



*Research article*

## Eco-label selection and information sharing in green supply chains under demand uncertainty: A perspective on two types of green products

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### Appendix A

#### 1. The solution procedure for Model NDS

In NDS scenario, the market size observed by the government and manufacturer is  $E[A]$  (i.e.,  $a$ ), whereas the market size of the retailer is  $E[A|F]$  (i.e.,  $(1 - \rho)a + \rho F$ ).

First, the retailer’s profit concave on  $p$  requires  $\frac{\partial^2 E[\pi_r^{NDS}|F]}{\partial p^2} = -2 < 0$ . Using the first-order condition in Eq. (3), we have  $p = \frac{E[A|F]+bg-t+w}{2}$ .

Since the retailer does not share the DFI with the manufacturer, the manufacturer’s belief for the retail price is  $p = \frac{E[A]+bg-t+w}{2}$ . Substituting it into Eq. (2), the manufacturer’s profit concave on  $g$  and  $w$  requires that the Hessian matrix of  $E[\pi_m^{NDS}]$  is negative definite and the determinant of it is non-negative. Thus, we get  $H(g, w) = \begin{bmatrix} -2f & \frac{b}{2} \\ \frac{b}{2} & -1 \end{bmatrix}$ ,  $\det[H(g, w)] = \frac{8f-b^2}{4} > 0$ .

Using the first-order conditions in Eq. (2), we have  $g = \frac{b(E[A]-t)}{8f-b^2}$  and  $w = \frac{4f(E[A]-t)}{8f-b^2}$ . Substituting them and  $p = \frac{E[A]+bg-t+w}{2}$  into Eq. (1),  $SW$  concave on  $t$  requires  $\frac{\partial^2 E[SW^{NDS}]}{\partial t^2} =$

$\frac{2f(2b+b^2-2f)}{(8f-b^2)^2} < 0$ . Using the first-order condition in Eq. (1), we have  $t = \frac{2E[A](b+3f)}{2b+b^2-2f}$ .

Finally, using the first-order conditions in Eqs. (1)-(3), we obtain the optimal solutions and then get the expected demand, enterprise profits and  $SW$ .

## 2. The solution procedure for Model SDS

In SDS scenario, the market size observed by the government is  $E[A]$  (i.e.,  $a$ ), whereas the market size of the manufacturer and retailer is  $E[A|F]$  (i.e.,  $(1-\rho)a + \rho F$ ).

First, the retailer's profit concave on  $p$  requires  $\frac{\partial^2 E[\pi_r^{SDS}|F]}{\partial p^2} = -2 < 0$ . Using the first-order condition in Eq. (3), we have  $p = \frac{E[A|F]+bg-t+w}{2}$ .

Since the retailer shares the DFI with the manufacturer, the manufacturer's belief for the retail price is  $p = \frac{E[A|F]+bg-t+w}{2}$ . Substituting it into Eq. (2), the manufacturer's profit concave on  $g$  and  $w$  requires that the Hessian matrix of  $E[\pi_m^{SDS}|F]$  is negative definite and the determinant of it is non-negative. Thus, we get  $H(g, w) = \begin{bmatrix} -2f & \frac{b}{2} \\ \frac{b}{2} & -1 \end{bmatrix}$ ,  $\det[H(g, w)] = \frac{8f-b^2}{4} > 0$ .

Using the first-order conditions in Eq. (2), we have  $g = \frac{b(E[A|F]-t)}{8f-b^2}$  and  $w = \frac{4f(E[A|F]-t)}{8f-b^2}$ . The government's beliefs for the retail price, wholesale price and greenness are  $p = \frac{E[A]+bg-t+w}{2}$ ,  $w = \frac{4f(E[A]-t)}{8f-b^2}$  and  $g = \frac{b(E[A]-t)}{8f-b^2}$  respectively. Substituting them into Eq. (1),  $SW$  concave on  $t$  requires  $\frac{\partial^2 E[SW^{SDS}]}{\partial t^2} = \frac{2f(2b+b^2-2f)}{(8f-b^2)^2} < 0$ . Using the first-order condition in Eq. (1), we have  $t = \frac{2E[A](b+3f)}{2b+b^2-2f}$ .

Finally, using the first-order conditions in Eqs. (1)-(3), we obtain the optimal solutions and then get the expected demand, enterprise profits and  $SW$ .

## 3. The solution procedure for Model NMS

First, the retailer's profit concave on  $p$  requires  $\frac{\partial^2 E[\pi_r^{NMS}|F]}{\partial p^2} = -2 < 0$ . Using the first-order condition in Eq. (6), we have  $p = \frac{E[A|F]+bg-t+w}{2}$ .

The manufacturer's belief for the retail price is  $p = \frac{E[A]+bg-t+w}{2}$ . Substituting it into Eq. (5), the manufacturer's profit concave on  $g$  and  $w$  requires that the Hessian matrix of  $E[\pi_m^{NMS}]$  is negative definite and the determinant of it is non-negative. Thus, we get  $H(g, w) = \begin{bmatrix} -v(E[A] + 3bg - t - w) & vg + \frac{b}{2} \\ vg + \frac{b}{2} & -1 \end{bmatrix}$ ,  $\det[H(g, w)] = \frac{b(8vg-b)}{4} + v(E[A] - t - w - vg^2) > 0$ .

Using the first-order conditions in Eq. (5), we have  $g = \frac{b}{2v}$  and  $w = \frac{4v(E[A]-t)+3b^2}{8v}$ . Substituting them and  $p = \frac{E[A]+bg-t+w}{2}$  into Eq. (4),  $SW$  concave on  $t$  requires  $\frac{\partial^2 E[SW^{NMS}]}{\partial t^2} = -\frac{1}{16} < 0$ . Using the first-order condition in Eq. (4), we have  $t = -\frac{12vE[A]+8b+3b^2}{4v}$ .

Finally, using the first-order conditions in Eqs. (4)-(6), we obtain the optimal solutions and get  $\det[H(g, w)] = \frac{4vE[A]+2b+b^2}{2} > 0$ , and then the expected demand, enterprise profits and  $SW$  can be directly calculated.

#### 4. The solution procedure for Model SMS

First, the retailer's profit concave on  $p$  requires  $\frac{\partial^2 E[\pi_r^{SMS}|F]}{\partial p^2} = -2 < 0$ . Using the first-order condition in Eq. (6), we have  $p = \frac{E[A|F]+bg-t+w}{2}$ . Substituting it into Eq. (5), the manufacturer's profit concave on  $g$  and  $w$  requires that the Hessian matrix of  $E[\pi_m^{SMS}|F]$  is negative definite and the determinant of it is non-negative. Thus, we get  $H(g, w) = \begin{bmatrix} -v(E[A|F] + 3bg - t - w) & vg + \frac{b}{2} \\ vg + \frac{b}{2} & -1 \end{bmatrix}$ ,  $\det[H(g, w)] = \frac{b(8vg-b)}{4} + v(E[A|F] - t - w - vg^2) > 0$ .

Using the first-order conditions in Eq. (5), we have  $g = \frac{b}{2v}$  and  $w = \frac{4v(E[A|F]-t)+3b^2}{8v}$ . The government's beliefs for the retail price and wholesale price are  $p = \frac{E[A]+bg-t+w}{2}$  and  $w = \frac{4v(E[A]-t)+3b^2}{8v}$  respectively. Substituting them and  $g = \frac{b}{2v}$  into Eq. (4),  $SW$  concave on  $t$  requires  $\frac{\partial^2 E[SW^{SMS}]}{\partial t^2} = -\frac{1}{16} < 0$ . Using the first-order condition in Eq. (4), we have  $t = -\frac{12vE[A]+8b+3b^2}{4v}$ .

Finally, using the first-order conditions in Eqs. (4)-(6), we obtain the optimal solutions and get  $\det[H(g, w)] = \frac{vE[A|F]+3vE[A]+2b+b^2}{2} > 0$ , and then the expected demand, enterprise profits and  $SW$  can be directly calculated.

#### 5. The solution procedure for Model NDC

First, the retailer's profit concave on  $p$  requires  $\frac{\partial^2 E[\pi_r^{NDC}|F]}{\partial p^2} = -2 < 0$ . Using the first-order condition in Eq. (9), we have  $p = \frac{E[A|F]+bg-t+w}{2}$ . The manufacturer's belief for the retail price is  $p = \frac{E[A]+bg-t+w}{2}$ . Substituting it into Eq. (8), the manufacturer's profit concave on  $w$  requires  $\frac{\partial^2 E[\pi_m^{NDC}]}{\partial w^2} = -1 < 0$ .

Using the first-order condition in Eq. (8), we have  $w = \frac{E[A]+bg-t}{2}$ . Substituting it and  $p =$

$\frac{E[A]+bg-t+w}{2}$  into Eq. (7),  $SW$  concave on  $t$  and  $g$  requires that the Hessian matrix of  $E[SW^{NDC}]$  is negative definite and the determinant of it is non-negative. Thus, we get  $H(t, g) = \begin{bmatrix} -\frac{1}{16} & -\frac{1}{4} - \frac{3b}{16} \\ -\frac{1}{4} - \frac{3b}{16} & \frac{b}{2} + \frac{7b^2}{16} - 2f \end{bmatrix}$ ,  $\det[H(t, g)] = -\frac{1+2b+b^2-2f}{16} > 0$ . Using the first-order conditions in Eq. (7), we have  $t = \frac{E[A](1+b+6f)}{1+2b+b^2-2f}$  and  $g = -\frac{E[A](1+b)}{1+2b+b^2-2f}$ .

Finally, using the first-order conditions in Eqs. (7)-(9), we obtain the optimal solutions and then get the expected demand, enterprise profits and  $SW$ .

## 6. The solution procedure for Model SDC

First, the retailer's profit concave on  $p$  requires  $\frac{\partial^2 E[\pi_r^{SDC}|F]}{\partial p^2} = -2 < 0$ . Using the first-order condition in Eq. (9), we have  $p = \frac{E[A|F]+bg-t+w}{2}$ . Substituting it into Eq. (8), the manufacturer's profit concave on  $w$  requires  $\frac{\partial^2 E[\pi_m^{SDC}|F]}{\partial w^2} = -1 < 0$ .

Using the first-order condition in Eq. (8), we have  $w = \frac{E[A|F]+bg-t}{2}$ . The government's beliefs for the retail price and wholesale price are  $p = \frac{E[A]+bg-t+w}{2}$  and  $w = \frac{E[A]+bg-t}{2}$  respectively. Substituting them into Eq. (7),  $SW$  concave on  $t$  and  $g$  requires that the Hessian matrix of  $E[SW^{SDC}]$  is negative definite and the determinant of it is non-negative. Thus, we get  $H(t, g) = \begin{bmatrix} -\frac{1}{16} & -\frac{1}{4} - \frac{3b}{16} \\ -\frac{1}{4} - \frac{3b}{16} & \frac{b}{2} + \frac{7b^2}{16} - 2f \end{bmatrix}$ ,  $\det[H(t, g)] = -\frac{1+2b+b^2-2f}{16} > 0$ . Using the first-order conditions in Eq. (7), we have  $t = \frac{E[A](1+b+6f)}{1+2b+b^2-2f}$  and  $g = -\frac{E[A](1+b)}{1+2b+b^2-2f}$ .

Finally, using the first-order conditions in Eqs. (7)-(9), we obtain the optimal solutions and then get the expected demand, enterprise profits and  $SW$ .

## 7. The solution procedure for Model NMC

First, the retailer's profit concave on  $p$  requires  $\frac{\partial^2 E[\pi_r^{NMC}|F]}{\partial p^2} = -2 < 0$ . Using the first-order condition in Eq. (12), we have  $p = \frac{E[A|F]+bg-t+w}{2}$ . The manufacturer's belief for the retail price is  $p = \frac{E[A]+bg-t+w}{2}$ . Substituting it into Eq. (11), the manufacturer's profit concave on  $w$  requires  $\frac{\partial^2 E[\pi_m^{NMC}]}{\partial w^2} = -1 < 0$ .

Using the first-order condition in Eq. (11), we have  $w = \frac{E[A]+bg-t+vg^2}{2}$ . Substituting it and  $p = \frac{E[A]+bg-t+w}{2}$  into Eq. (10),  $SW$  concave on  $t$  and  $g$  requires that the Hessian matrix of  $E[SW^{NMC}]$

is negative definite and the determinant of it is non-negative. Thus, we get  $H(t, g) =$

$$\begin{bmatrix} -\frac{1}{16} & \frac{-4-3b+6vg}{16} \\ \frac{-4-3b+6vg}{16} & \frac{-14vE[A]+6v(t-4g+7vg^2)+7b^2+(8-42vg)b}{16} \end{bmatrix}, \quad \det [H(t, g)] =$$

$$\frac{7vE[A]+v(36g-3t-39vg^2)+(39vg-16)b-8b^2-8}{128} > 0.$$

Using the first-order conditions in Eq. (10), we have

$$t = -\frac{12vE[A]+5+8b+3b^2}{4v} \quad \text{and} \quad g = \frac{1+b}{2v}.$$

Finally, using the first-order conditions in Eqs. (10)-(12), we obtain the optimal solutions and get  $\det[H(t, g)] = \frac{4vE[A]+1+2b+b^2}{32} > 0$ , and then the expected demand, enterprise profits and  $SW$  can be directly calculated.

## 8. The solution procedure for Model SMC

First, the retailer's profit concave on  $p$  requires  $\frac{\partial^2 E[\pi_r^{SMC}|F]}{\partial p^2} = -2 < 0$ . Using the first-order condition in Eq. (12), we have  $p = \frac{E[A|F]+bg-t+w}{2}$ . Substituting it into Eq. (11), the manufacturer's profit concave on  $w$  requires  $\frac{\partial^2 E[\pi_m^{SMC}|F]}{\partial w^2} = -1 < 0$ .

Using the first-order condition in Eq. (11), we have  $w = \frac{E[A|F]+bg-t+vg^2}{2}$ . The government's beliefs for the retail price and wholesale price are  $p = \frac{E[A]+bg-t+w}{2}$  and  $w = \frac{E[A]+bg-t+vg^2}{2}$  respectively. Substituting them into Eq. (10),  $SW$  concave on  $t$  and  $g$  requires that the Hessian matrix of  $E[SW^{SMC}]$  is negative definite and the determinant of it is non-negative. Thus, we get

$$H(t, g) = \begin{bmatrix} -\frac{1}{16} & \frac{-4-3b+6vg}{16} \\ \frac{-4-3b+6vg}{16} & \frac{-14vE[A]+6v(t-4g+7vg^2)+7b^2+(8-42vg)b}{16} \end{bmatrix}, \quad \det [H(t, g)] =$$

$$\frac{7vE[A]+v(36g-3t-39vg^2)+(39vg-16)b-8b^2-8}{128} > 0.$$

Using the first-order conditions in Eq. (10), we have

$$t = -\frac{12vE[A]+5+8b+3b^2}{4v} \quad \text{and} \quad g = \frac{1+b}{2v}.$$

Finally, using the first-order conditions in Eqs. (10)-(12), we obtain the optimal solutions and get  $\det[H(t, g)] = \frac{4vE[A]+1+2b+b^2}{32} > 0$ , and then the expected demand, enterprise profits and  $SW$  can be directly calculated.

## Appendix B

**Proof of Lemma 1.** According to Table 4, we can calculate that  $\frac{\partial g^{SDS^*}}{\partial \rho} = \frac{b(F-a)}{8f-b^2}$ ,  $\frac{\partial w^{SDS^*}}{\partial \rho} = \frac{4f(F-a)}{8f-b^2}$ ,  $\frac{\partial p^{SDS^*}}{\partial \rho} = \frac{6f(F-a)}{8f-b^2}$ ,  $\frac{\partial D^{SDS^*}}{\partial \rho} = \frac{2f(F-a)}{8f-b^2}$ ,  $\frac{\partial E[\pi_r^{SDS^*}]}{\partial \rho} = \frac{4f^2\sigma_0^2}{(8f-b^2)^2}$  and  $\frac{\partial E[\pi_m^{SDS^*}]}{\partial \rho} = \frac{f\sigma_0^2}{8f-b^2}$ . Based on the

second-order conditions, it is straightforward to show that the coefficient of  $\rho$  in  $E[\pi_r^{SDS^*}]$  and  $E[\pi_m^{SDS^*}]$  is positive, while the coefficient of  $\rho$  in  $g^{SDS^*}$ ,  $w^{SDS^*}$ ,  $p^{SDS^*}$  and  $D^{SDS^*}$  is positive if  $F > a$  and negative otherwise.

**Proof of Lemma 2.** According to Table 5, we can get that  $g^{SMS^*}$  is determined only by the ratio of the market appeal of the greenness ( $b$ ) and the marginal variable cost of the greenness ( $v$ ),  $\frac{b}{2v}$ , which

is independent of  $\rho$ . Moreover, we can calculate that  $\frac{\partial w^{SMS^*}}{\partial \rho} = \frac{F-a}{2}$ ,  $\frac{\partial p^{SMS^*}}{\partial \rho} = \frac{3(F-a)}{4}$ ,  $\frac{\partial D^{SMS^*}}{\partial \rho} = \frac{F-a}{4}$ ,

$\frac{\partial E[\pi_r^{SMS^*}]}{\partial \rho} = \frac{\sigma_0^2}{16}$  and  $\frac{\partial E[\pi_m^{SMS^*}]}{\partial \rho} = \frac{\sigma_0^2}{8}$ . It is straightforward to show that the coefficient of  $\rho$  in  $E[\pi_r^{SMS^*}]$  and  $E[\pi_m^{SMS^*}]$  is positive, while the coefficient of  $\rho$  in  $w^{SMS^*}$ ,  $p^{SMS^*}$  and  $D^{SMS^*}$  is positive when  $F > a$  and negative otherwise.

**Proof of Proposition 1.** According to Table 4, we can calculate that  $g^{SDS^*} - g^{NDS^*} = \frac{b(F-a)\rho}{8f-b^2}$ ,

$w^{SDS^*} - w^{NDS^*} = \frac{4f(F-a)\rho}{8f-b^2}$ ,  $p^{SDS^*} - p^{NDS^*} = \frac{(4f+b^2)(F-a)\rho}{2(8f-b^2)}$  and  $D^{SDS^*} - D^{NDS^*} = -\frac{(4f-b^2)(F-a)\rho}{2(8f-b^2)}$ .

Based on the second-order conditions, when  $F > a$ , we can get that  $g^{SDS^*} > g^{NDS^*}$ ,  $w^{SDS^*} > w^{NDS^*}$ ,  $p^{SDS^*} > p^{NDS^*}$  and  $D^{SDS^*} < D^{NDS^*}$ ; otherwise,  $g^{SDS^*} < g^{NDS^*}$ ,  $w^{SDS^*} < w^{NDS^*}$ ,  $p^{SDS^*} < p^{NDS^*}$ ,  $D^{SDS^*} > D^{NDS^*}$ .

**Proof of Proposition 2.** According to the optimal ex-ante expected profit expressions in Table 4, we can calculate that the values of information sharing for the retailer and manufacturer are  $V_r^{DS} =$

$E[\pi_r^{SDS^*}] - E[\pi_r^{NDS^*}] = -\frac{(4f-b^2)(12f-b^2)\rho\sigma_0^2}{4(8f-b^2)^2}$  and  $V_m^{DS} = E[\pi_m^{SDS^*}] - E[\pi_m^{NDS^*}] = \frac{f\rho\sigma_0^2}{8f-b^2}$

respectively. Based on the second-order conditions, we know  $V_r^{DS} < 0$ ,  $V_m^{DS} > 0$ . Moreover, we can

derive  $\frac{\partial V_r^{DS}}{\partial \rho} = -\frac{(4f-b^2)(12f-b^2)\sigma_0^2}{4(8f-b^2)^2} < 0$ ,  $\frac{\partial V_m^{DS}}{\partial \rho} = \frac{f\sigma_0^2}{8f-b^2} > 0$ .

**Proof of Proposition 3.** According to Table 5, we can calculate that  $g^{SMS^*} - g^{NMS^*} = 0$ ,  $w^{SMS^*} - w^{NMS^*} = \frac{(F-a)\rho}{2}$ ,  $p^{SMS^*} - p^{NMS^*} = \frac{(F-a)\rho}{4}$  and  $D^{SMS^*} - D^{NMS^*} = -\frac{(F-a)\rho}{4}$ . Therefore, it is obvious that  $g^{SMS^*} = g^{NMS^*}$ . When  $F > a$ , we can get that  $w^{SMS^*} > w^{NMS^*}$ ,  $p^{SMS^*} > p^{NMS^*}$  and  $D^{SMS^*} < D^{NMS^*}$ ; otherwise,  $w^{SMS^*} < w^{NMS^*}$ ,  $p^{SMS^*} < p^{NMS^*}$ ,  $D^{SMS^*} > D^{NMS^*}$ .

**Proof of Proposition 4.** According to the optimal ex-ante expected profit expressions in Table 5, we can calculate that the values of information sharing for the retailer and manufacturer are  $V_r^{MS} =$

$E[\pi_r^{SMS^*}] - E[\pi_r^{NMS^*}] = -\frac{3\rho\sigma_0^2}{16}$  and  $V_m^{MS} = E[\pi_m^{SMS^*}] - E[\pi_m^{NMS^*}] = \frac{\rho\sigma_0^2}{8}$  respectively. Thus, we

can easily conclude that  $V_r^{MS} < 0$  and  $V_m^{MS} > 0$ . Moreover, we can derive  $\frac{\partial V_r^{MS}}{\partial \rho} = -\frac{3\sigma_0^2}{16} < 0$  and

$\frac{\partial V_m^{MS}}{\partial \rho} = \frac{\sigma_0^2}{8} > 0$ .

**Proof of Lemma 3.** For DIGP, according to Table 6, we can calculate that  $\frac{\partial w^{SDC^*}}{\partial \rho} = \frac{F-a}{2}$ ,  $\frac{\partial p^{SDC^*}}{\partial \rho} =$

$\frac{3(F-a)}{4}$ ,  $\frac{\partial D^{SDC^*}}{\partial \rho} = \frac{F-a}{4}$ ,  $\frac{\partial E[\pi_r^{SDC^*}]}{\partial \rho} = \frac{\sigma_0^2}{16}$  and  $\frac{\partial E[\pi_m^{SDC^*}]}{\partial \rho} = \frac{\sigma_0^2}{8}$ . Thus, it is obvious that the coefficient of

$\rho$  in  $E[\pi_r^{SDC^*}]$  and  $E[\pi_m^{SDC^*}]$  is positive, while the coefficient of  $\rho$  in  $w^{SDC^*}$ ,  $p^{SDC^*}$  and  $D^{SDC^*}$  is positive when  $F > a$  and negative otherwise. Similarly, according to Table 7, the proof for MIGP is analogous to that for DIGP.

**Proof of Proposition 5.** For DIGP, according to Table 6, we can calculate that  $w^{SDC^*} - w^{NDC^*} = \frac{(F-a)\rho}{2}$ ,  $p^{SDC^*} - p^{NDC^*} = \frac{(F-a)\rho}{4}$  and  $D^{SDC^*} - D^{NDC^*} = -\frac{(F-a)\rho}{4}$ . So when  $F > a$ , we can get that  $w^{SDC^*} > w^{NDC^*}$ ,  $p^{SDC^*} > p^{NDC^*}$  and  $D^{SDC^*} < D^{NDC^*}$ ; otherwise,  $w^{SDC^*} < w^{NDC^*}$ ,  $p^{SDC^*} < p^{NDC^*}$ ,  $D^{SDC^*} > D^{NDC^*}$ . Similarly, according to Table 7, the proof for MIGP is analogous to that for DIGP.

**Proof of Proposition 6.** For DIGP, according to Table 6, we can easily conclude that the values of information sharing for the retailer and manufacturer are  $V_r^{DC} = E[\pi_r^{SDC^*}] - E[\pi_r^{NDC^*}] = -\frac{3\rho\sigma_0^2}{16} < 0$  and  $V_m^{DC} = E[\pi_m^{SDC^*}] - E[\pi_m^{NDC^*}] = \frac{\rho\sigma_0^2}{8} > 0$  respectively. Moreover, we can derive  $\frac{\partial V_r^{DC}}{\partial \rho} = -\frac{3\sigma_0^2}{16} < 0$ ,  $\frac{\partial V_m^{DC}}{\partial \rho} = \frac{\sigma_0^2}{8} > 0$ . Similarly, according to Table 7, the proof for MIGP is analogous to that for DIGP.

**Proof of Proposition 7.** According to Tables 4 and 6, based on the second-order conditions, we can calculate that  $t^{NDC^*} - t^{NDS^*} = \frac{(2f(b-4)-b^2-b^3)a}{(2b+b^2-2f)(1+2b+b^2-2f)}$ ,  $g^{NDC^*} - g^{NDS^*} = \frac{(2f-b-b^2)a}{(2b+b^2-2f)(1+2b+b^2-2f)} > 0$ ,  $w^{NDC^*} - w^{NDS^*} = \frac{4af}{(2b+b^2-2f)(1+2b+b^2-2f)} > 0$ ,  $p^{NDC^*} - p^{NDS^*} = \frac{6af}{(2b+b^2-2f)(1+2b+b^2-2f)} > 0$  and  $D^{NDC^*} - D^{NDS^*} = \frac{2af}{(2b+b^2-2f)(1+2b+b^2-2f)} > 0$ . According to the second-order conditions, the

lower bound of the feasible region is  $f > f_0 = \frac{1+2b+b^2}{2}$ . Setting  $t^{NDC^*} - t^{NDS^*} = 0$  yields  $f_1 = \frac{b^2+b^3}{2b-8}$ . From the above equation, we know that  $t^{NDC^*} - t^{NDS^*} = \frac{\phi(f)a}{(2b+b^2-2f)(1+2b+b^2-2f)}$ , where  $\phi(f) = 2(b-4)f - b^2 - b^3$ . It is easy to get  $\phi'(f) = 2(b-4)$  and  $\phi(f_0) = -4 - 7b - 3b^3 < 0$ . When  $b \leq 4$ , we have  $\phi'(f) \leq 0$ , implying that  $\phi(f) \leq 0$  for all  $f \in (f_0, +\infty)$ . Therefore, when  $b \leq 4$  and  $f > f_0$ ,  $t^{NDC^*} \leq t^{NDS^*}$ . Conversely, when  $b > 4$ , we have  $\phi'(f) > 0$ , indicating that  $\phi(f)$  is increasing in  $f \in (f_0, +\infty)$ . Therefore, there exists a threshold  $f_1 \in (f_0, +\infty)$  such that  $\phi(f) = 0$ . Specifically, when  $b > 4$ , if  $f_0 < f \leq f_1$ , then  $t^{NDC^*} \leq t^{NDS^*}$ ; if  $f > f_1$ , then  $t^{NDC^*} > t^{NDS^*}$ . Moreover, since  $t^{NDC^*} = \frac{(1+b+6f)a}{1+2b+b^2-2f} < 0$  and  $t^{NDS^*} = \frac{2(3f+b)a}{b^2+2b-2f} < 0$ , the condition  $t^{NDC^*} > t^{NDS^*}$  implies that the subsidy for the CLS is lower than that for the SLS, whereas  $t^{NDC^*} \leq t^{NDS^*}$  indicates that the subsidy for the CLS is higher than that for the SLS.

**Proof of Proposition 8.** According to Tables 4 and 6, based on the second-order conditions, we can calculate that  $E[\pi_r^{NDC^*}] - E[\pi_r^{NDS^*}] = \frac{4a^2f^2(4f-1-4b-2b^2)}{(2b+b^2-2f)^2(1+2b+b^2-2f)^2} > 0$ ,  $E[SW^{NDC^*}] - E[SW^{NDS^*}] = \frac{a^2f}{(2b+b^2-2f)(1+2b+b^2-2f)} > 0$  and  $E[\pi_m^{NDC^*}] - E[\pi_m^{NDS^*}] =$

$\frac{a^2 f((28-8b)f^2 - (8+24b-8b^3)f - 3b^2 - 8b^3 - 7b^4 - 2b^5)}{(2b+b^2-2f)^2(1+2b+b^2-2f)^2}$ . According to the second-order conditions, the lower

bound of the feasible region is  $f > f_0 = \frac{1+2b+b^2}{2}$ . From the above equation, we know that

$$E[\pi_m^{NDC^*}] - E[\pi_m^{NDS^*}] = \frac{\varphi(f)a^2 f}{(2b+b^2-2f)^2(1+2b+b^2-2f)^2}, \quad \text{where } \varphi(f) = (28-8b)f^2 - (8+24b -$$

$8b^3)f - 3b^2 - 8b^3 - 7b^4 - 2b^5$ . The function  $\varphi(f)$  is convex when  $b \leq \frac{7}{2}$  and concave when

$b > \frac{7}{2}$ . When  $b \leq \frac{7}{2}$ , Solving  $\varphi(f) = 0$  yields  $f = \frac{-2-6b+2b^3 - \sqrt{4+24b+57b^2+42b^3+9b^4}}{2(2b-7)} < f_0$  or  $f =$

$\frac{-2-6b+2b^3 + \sqrt{4+24b+57b^2+42b^3+9b^4}}{2(2b-7)} < 0$ . Therefore, when  $b \leq \frac{7}{2}$  and  $f > f_0$ , we have  $E[\pi_m^{NDC^*}] \geq$

$E[\pi_m^{NDS^*}]$ . In contrast, when  $b > \frac{7}{2}$ , Solving  $\varphi(f) = 0$  yields  $f =$

$\frac{-2-6b+2b^3 - \sqrt{4+24b+57b^2+42b^3+9b^4}}{2(2b-7)} < f_0$  or  $f_2 = \frac{-2-6b+2b^3 + \sqrt{4+24b+57b^2+42b^3+9b^4}}{2(2b-7)} > f_0$ . Thus, when

$b > \frac{7}{2}$ , if  $f_0 < f \leq f_2$ , then  $E[\pi_m^{NDC^*}] \geq E[\pi_m^{NDS^*}]$ ; if  $f > f_2$ , then  $E[\pi_m^{NDC^*}] < E[\pi_m^{NDS^*}]$ .

**Proof of Proposition 9.** According to Tables 4 and 6, based on the second-order conditions, we can

calculate that  $g^{SDC^*} - g^{SDS^*} = \frac{(2f-b-b^2)a}{(2b+b^2-2f)(1+2b+b^2-2f)} - \frac{b(F-a)\rho}{8f-b^2}$ ,  $w^{SDC^*} - w^{SDS^*} =$

$\frac{4af}{(2b+b^2-2f)(1+2b+b^2-2f)} - \frac{b^2(F-a)\rho}{2(8f-b^2)}$ ,  $p^{SDC^*} - p^{SDS^*} = \frac{6af}{(2b+b^2-2f)(1+2b+b^2-2f)} - \frac{3b^2(F-a)\rho}{4(8f-b^2)}$  and

$D^{SDC^*} - D^{SDS^*} = \frac{2af}{(2b+b^2-2f)(1+2b+b^2-2f)} - \frac{b^2(F-a)\rho}{4(8f-b^2)}$ .

**Proof of Proposition 10.** According to Tables 4 and 6, based on the second-order conditions, we can

calculate that  $E[\pi_r^{SDC^*}] - E[\pi_r^{SDS^*}] = \frac{4a^2 f^2(4f-1-4b-2b^2)}{(2b+b^2-2f)^2(1+2b+b^2-2f)^2} - \frac{b^2(16f-b^2)\rho\sigma_0^2}{16(8f-b^2)^2} > 0$ ,

$E[SW^{SDC^*}] - E[SW^{SDS^*}] = \frac{a^2 f}{(2b+b^2-2f)(1+2b+b^2-2f)} > 0$  and  $E[\pi_m^{SDC^*}] - E[\pi_m^{SDS^*}] =$

$\frac{a^2 f((28-8b)f^2 - (8+24b-8b^3)f - 3b^2 - 8b^3 - 7b^4 - 2b^5)}{(2b+b^2-2f)^2(1+2b+b^2-2f)^2} - \frac{b^2 \rho \sigma_0^2}{8(8f-b^2)}$ . According to the second-order conditions,

the lower bound of the feasible region is  $f > f_0 = \frac{1+2b+b^2}{2}$ . From the above equation, we know that

$E[\pi_m^{SDC^*}] - E[\pi_m^{SDS^*}] = \frac{a^2 f \varphi(f)}{(2b+b^2-2f)^2(1+2b+b^2-2f)^2} - \frac{b^2 \rho \sigma_0^2}{8(8f-b^2)}$ , where  $\varphi(f) = (28-8b)f^2 - (8+$

$24b - 8b^3)f - 3b^2 - 8b^3 - 7b^4 - 2b^5$ . It is obvious that  $\frac{b^2 \rho \sigma_0^2}{8(8f-b^2)} < 0$  for all  $f \in (f_0, +\infty)$ .

According to the Proof of Proposition 8, if  $b \leq \frac{7}{2}$ , then  $\varphi(f) > 0$  for all  $f \in (f_0, +\infty)$ , and the sign

of  $E[\pi_m^{SDC^*}] - E[\pi_m^{SDS^*}]$  depends on whether the positive deterministic gain (i.e.,

$\frac{a^2 f \varphi(f)}{(2b+b^2-2f)^2(1+2b+b^2-2f)^2}$ ) outweighs the negative stochastic term (i.e.,  $\frac{b^2 \rho \sigma_0^2}{8(8f-b^2)}$ ). If  $b > \frac{7}{2}$ , then the

sign of  $\varphi(f)$  changes at  $f_2 = \frac{-2-6b+2b^3+\sqrt{4+24b+57b^2+42b^3+9b^4}}{2(2b-7)} > f_0$ . In particular, for  $f > f_2$ , we have  $E[\pi_m^{SDC^*}] < E[\pi_m^{SDS^*}]$ . Moreover,  $\lim_{f \rightarrow f_0} (E[\pi_m^{SDC^*}] - E[\pi_m^{SDS^*}]) > 0$ . Therefore, there exists a threshold  $f_3$  such that  $E[\pi_m^{SDC^*}] = E[\pi_m^{SDS^*}]$ , where  $f_0 < f_3 \leq f_2$ .

**Proof of Proposition 11.** Based on Tables 5 and 7, the proof for MIGP is similar to that of DIGP in Propositions 7 and 9, which we omit.

**Proof of Proposition 12.** Based on Tables 5 and 7, the proof for MIGP is similar to that of DIGP in Propositions 8 and 10, which we omit.



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