Journal of Industrial and Management Optimization

https://www.aimspress.com/journal/jimo

JIMO, 22(6): 2647–2682.
DOI:10.3934/jimo.2026097
Received: 19 December 2025
Revised: 08 April 2026
Accepted: 13 April 2026
Published: 13 May 2026

Research article

mCube: A multinomial micro-level reserving model

Emmanuel Jordy Menvouta[1], **Robin Van Oirbeek**[2,*] **and Tim Verdonck**[1,3]

[1] KU Leuven, Department of Mathematics, Leuven, Belgium
[2] Allianz Benelux, Boulevard Roi Albert II 32, Brussels, Belgium
[3] University of Antwerp – imec, Department of Mathematics, Antwerp, Belgium

* **Correspondence:** Email: robin.vanoirbeek@gmail.com.

Appendix

A. Variables in the data set

The original data set obtained from the European insurer was further preprocessed by the following steps:

- A transition occurs in the multi-state model when a payment was recorded higher than *"minPayVal"* (or lower than *"minPayVal"*, in the case of a reimbursement). Therefore, payments are lumped together when necessary.
- Several variables were created based on the time at which the claim occurred, the time of reporting of the claim, and the time at which payments were made. These variables include fastRep, which is an indicator of whether the claim was reported less than 30 days from when it occurred, and finYear, which is the financial year in which a payment happens. Other variables that are also created are deltRep, representing the reporting delay, inStateTime, which is the time a claim spends in a specific state, and inProcTime, which is the time a claim spends in the entire multi-state process. The variable delt1PayTimeTrans is the time since the previous payment. All these time variables are expressed in the equivalent number periods of 30 days. Moreover, a maximum of 15 periods have been set such that if a claim has more than 15 periods, it is either forced to move to the next state or out of the multi-state process if it has reached the maximum number of transitions.
- Variables are created from the payment amount: The variable delt0Pay is the amount of the payment in the current state, the variable delt1Pay is the payment amount of the previous state,

and the variable cumDelt1Pay is the cumulative payment amount from all the previous states of the claim.

B. Building the IBNR data set

Once the yearly number of IBNR claims have been estimated and the monthly reporting delay evaluated, an IBNR multi-state data set must be constructed and passed through the multi-state process starting from the reporting state. The variables from Appendix A are constructed in the following way: The reporting delay (deltRep) depends on the accident year and the estimated reporting month, fastRep is 0 as the claim is IBNR, procTime, and stateTime are both 1, deltPay, deltPayTime, and cumDeltPay are NA as there has been no previous payment.

C. Algorithm for simulating open claim reserves trajectories

This section presents the algorithm for simulating claims reserves as illustrated and explained in Section 4.4 of the main manuscript. For this algorithm, We need the following elements:

- **timeMods**: The fitted time models. This should be a list of length *"maxMod"*.
- **payMods**: The fitted payment models. This should be a list of length *"maxMod"*.
- **testData**: The test data on which to simulate reserves.
- **splits**: Splitting points for the numeric variables that were binned. This should also contain the levels for the time variables that are categorized.
- **fixedTimeMax**: Maximum amount of time a claim is allowed to stay in a state. This should be of length *"maxMod"*.
- **nSims**: Number of trajectories to be simulated for each claim.
- **npmax**: Maximum number of transitions we allow a claim to make. This is used to capture claims with longer developments.

We note that when a claim has stayed for too long in a state as defined by *"fixedStateTimeMax"*, we modify the estimated discrete-time hazard functions to

$$
\begin{aligned}
\tilde{\lambda}_{j,j+1}(t \mid \mathbf{x}_{k,t}) &= \hat{\lambda}_{j,j+1}(t \mid \mathbf{x}_{k,t}) + \frac{1 - \hat{\lambda}_{j,j+1}(t \mid \mathbf{x}_{k,t}) - \hat{\lambda}_{j,tp}(t \mid \mathbf{x}_{k,t}) - \hat{\lambda}_{j,tn}(t \mid \mathbf{x}_{k,t})}{3}, \\
\tilde{\lambda}_{j,j}(t \mid \mathbf{x}_{k,t}) &= \hat{\lambda}_{j,tp}(t \mid \mathbf{x}_{k,t}) + \frac{1 - \hat{\lambda}_{j,j+1}(t \mid \mathbf{x}_{k,t}) - \hat{\lambda}_{j,tp}(t \mid \mathbf{x}_{k,t}) - \hat{\lambda}_{j,tn}(t \mid \mathbf{x}_{k,t})}{3}, \\
\tilde{\lambda}_{j,tn}(t \mid \mathbf{x}_{k,t}) &= \hat{\lambda}_{j,tn}(t \mid \mathbf{x}_{k,t}) + \frac{1 - \hat{\lambda}_{j,j+1}(t \mid \mathbf{x}_{k,t}) - \hat{\lambda}_{j,tp}(t \mid \mathbf{x}_{k,t}) - \hat{\lambda}_{j,tn}(t \mid \mathbf{x}_{k,t})}{3}, \\
\tilde{\lambda}_{j}, j(t \mid \mathbf{x}_{k,t}) &= 0.
\end{aligned}
\tag{1}
$$

Similarly, when a claim has reached state *"npmax"*-1, we need to modify the transition probabilities in the following way:

$$
\tilde{\tilde{\lambda}}_{npmax-1,npmax}(t \mid \mathbf{x}_{k,t}) = 0,
$$

$$
\begin{aligned}
\tilde{\tilde{\lambda}}_{npmax-1,tp}(t \mid \mathbf{x}_{k,t}) &= \hat{\lambda}_{npmax-1,tp}(t \mid \mathbf{x}_{k,t}) + \hat{\lambda}_{npmax-1,npmax}(t \mid \mathbf{x}_{k,t}),\\
\tilde{\tilde{\lambda}}_{npmax-1,tn}(t \mid \mathbf{x}_{k,t}) &= \hat{\lambda}_{npmax-1,tn}(t \mid \mathbf{x}_{k,t}),\\
\tilde{\tilde{\lambda}}_{npmax-1,npmax-1}(t \mid \mathbf{x}_{k,t}) &= \hat{\lambda}_{npmax-1,npmax-1}(t \mid \mathbf{x}_{k,t}).
\end{aligned}
\tag{2}
$$

If a claim stays too long in state *"npmax"*-1, we can also modify (2) using (1).

We now give in Algorithm 1 the pseudocode for the simulation of open claim trajectories.

Algorithm 1 pseudocode for the simulation of open claims trajectories

```
Require: timeMods, payMods,testData, splits, fixedTimeMax, nSims, npmax.
Ensure: trajectories for each open claim
  for each claim k in testData do
      for trajectory ← 1 . . . , nSims do
          while trajectory is still open do
              Estimate transition probabilities
              if stateTime ≥ fixedStateTimeMax or state = npmax-1 then
                  Update transition probabilities
              end if
              Simulate next state using timeMods
              if transitioned to a state with payment then
                  Estimate payment in current state using payMods
              end if
              if not transitioned to a terminal state then
                  Update covariates using splits
              end if
          end while
          Compute reserve for that trajectory
      end for
  end for
```

D. Selecting hyper-parameters

During the preprocessing stage, we need to decide on the value for the hyper-parameters *"nMinLev"* and *"nGroups"* that are necessary for binning the continuous predictors. We suggest to set *"nMinLev"* to at least 30 for statistical significance of estimated parameters. We suggest to set *"nGroups"* between 5 and 15. We also have to define *"minPayVal"*, which is the minimum amount paid for a non-terminal payment to be considered. Intermediate payments that are lower in absolute value than this amount, will be aggregated and considered as a single payment. We suggest to discuss with the business to determine what is a meaningful payment amount. We also define *"perLen"* as the number of days in one time period. We recommend to choose *"perLen"* so that it represents either monthly, quarterly, or yearly information. For binning the continuous variable inStateTime, representing the time a claim spends in a specific state, the minimum number of observations in each category should be *"nMinTimeLev"*. Note that this hyper-parameter can differ from *"nMinLev"*. We chose a value of 30 for statistical significance of estimated parameters. Other hyper-parameters include *"nMaxLevInstate"*, the maximum number of time periods a claim is allowed to stay in the same state, and *"nMaxLevInProc"*, the maximum number of periods a claim is allowed to stay in the whole multi-state process. These two hyper-parameters should be chosen so that only a small percentage, say 1%, of the claims stay for more than these hyper-parameters in the state or in the process. In order to have valid statistical models, we need a sufficient number of observations. Therefore, we define *"nMinModT"* as the minimum number of observations required to fit a multinomial model with predictors in the time process, and we define *"nMinNoModT"* similarly in the case of no predictors. Note that when the chosen value for *"nMinModT"* is smaller than the number of predictors multiplied by *"nTimesParamT"*, it is replaced by this product. However, it is quite likely that the number of required observations is not met for the time model in state $S_{npmax-1}$, since claims with a large number of payments are rare. Therefore, it was decided to construct *"maxMod"* unique time models for states $S_1, \ldots, S_{maxMod}$. For the states $S_{maxMod+1}, \ldots, S_{npmax-1}$, the model of state S_{maxMod} will be reused. This implies that the model of state S_{maxMod} is based on payments that happened from the $maxMod^{th}$ payment on for each claim.

During the modelling of the payment process, we have to choose *"nBins"*, which equals $L + 1$ and thus represents the number of bins during the splicing procedure. Besides the number of bins, the splitting points **b** themselves need to be determined as well. Finally, we define *"nMinModP"* as the minimum number of observations to fit a multinomial model with predictors in the payment process, and we define *"nMinNoModP"* similarly in the case of no predictors. Once again, when the chosen value for *"nMinModP"* is smaller than the number of predictors multiplied with *"nTimesParamP"*, it is replaced by this product.

When simulating trajectories for open claims, we need to choose a value for *"nSims"*, *"fixedTimeMax"*, and *"npmax"*. The value for *"fixedTimeMax"* should make business sense, and should be such that only a small percentage of the claims stay in a state for longer than this; hence, we set it to 24. The value for *"npmax"* should be large enough to capture claims with longer developments, and hence we set it to 50.

Taking into account the time necessary to fit our multi-state process, a cross-validation strategy for hyper-parameter tuning proved to be extremely time consuming. We therefore propose to set values for these hyper-parameters based on actuarial experience and observed results.

Table 1. Hyper-parameters for the multinomial multi-state model.

	Hyper-parameters
Pre-processing	$nMinLev = 30$; $nGroups = 5$; $minPayVal = 200$; $perLen = 30$
Time process	$nMinTimeLev = 30$; $nMaxLevInState = 12$; $nMaxLevInProc = 24$; $maxMod = 6$ $nMinModT = 500$; $nMinNoModT = 50$ $nTimesParamsT = 5$; $npmax = 30$
Payment process	$nBins = 4$; $nMinModP = 500$; $nMinNoModP = 50$; $nTimesParamsP = 5$
Open claims simulations	$N_{sim} = 100$; $fixedTimeMax = 24$; $npmax = 50$

E. Splitting points for past payment information

The split obtained after binning the continuous predictors deltPay and cumDeltPay are shown, respectively, in Tables 2 and 3. The binning thresholds in Tables 2 and 3 correspond to economically meaningful categories in bodily injury claims management. For the payment distribution, the bins represent: (1) small administrative or provisional payments, typically below 1,000; (2) moderate partial payments associated with initial medical costs or temporary disability compensation; (3) large payments reflecting significant medical procedures, permanent disability settlements, or legal costs; and (4) extreme payments in the GPD tail, often corresponding to catastrophic injury or fatality claims.

For the time covariates, the bins for `inStateTime` reflect typical claim management cycles: the initial assessment phase (0–3 months), active negotiation or treatment (3–12 months), and extended litigation or complex medical cases (>12 months). The `inProcTime` bins capture the overall claim duration from reporting to current evaluation, with longer durations indicating claims that have persisted through multiple payment states.

These interpretations are consistent with standard actuarial practices for European bodily injury portfolios and should be adapted when applying mCube to other lines of business.

Table 2. Splitting points for the previous payment (deltPay) in each data set.

State				
S_1	S_2	S_3	S_4	S_{5+}
586.20	-4045.03	-657.02	0.00	0.00
1,247.06	-1,169.81	1,179.08	965.04	611.74
2,584.57	1,669.23	3,615.08	3,576.90	2,596.18
7,955.07	3,805.08	4,775.55	5,070.10	3,501.23

Table 3. Splitting points for the cumulative previous payment (cumDeltPay) in each data set.

State				
S_1	S_2	S_3	S_4	S_{5+}
80.00	24.62	146.26	249.78	761.56
380.70	258.52	1,284.04	4,651.18	4,630.25
1,416.29	634.86	5,048.92	8,626.82	10,292.00
2,631.85	7285.79	6,954.32	12,356.65	18,322.42

F. Partial dependence plots of the time models

In this section, we present the partial dependence plots of the time models for states S_3, S_4, and S_{5+}. From Figures 1, 2, 3, we observe similar marginal effects of covariates as in state S_2.

G. Partial dependence plots of the payment models

This section shows the marginal effect of predictors in the payment models for states S_3, S_4, and S_{5+} using partial dependence plots. From Figures 4, 5, 6, we observe similar marginal effects of covariates.

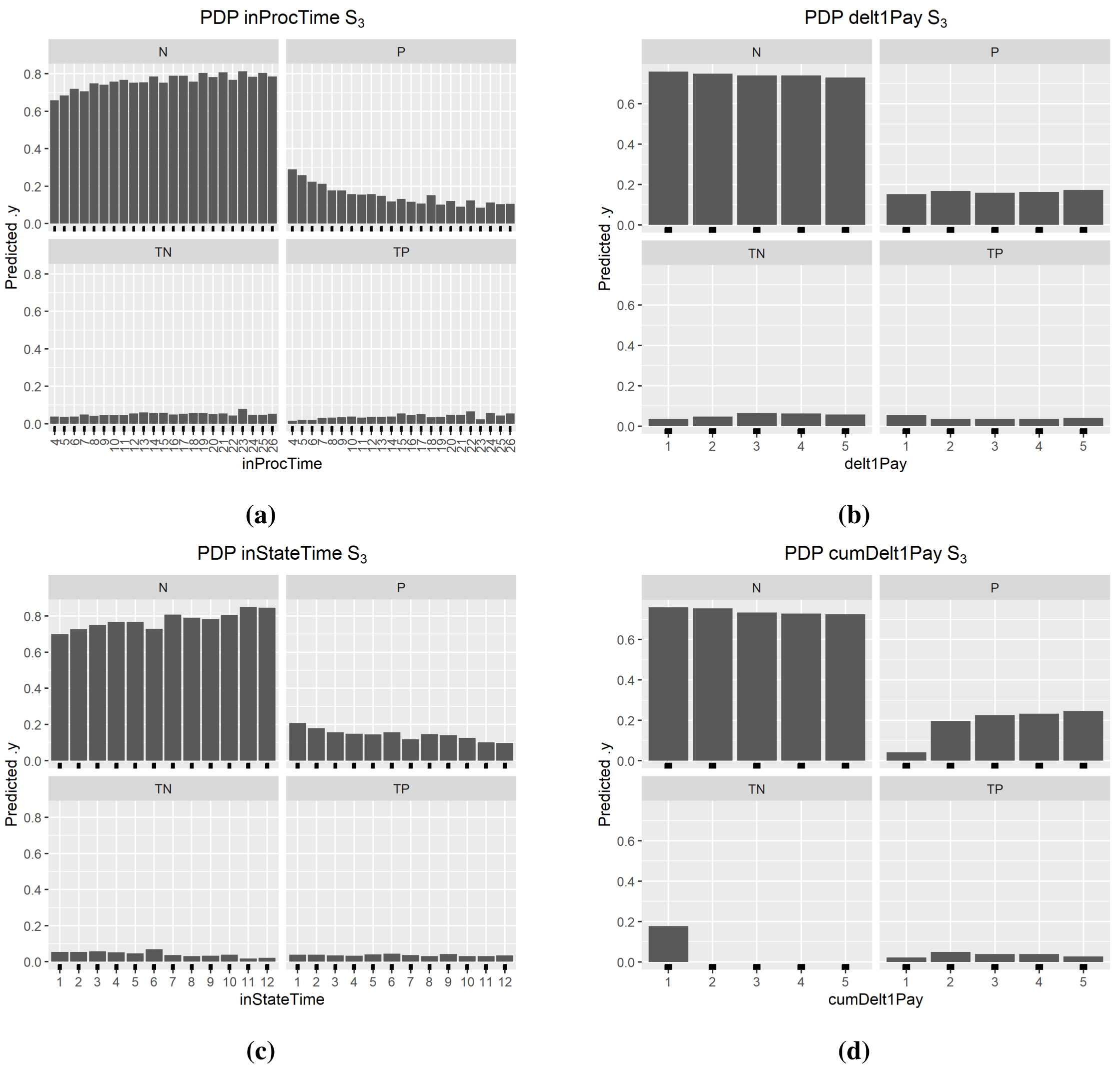


Figure 1. Partial dependence plots representing the marginal effect on transition probabilities of the time spent in the process (a), the previous payment size (b), the time spent in the state (c), and the cumulative previous payment size (d) for claims in S_3.

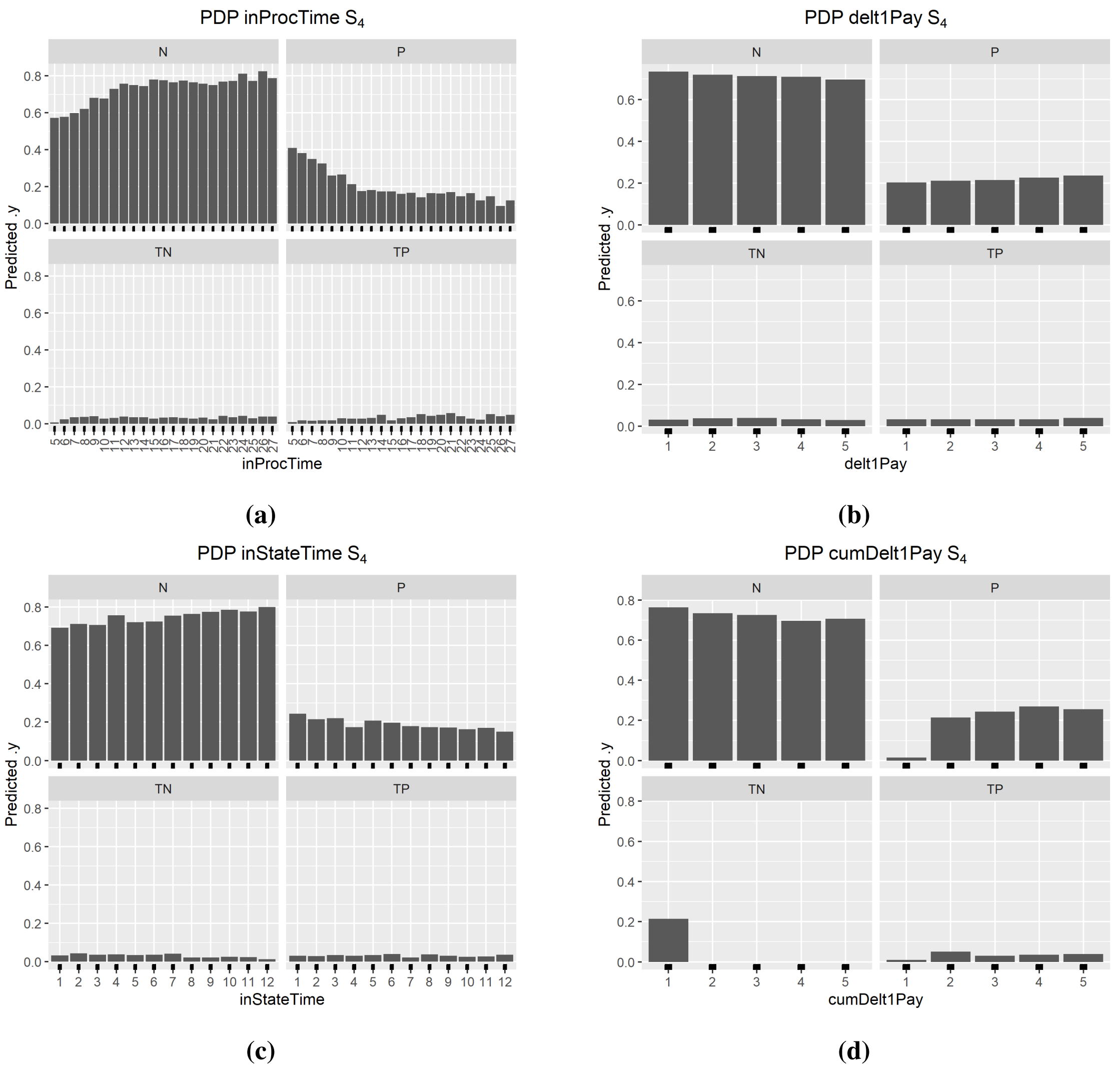
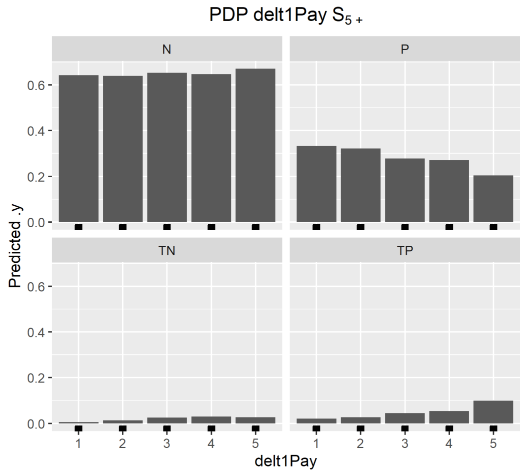
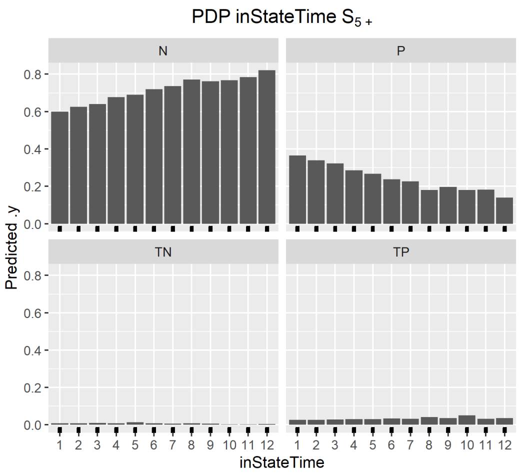


Figure 2. Partial dependence plots representing the marginal effect on transition probabilities of the time spent in the process (a), the previous payment size (b), the time spent in the state (c), and the cumulative previous payment size (d) for claims in S_4.

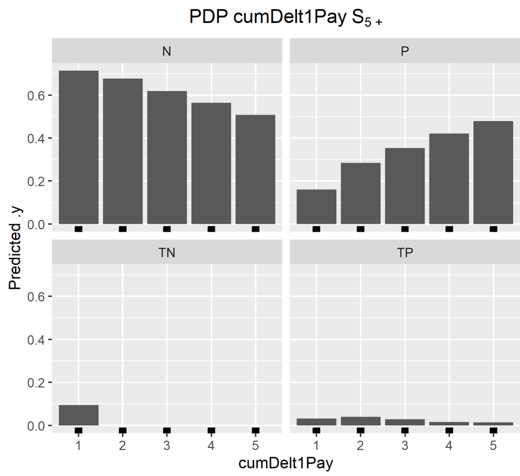
(a)



(b)



(c)



(d)

Figure 3. Partial dependence plots representing the marginal effect on transition probabilities of the time spent in the process (a), the previous payment size (b), the time spent in the state (c), and the cumulative previous payment size (d) for claims in S_{5+}.

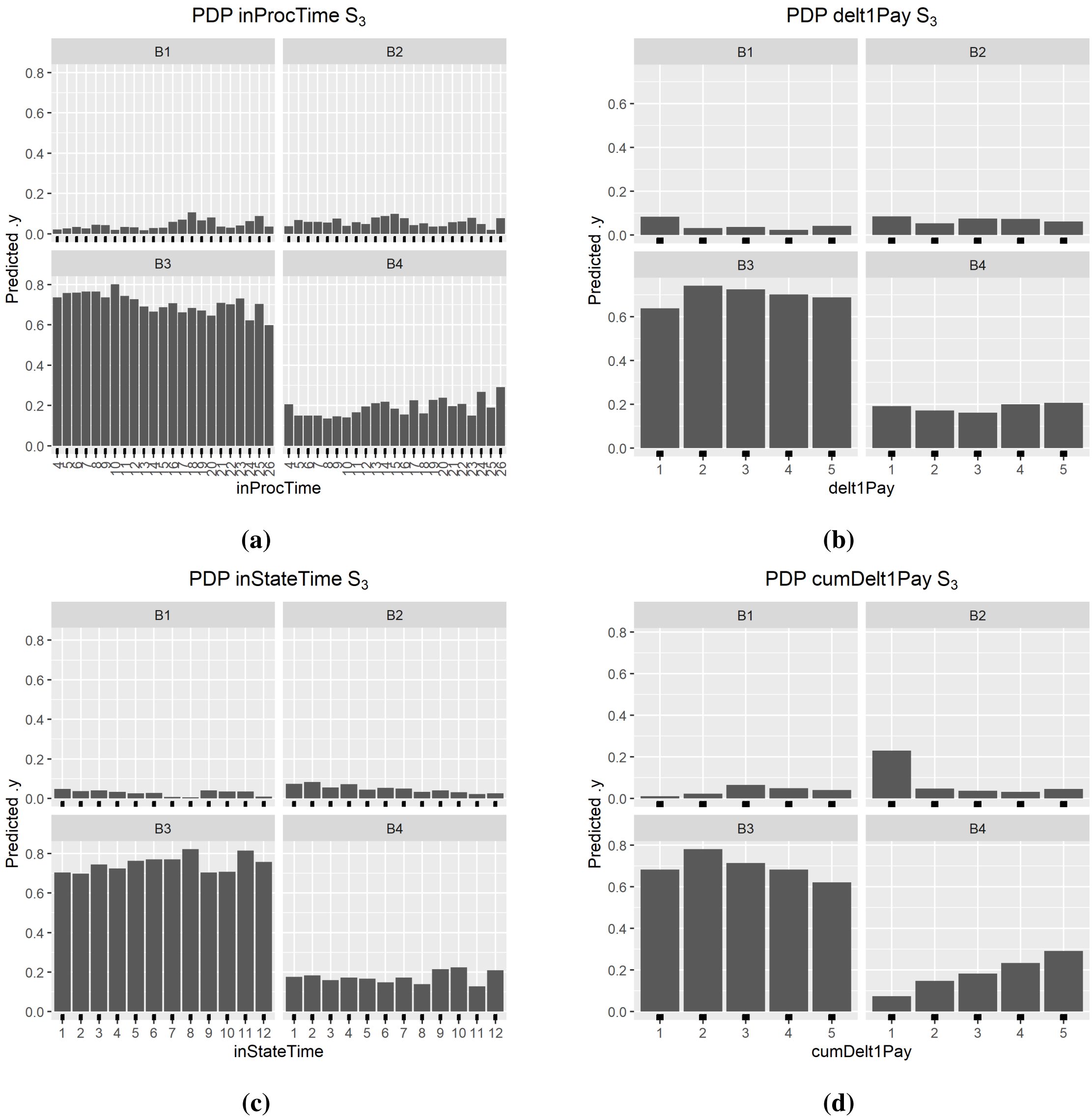


Figure 4. Partial dependence plots representing the marginal effect on probabilities to belong to a payment bin of the time spent in the process (a), the previous payment size (b), the time spent in the state (c), and the cumulative previous payment size (d) for claims in S_3.

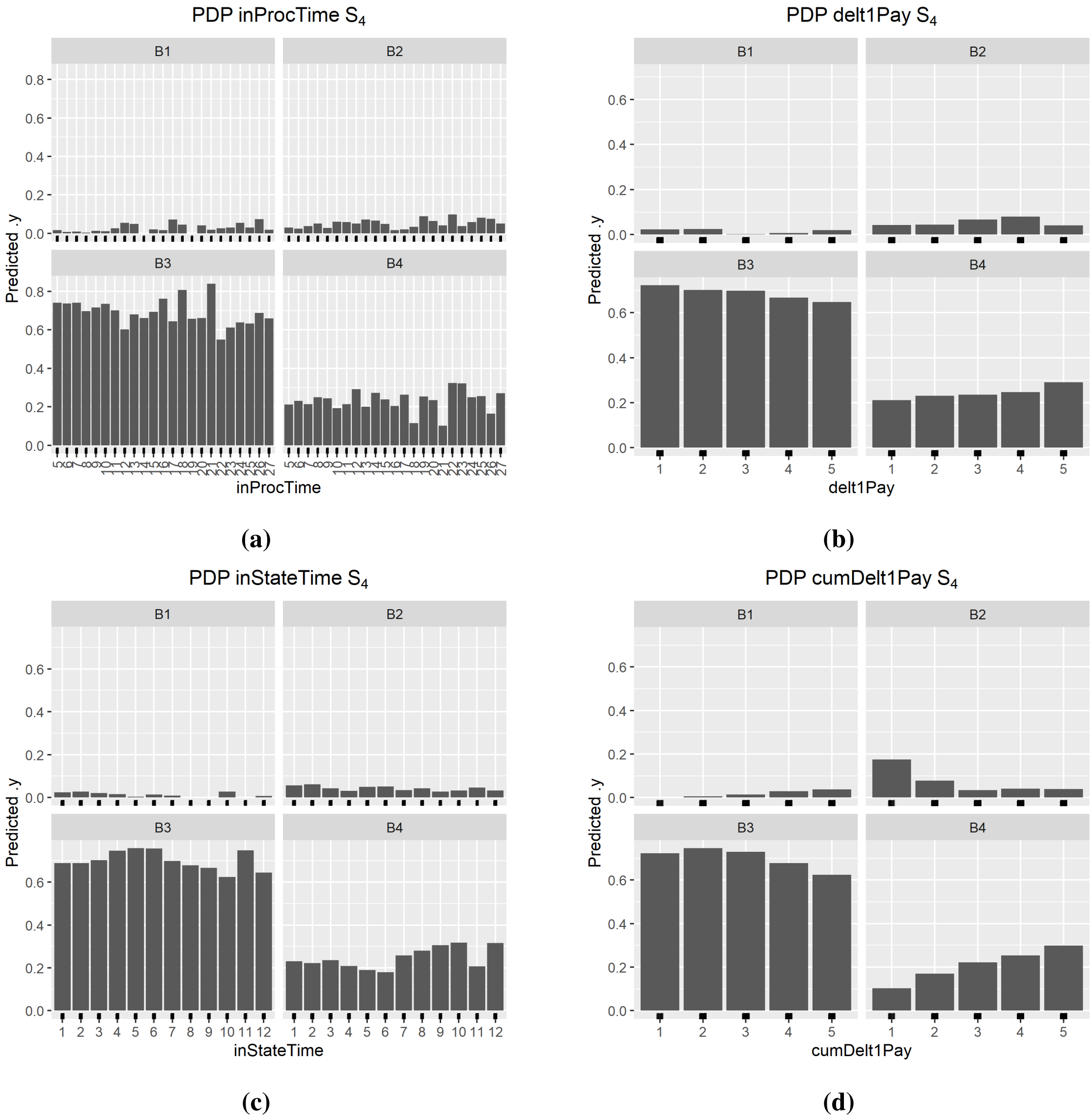
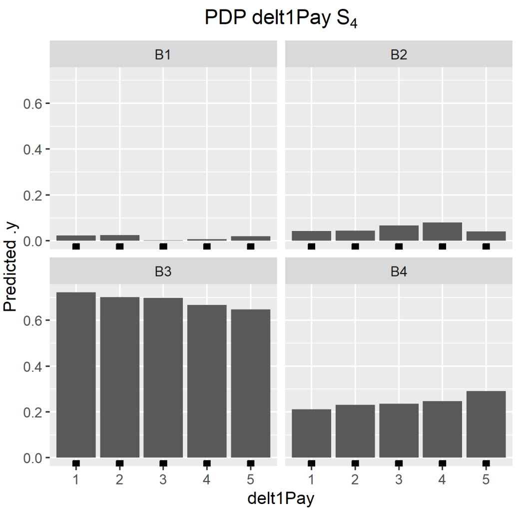
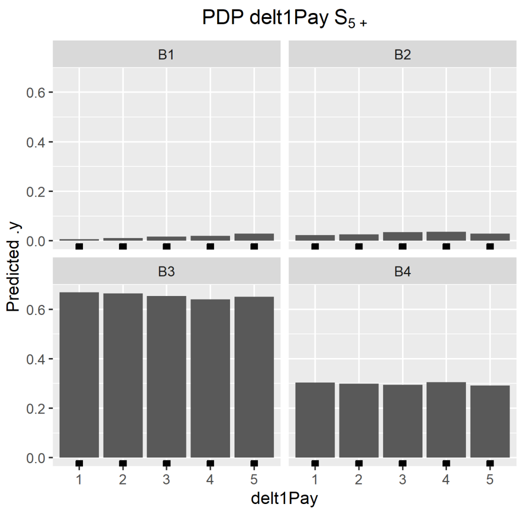


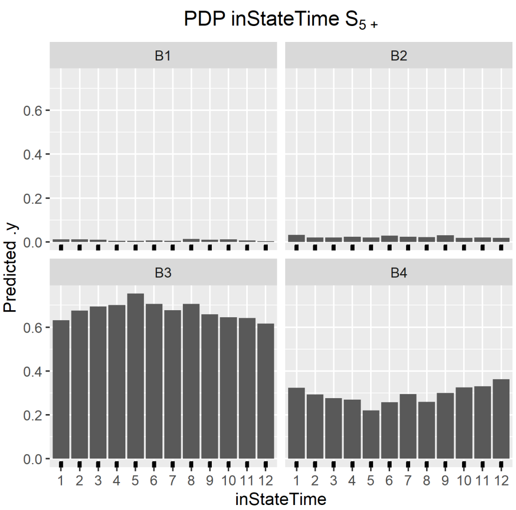
Figure 5. Partial dependence plots representing the marginal effect on probabilities to belong to a payment bin of the time spent in the process (a), the previous payment size (b), the time spent in the state (c), and the cumulative previous payment size (d) for claims in S_4.



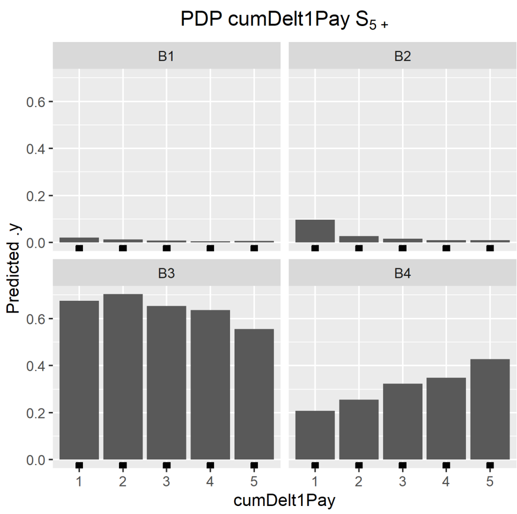
(a)



(b)



(c)



(d)

Figure 6. Partial dependence plots representing the marginal effect on probabilities to belong to a payment bin of the time spent in the process (a), the previous payment size (b), the time spent in the state (c), and the cumulative previous payment size (d) for claims in S_{5+}.

H. Assumptions of the Chain-Ladder.

Figure 8 shows the exponential quantile-quantile (Q-Q) plot for the reporting delay for each of the accident years. We observe that for the accident years 2011 and 2012, there is a change in the pattern of reporting for the claims. Figure 7 shows the residual plots for the Mack chain-ladder model used to compute the reserves. We observe patterns in the residual plots, which hint to the assumptions not being satisfied and which explain the under-reserving.

Table 4. Predicted yearly IBNR claim counts when removing accident years 2011–2012 and using accident year 2010 as reference.

	2006	2007	2008	2009	2010	Total
Database	0	0	1	7	176	184
CL 2.5%	0	0	0	2	134	136
CL mean	0	0	3	18	201	222
CL 97.5%	0	0	15	43	273	331

Table 5. Observed reserves and mean of the predicted reserves for the subset of bodily injury claims when the accident years 2011–2012 are removed and the year 2010 is used as reference year.

	Database	mCube	chain-ladder
RBNS reserve	30,196,984	34,701,666	-
IBNR reserve	3,808,605	1,506,680	-
Total reserve	31,388,212	36,208,346	34,629,271
PE	0	15.4%	10.3%

I. Summary statistics Allianz dataset.

Table 6. Absolute and relative frequencies of the total number of transitions for each claim.

	Total number of transitions						
	1	2	3	4	5	6	> 6
abs.freq	2,275	7,429	7,250	3,125	1,481	848	3,413
rel.freq	8.81 %	28.77 %	28.08 %	12.10 %	5.74 %	3.28%	13.22%

J. probability-probability (P-P) plots of the fitted payment distributions.

In this section, we present probability-probability (P-P) plots of the fitted components in our spliced modelling of the payment distribution in each state.

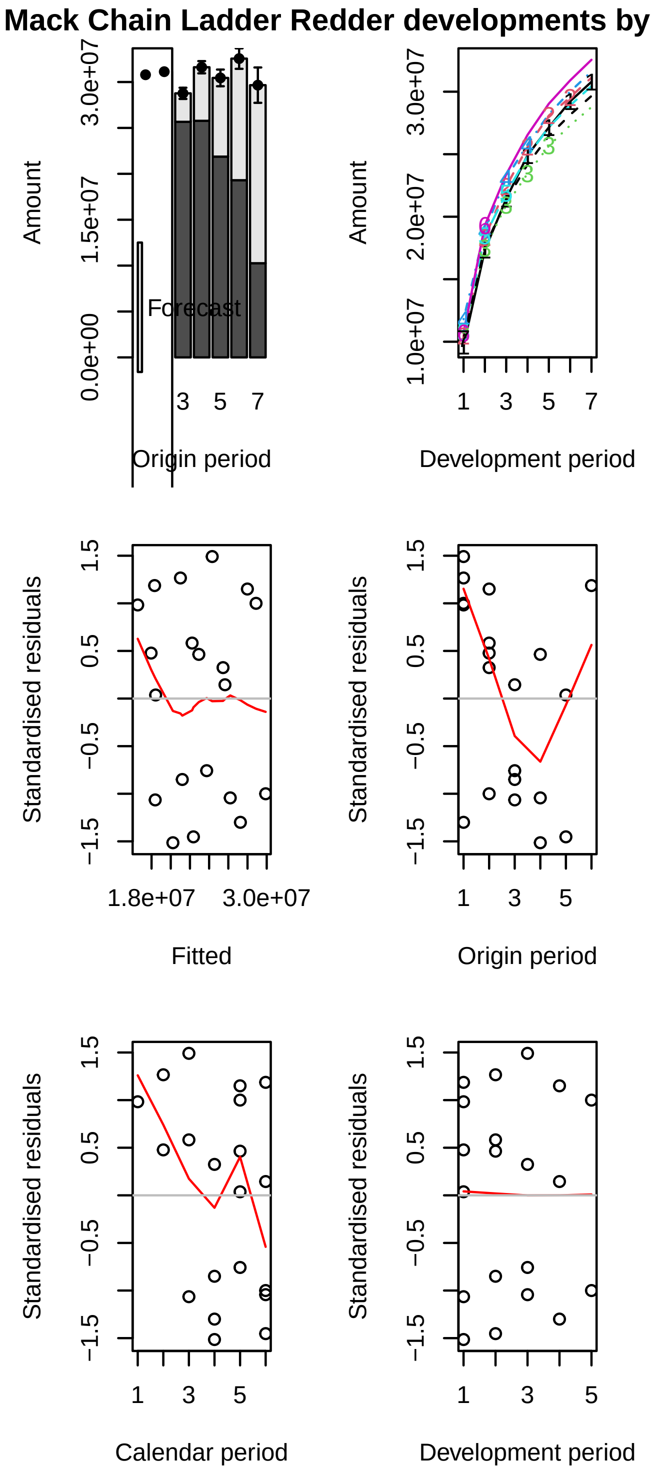


Figure 7. Assumptions of the Mack chain-ladder used to compute the reserves.

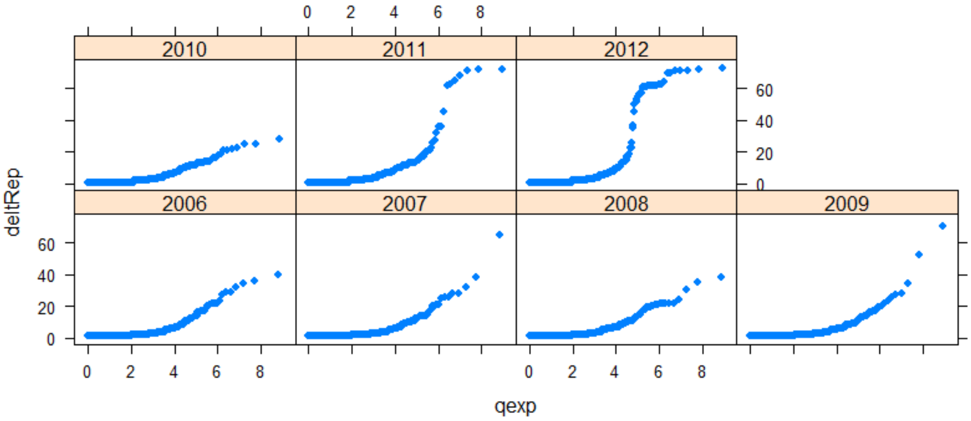


Figure 8. Exponential quantile-quantile (Q-Q) plot for the reporting delay by accident year.

Table 7. Summary statistics for the time spent in each state.

State	Min	Median	Mean	Max	IQR	Skewness	Kurtosis
S_0	1.00	2.00	5.36	72.00	5.00	3.36	14.46
S_1	1.00	3.00	5.89	72.00	5.00	3.07	12.55
S_2	1.00	4.00	7.02	72.00	6.00	2.82	10.14
S_3	1.00	4.00	6.44	72.00	6.00	2.59	8.50
S_4	1.00	3.00	6.16	61.00	6.00	2.68	9.07
S_{5+}	1.00	3.00	4.64	61.00	4.00	3.10	12.91

Table 8. Summary statistics for the payment distribution in each state.

State	Min	Median	Mean	Max	IQR	Skewness	Kurtosis
S_0	-18,365.69	1,149.30	2,808.84	343,463.26	2,360.00	22.35	679.28
S_1	-45,968.06	662.06	1,390.16	198,923.65	1,747.04	10.19	256.32
S_2	-58,154.22	808.90	2041.66	165,782.32	1,936.61	8.89	131.32
S_3	-53,078.64	924.56	2,549.96	273,638.48	2,291.40	8.67	154.79
S_4	-33,892.65	1,013.95	2,550.49	97,253.20	2,622.99	5.19	57.49
S_{5+}	-123,556.32	938.34	4,088.08	586,846.69	2,932.84	16.84	472.04

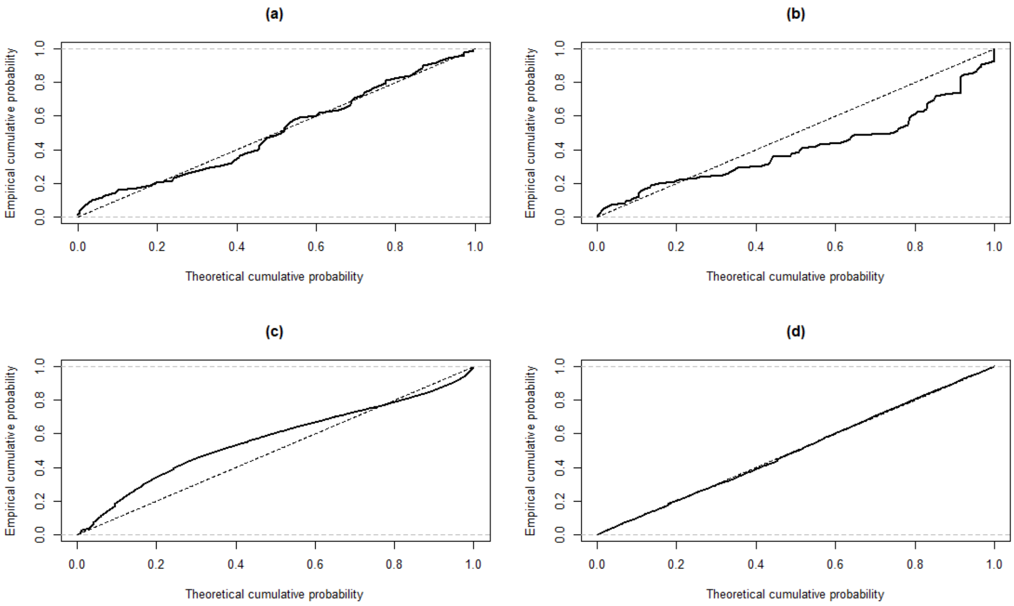


Figure 9. Empirical distribution of the ranks in the fitted model for the first payment, which consists of (a) large negative payments, (b) small negative payments, (c) small positive payments, and (d) large positive payments.

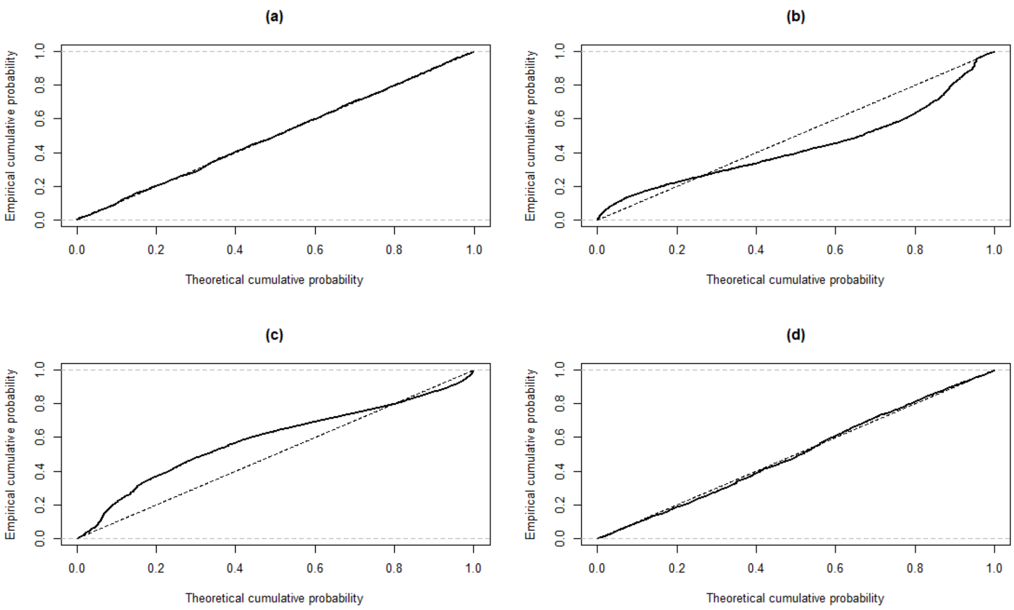


Figure 10. Empirical distribution of the ranks in the fitted model for the second payment, which consists of (a) large negative payments, (b) small negative payments, (c) small positive payments, and (d) large positive payments.

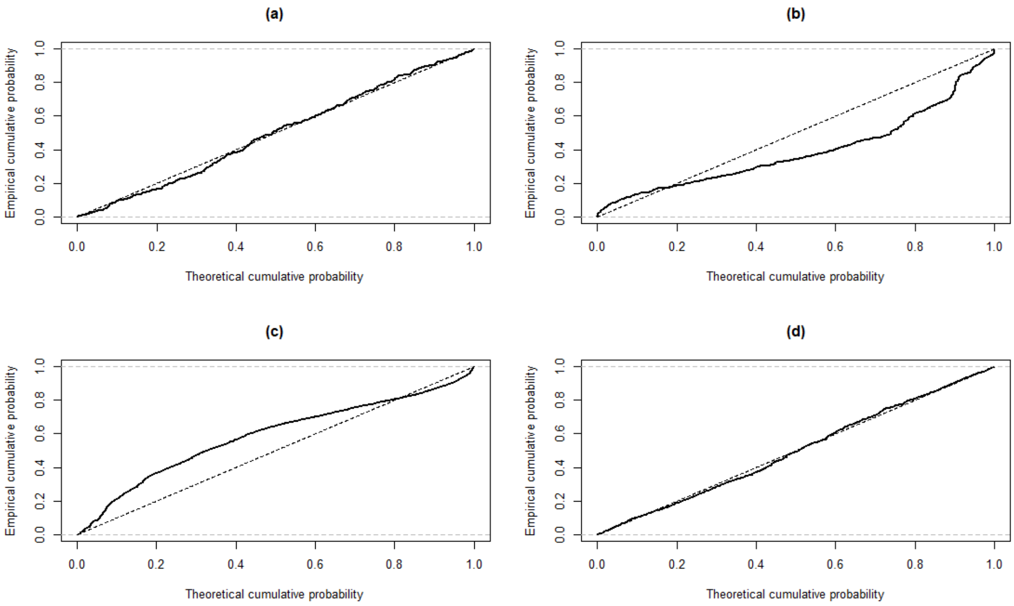


Figure 11. Empirical distribution of the ranks in the fitted model for the third payment, which consists of (a) large negative payments, (b) small negative payments, (c) small positive payments, and (d) large positive payments.

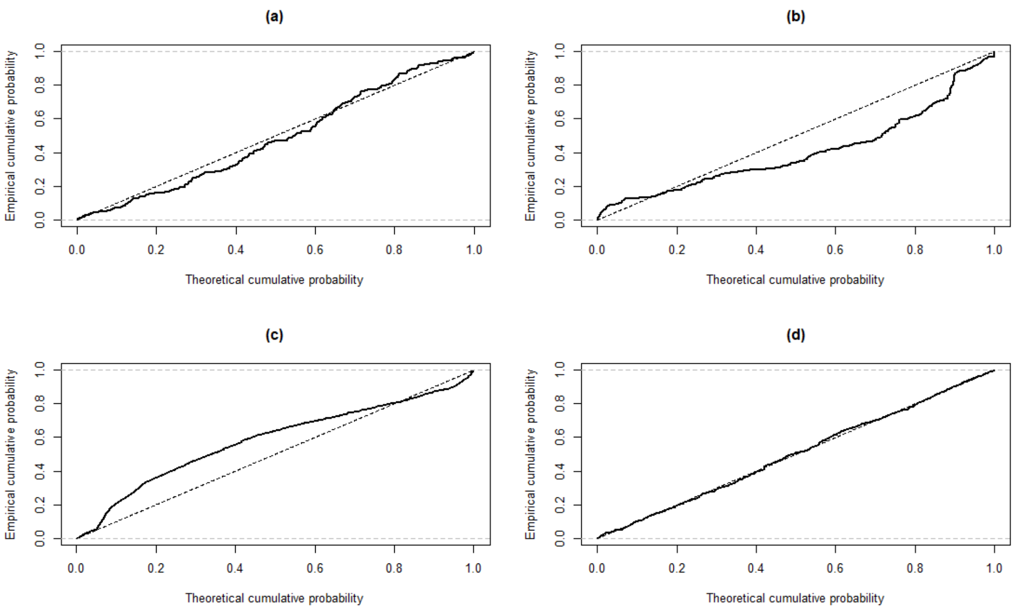


Figure 12. Empirical distribution of the ranks in the fitted model for the fourth payment, which consists of (a) large negative payments, (b) small negative payments, (c) small positive payments, and (d) large positive payments.

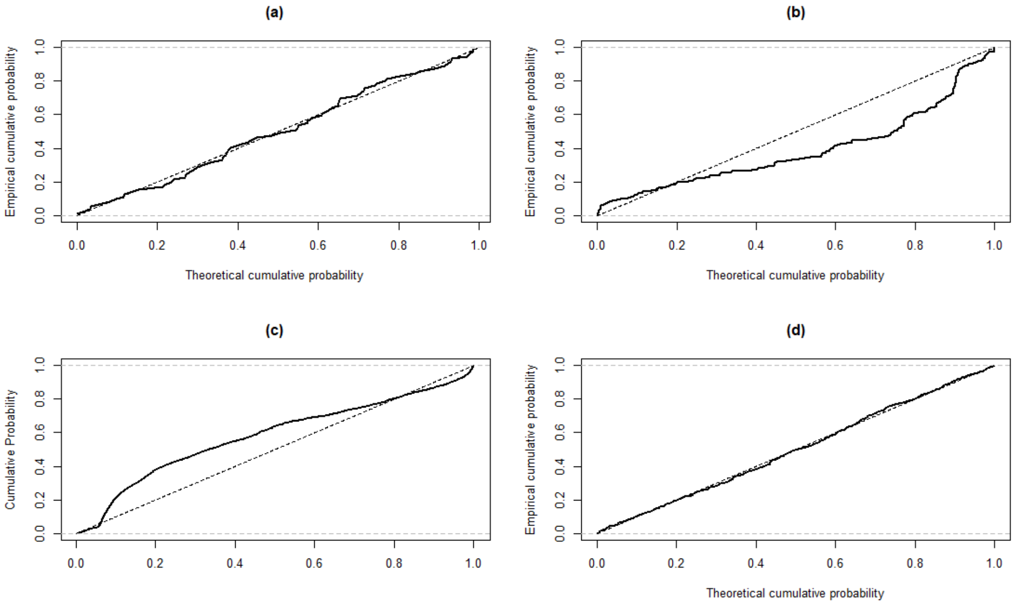


Figure 13. Empirical distribution of the ranks in the fitted model for the fifth payment, which consists of (a) large negative payments, (b) small negative payments, (c) small positive payments, and (d) large positive payments.

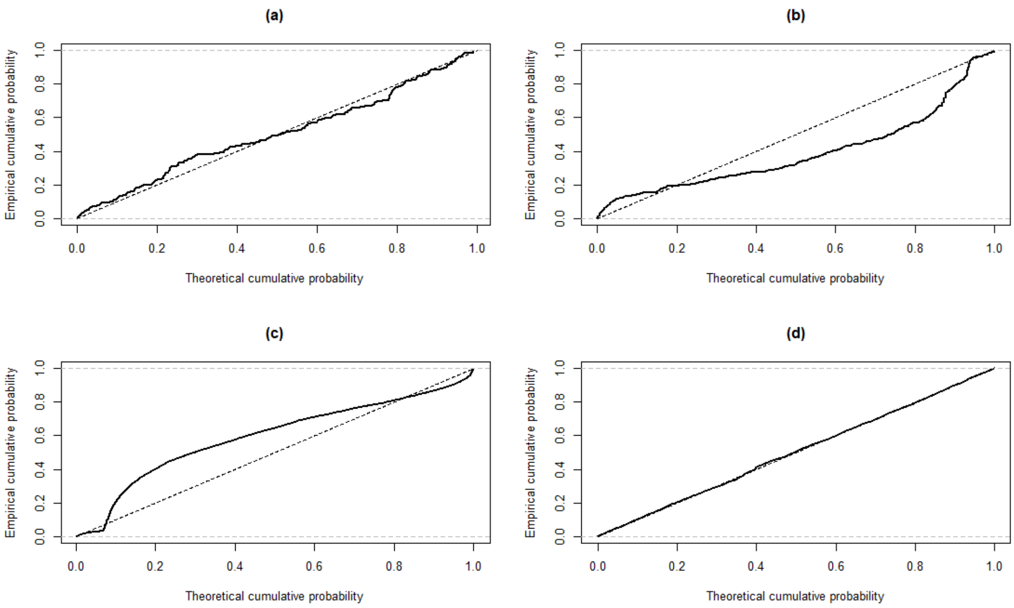


Figure 14. Empirical distribution of the ranks in the fitted model for the sixth payment, which consists of (a) large negative payments, (b) small negative payments, (c) small positive payments, and (d) large positive payments.

AIMS Press