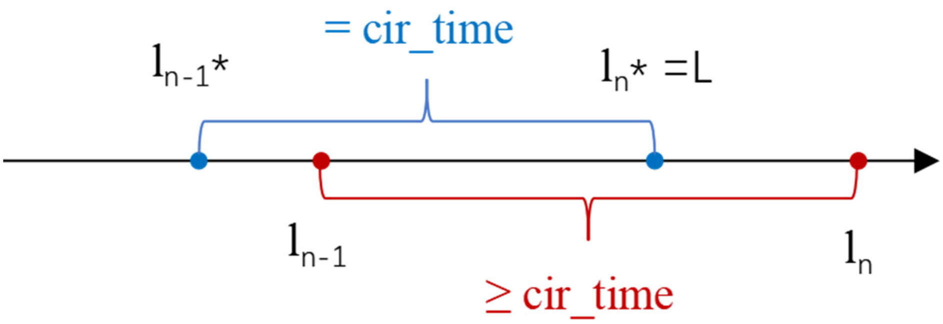

Figure 11. Comparison of timelines for Case 1 and Case 2/Case 4.

For Case 3), the backlog time occurs for the testing institution when it receives the last batch of samples. In addition, by the same reasoning as in Case 1), it is satisfied that

$$L-l_{n-1} \geq cir\_time$$

The objective function can be rewritten as:

$$\min ft_n = L + c + q_n / v^2 + ft_{n-1} - at_n$$

Additionally, provided that

$$ft_n - ft_n^* = q_n - q_n^* + ft_{n-1} - at_n$$

Since

$$ft_{n-1} - at_n > 0$$
$$q_n = (L\text{-}l_{n-1}) \cdot v^1 \geq cir\_time \cdot v^1$$
$$q_n - q_n^* \geq 0$$

Therefore,

$$ft_n - ft_n^* > 0$$

Consequently, any solution obtained in case 3) is inferior to the optimal solution in Case 1).

For Case 4), backlog time occurs for the testing institution when it receives the last batch of samples. However, the time at which the last trip picks up samples at the collection point exceeds the time at which the collection point finishes its sample collection. In this case,

$$ft_{n-1} > at_n$$

Equivalently,

$$ft_{n-1} > l_n + c$$

By the same reasoning as Case 2), it is satisfied that

$$L - l_{n-1} < cir\_time$$

In this case, the objective function can be rewritten as:

$$\min ft_n = ft_{n-1} + q_n / v^2$$

Compare ft_n here with the optimal solution of Case 1):

$$ft_n - ft_n^* = ft_{n-1} - (L+c) + \left(q_n - q_n^*\right) / v^2$$

Since

$$q_n = \min\left\{Q, (L-l_{n-1}) \cdot v^1\right\}$$
$$(L-l_{n-1}) \cdot v^1 < cir\_time \cdot v^1 < Q$$

This ensures that, within the optimal solution of Case 4),

$$q_n = (L-l_{n-1}) \cdot v^1$$

Additionally, provided that

$$ft_{n-1} - (L+c) > l_n + c - (L+c) = l_n - L$$

It can be obtained that

$$ft_n - ft_n^* > l_n - L + \left[L - l_{n-1} - \left(L - l_{n-1}^*\right)\right] \cdot v^1/v^2 = l_n - l_n^* - \left(l_{n-1} - l_{n-1}^*\right) \cdot v^1/v^2$$

As shown in Figure 11,

$$l_n - l_n^* \geq l_{n-1} - l_{n-1}^*$$

Consider that

$$v^1/v^2 \leq 1$$

Therefore,

$$l_n - l_n^* \text{-} \left(l_{n-1} - l_{n-1}^*\right) \cdot v^1/v^2 \geq 0$$

$$ft_n - ft_n^* > 0$$

Consequently, any solution obtained in Case 4) is inferior to the optimal solution in Case 1).

In conclusion, when

$$v^1 \leq v^2$$

$$cir\_time \cdot v^1 \leq Q$$

The optimal decision solution on the temporal dimension can be obtained as:

$$\widehat{l_n} = l_n^*$$

$$\widehat{l_{n-1}} = l_{n-1}^*$$

$$\widehat{q_n} = q_n^*$$

$$\widehat{ft_n} = ft_n^*$$

To satisfy the constraints of Case 1), multiple strategies can be adopted, one of which is provided here:

$$q_p = \max\left\{Q, v^2/v^1 \cdot q_{p+1}\right\}$$

$$\forall p \in \left\{p \middle| 2 \leq p \leq n-1\right\}$$

$$q_1 = G - \sum_{p=2}^{n} q_p$$

$$l_1 = q_1/v^1$$

$$l_{p+1} = l_p + (q_{p+1} - q_p)/v^1$$

Lemma 2: If $v^1 > v^2$ and $cir\_time \cdot v^1 \leq Q$, then the optimal solution for the restricted decision on the temporal dimension satisfies

$$\hat{l}_1 = cir\_time \cdot v^2/v^1$$

$$\hat{l}_2 = cir\_time \cdot (1 + v^2/v^1)$$

The minimized objective function value is

$$\widehat{ft_n} = cir\_time \cdot v^2/v^1 + c + G/v^2$$

Proof: When $v^1 > v^2$, the sampling speed exceeds the testing speed, and the testing institution evidently experiences little idle time. Under this condition, the objective function can be expressed as:

$$ft_n = l_1 + c + G/v^2 + \max\left\{0, \sum_{i \in P, i > 1} \left(at_i - ft_{i-1}\right)\right\}$$

Among them, the unknown terms comprise l_1, q_n, and idle time $\max\left\{0, \sum_{i \in P, i > 1} (at_i - ft_{i-1})\right\}$. Since $v^1 > v^2$, provided that

$$l_p - l_{p-1} \leq q_{p-1}/v^2 \qquad p \in P$$

Equivalently,

$$q_p/v^1 \leq q_{p-1}/v^2$$

Alternatively expressed as

$$l_{p+1}-l_p \le \left(l_p-l_{p-1}\right)\cdot v^1/v^2$$

This ensures that

$$at_p \le ft_{p-1}$$

Consequently, the proof of Lemma 2 is partitioned into two distinct cases:
Case 1), where $\forall p \in P$, $q_p/v^1 \le q_{p-1}/v^2$;
Case 2), where $\exists p \in P$ makes $q_p/v^1 > q_{p-1}/v^2$ occur.

For Case 1), the testing institution exhibits no idle time after receiving the first batch of samples. The objective function can be expressed as

$$\min ft_n = l_1 + c + G/v^2$$

where [l_1] is the only variable. Accordingly, the objective function $\min ft_n$ can be rewritten as:

$$\min l_1$$

Consider that

$$l_2-l_1 \le \left(l_1-0\right)\cdot v^1/v^2$$

For two consecutive trips, we have:

$$l_2-l_1 \ge cir\_time$$

Therefore,

$$l_1 \ge cir\_time\cdot v^1/v^2$$

To minimize the objective function, let

$$l_1^* = cir\_time\cdot v^1/v^2$$

Under these conditions, Case 1) achieves the optimal solution, denoted as:

$$l_2^* = cir\_time\cdot\left(1+v^1/v^2\right)$$

$$ft_n^{\ *} = cir\_time\cdot v^1/v^2 + c + G/v^2$$

In the optimal solution of Case 1), the testing institution maintains no idle time after $at_1^{\ *} = l_1^* + c$. However, in Case 2), since $\exists p \in P$ makes $q_p/v^1 > q_{p-1}/v^2$ occur, the idle time necessarily occurs after $at_1 = l_1 + c$. Therefore, achieving $l_1 < l_1^*$ is only feasible when $ft_n < ft_n^{\ *}$.
Let

$$l_1 < l_1^*$$

That is,

$$l_1 < cir\_time\cdot v^1/v^2$$

Accounting for the temporal consumption of drone trips,

$$l_2-l_1 \ge cir\_time$$

We derive

$$l_2-l_1 \ge \left(l_1-0\right)\cdot v^1/v^2$$

At this point,

$$ft_1 = l_1 + c + q_1 = l_1 + c + l_1\cdot v^1/v^2$$

$$at_2 = l_2 + c = l_1 + c + \left(l_2 - l_1\right)$$

$$at_2 > ft_1$$

The objective function can be rewritten as:

$$\begin{aligned}&\min ft_n\\&=l_2+c+\left(G-q_1\right)/v^2+\max\left\{0,\sum_{i\in P,i>2}\left(at_i-ft_{i-1}\right)\right\}\\&=l_2+c+G/v^2-l_1\cdot v^1/v^2+\max\left\{0,\sum_{i\in P,i>2}\left(at_i-ft_{i-1}\right)\right\}\end{aligned}$$

Evidently, ft_n constitutes a lower bound ft_n':

$$ft_n'=l_2+c+G/v^2-l_1\cdot v^1/v^2$$

Moreover, the optimal solution ft_n^* from Case 1) can be rewritten as:

$$ft_n^*=l_2^*+c+G/v^2-l_1^*\cdot v^1/v^2$$

Compare ft_n' to ft_n^*:

$$ft_n'-ft_n^*=\left(l_1^*-l_1\right)\cdot v^1/v^2-\left(l_2^*-l_2\right)$$

As shown in Fig. 3.8, since

$$\begin{gathered}l_2^*-l_1^*=cir\_time\\l_2-l_1\geq cir\_time\\l_2^*-l_2\leq l_1^*-l_1\end{gathered}$$

Also,

$$v^1/v^2>1$$

Therefore,

$$\left(l_1^*-l_1\right)\cdot v^1/v^2-\left(l_2^*-l_2\right)>0$$

This proves that

$$ft_n>ft_n'>ft_n^*$$

Consequently, any solution obtained in Case 2) is inferior to the optimal solution in Case 1). Figure 12 shows the comparison of timelines for Case 1) and Case 2).

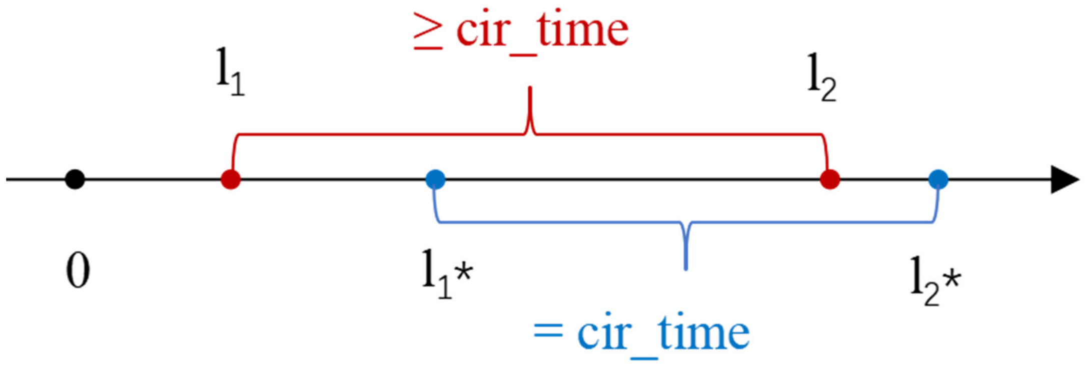

Figure 12. Timeline comparison between Case 1 and Case 2.

In conclusion, under the following conditions:

$$\begin{gathered}v^1>v^2\\cir\_time\cdot v^1\leq Q\end{gathered}$$

Case 1) achieves the optimal solution for the decision on the temporal dimension:

$$\hat{l}_1=l_1^*$$

$$\widehat{l_2} = l_2^*$$

$$\widehat{ft_n} = ft_n^*$$

The condition $l_{p+1} - l_p \le (l_p - l_{p-1}) \cdot v^1 / v^2$ required for Case 1) is relatively attainable. For instance, by adopting

$$l_p = \min\{l_p + cir\_time, L\} \qquad \forall p \in P, p > 2$$

we can achieve

$$l_{p+1} - l_p = l_p - l_{p-1} \le (l_p - l_{p-1}) \cdot v^1 / v^2$$

Multiple methodologies exist to satisfy this condition, and the specific implementation approach does not affect the optimality of the solution.