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Theory article

Green agricultural product operations strategies considering consumer skepticism under different supply chain modes

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Appendix

Proof for Proposition 1 Refer to the solution of optimal control. From Equation (3), we can obtain the optimal control problem of the supply chain as

$$J_{MR}^{C}(z,t)=\max_{s,g}\int_0^{\infty}e^{-\rho t}\{(\pi_M+\pi_R)[\theta(g-\varphi s)+\eta s-\sigma z]-\frac{1}{2}k_R s^2-\frac{1}{2}k_M g^2\}dt$$
$$=e^{-\rho t}\max_{s,g}\int_t^{\infty}e^{-\rho(x-t)}\{(\pi_M+\pi_R)[\theta(g-\varphi s)+\eta s-\sigma z]-\frac{1}{2}k_R s^2-\frac{1}{2}k_M g^2\}dx\,. \tag{A.1}$$

The total profit function is $V_{MR}^{C}(z)$, so we let

$$V_{MR}^{C}(z)=\max_{s,g}\int_t^{\infty}e^{-\rho(x-t)}\{(\pi_M+\pi_R)[\theta(g-\varphi s)+\eta s-\sigma z]-\frac{1}{2}k_R s^2-\frac{1}{2}k_M g^2\}dx \tag{A.2}$$

The optimal control problem of the supply chain is transformed into

$$J_{MR}^{C^*}(z,t)=e^{-\rho t}V_{MR}^{C}(z)\,. \tag{A.3}$$

At this point, the optimal control problem in the supply chain is satisfied by the following HJB equation for all $z>0$

$$\rho V_{MR}^{C}(z)=\max_{s,g}\left\{(\pi_M+\pi_R)[\theta(g-\varphi s)+\eta s-\sigma z]-\frac{1}{2}k_R s^2-\frac{1}{2}k_M g^2+V_{MR}^{C'}\dot{z}\right\}. \tag{A.4}$$

$V_{MR}^{C'}=\partial V_{MR}^{C}/\partial z$ is the marginal value of the state, and $\dot{z}$ is the state change rate given by Equation (2). Multiplying them gives the instantaneous change in the optimal value due to the evolution of consumer skepticism, which is a standard component of the HJB equation in continuous-time optimal control.

Expand Equation (A.4), we have

$$\rho V_{MR}^{C}(z)=\max_{s,g}\left\{(\pi_M+\pi_R)[\theta(g-\varphi s)+\eta s-\sigma z]-\frac{1}{2}k_R s^2-\frac{1}{2}k_M g^2+V_{MR}^{C'}((\alpha-\xi)z-\gamma g-\delta s)\right\}. \tag{A.5}$$

Calculating the Hessian matrix with respect to s and g for Equation (A.5), we have

$$H=\begin{bmatrix}-k_R & 0\\ 0 & -k_M\end{bmatrix}$$

By calculating the principal minor, it can be known that the Hessian matrix is negative definite. Thus, the variables s and g in Equation (A.5) can reach their maximum values. From the first-order condition, we can obtain

$$s^C=\frac{(\eta-\theta\varphi)(\pi_M+\pi_R)}{k_R}-\frac{\delta V_{MR}^{C'}}{k_R}, \tag{A.6}$$

$$g^C=\frac{\theta(\pi_M+\pi_R)}{k_M}-\frac{\gamma V_{MR}^{C'}}{k_M}. \tag{A.7}$$

By substituting Equation (A.6) and Equation (A.7) into Equation (A.5), we have

$$\rho V_{MR}^C(z)=[V_{MR}^{C'}(\alpha-\xi)-\sigma(\pi_M+\pi_R)]z+\frac{k_M[(\eta-\theta\varphi)(\pi_M+\pi_R)-\delta V_{MR}^{C'}]^2+k_R[\theta(\pi_M+\pi_R)-\gamma V_{MR}^{C'}]^2}{2k_Mk_R}. \tag{A.8}$$

By simplifying the structure of Equation (A.8), we can observe that the solution of the HJB equation can be expressed as a linear function of z. Thus, we set the specific expression of $V_{MR}^C(z)$

$$V_{MR}^C(z)=f_1z+f_2. \tag{A.9}$$

Among them, f_1 and f_2 are constants. Comparing Equation (A.8) and Equation (A.9), we have

$$f_1=\frac{(\alpha-\xi)f_1-\sigma(\pi_M+\pi_R)}{\rho}, \tag{A.10}$$

$$\rho f_2=\frac{k_M[(\eta-\theta\varphi)(\pi_M+\pi_R)-\delta f_1]^2+k_R[\theta(\pi_M+\pi_R)-\gamma f_1]^2}{2k_Mk_R}. \tag{A.11}$$

From Equation (A.10), we have

$$f_1^*=\frac{\sigma(\pi_M+\pi_R)}{\alpha-\xi-\rho}. \tag{A.12}$$

By substituting the first derivative of Equation (A.9) into Equation (A.6) and Equation (A.7), we can obtain the solutions of s and g as

$$s^C=\frac{(\eta-\theta\varphi)(\pi_M+\pi_R)-\delta f_1}{k_R}=\frac{(\pi_M+\pi_R)\left[(\eta-\theta\varphi)(\alpha-\xi-\rho)-\delta\sigma\right]}{k_R(\alpha-\xi-\rho)}, \tag{A.13}$$

$$g^C=\frac{\theta(\pi_M+\pi_R)-\gamma f_1}{k_M}=\frac{(\pi_M+\pi_R)\left[\theta(\alpha-\xi-\rho)-\gamma\sigma\right]}{k_M(\alpha-\xi-\rho)}. \tag{A.14}$$

We can further obtain

$$f_2^*=\frac{(\pi_M+\pi_R)^2[k_M((\eta-\theta\varphi)(\alpha-\xi-\rho)-\delta\sigma)^2+k_R(\theta(\alpha-\xi-\rho)-\gamma\sigma)^2]}{2(\alpha-\xi-\rho)^2\rho k_Mk_R}. \tag{A.15}$$

By substituting f_1 and f_2 into Equation (A.9), we have

$$V_{MR}^C(z)=f_1^*z+f_2^*. \tag{A.16}$$

Proof for Proposition 2 We set the supply chain members' profits functions as $V_M^N(z)$ and $V_R^N(z)$, and the HJB equations of Equation (8) and Equation (9) are

$$\rho V_M^N=\max_g\left\{\pi_M\left[\theta(g-\varphi s)+\eta s-\sigma z\right]-\frac{1}{2}k_Mg^2+V_M^{N'}[(\alpha-\xi)z-\gamma g-\delta s]\right\}, \tag{A.17}$$

$$\rho V_R^N=\max_s\left\{\pi_R\left[\theta(g-\varphi s)+\eta s-\sigma z\right]-\frac{1}{2}k_Rs^2+V_R^{N'}[(\alpha-\xi)z-\gamma g-\delta s]\right\}. \tag{A.18}$$

From the first derivative of Equation (A.17) and Equation (A.18) for s and g, we have

$$s^N = \frac{\pi_R(\eta - \theta\varphi) - \delta V_R^{N'}}{k_R}, \tag{A.19}$$

$$g^N = \frac{\theta\pi_M - \gamma V_M^{N'}}{k_M}. \tag{A.20}$$

By substituting Equation (A.19) and Equation (A.20) into Equation (A.17) and Equation (A.18), we have

$$\begin{aligned}\rho V_M^N(z) &= [V_M^{N'}(\alpha - \xi) - \sigma\pi_M]z + \\ &\frac{2k_M[(\eta-\theta\varphi)\pi_M + \delta V_M^{N'}][(\eta-\theta\varphi)\pi_R - \delta V_R^{N'}] + k_R(\theta\pi_M - \gamma V_M^{N'})^2}{2k_M k_R},\end{aligned} \tag{A.21}$$

$$\rho V_R^N(z) = [V_R^{N'}(\alpha - \xi) - \sigma\pi_R]z + \frac{k_M[\pi_R(\eta-\theta\varphi) - \delta V_R^{N'}]^2 + 2k_R(\theta\pi_M - \gamma V_M^{N'})(\theta\pi_R - \gamma V_R^{N'})}{2k_M k_R}. \tag{A.22}$$

Based on the simplified structures of Equation (A.21) and Equation (A.22), the HJB equation can be expressed as a linear function of z. Thus, we set

$$V_M^N = m_1 z + n_1, \tag{A.23}$$

$$V_R^N = m_2 z + n_2. \tag{A.24}$$

By comparing the coefficients of Equation (A.21), Equation (A.22), Equation (A.23), and Equation (A.24), we can obtain

$$m_1^* = \frac{\sigma\pi_M}{\alpha - \xi - \rho},$$

$$n_1^* = \frac{2k_M\pi_M\pi_R[(\eta-\theta\varphi)(\alpha-\xi-\rho) - \delta\sigma]^2 + k_R\pi_M^2[\theta(\alpha-\xi-\rho) - \gamma\sigma]^2}{2(\alpha-\xi-\rho)^2 k_M k_R},$$

$$m_2^* = \frac{\sigma\pi_R}{\alpha - \xi - \rho},$$

$$n_2^* = \frac{k_M\pi_R^2[(\eta-\theta\varphi)(\alpha-\xi-\rho) - \delta\sigma]^2 + 2k_R\pi_M\pi_R[\theta(\alpha-\xi-\rho) - \gamma\sigma]^2}{2(\alpha-\xi-\rho)^2 k_M k_R}.$$

By substituting m_1^*, n_1^*, m_2^*, and n_2^* into Equation (A.23) and Equation (A.24), we can obtain the expressions of profit functions $V_M^N(z)$ and $V_R^N(z)$ as

$$V_M^N = m_1^* z + n_1^*, \tag{A.30}$$

$$V_R^N = m_2^* z + n_2^*. \tag{A.31}$$

By substituting the first derivative of Equation (A.30) and Equation (A.31) into Equation (A.19) and Equation (A.20), we can obtain the solutions of s and g as

$$g^{N^*} = \frac{[\theta(\alpha-\xi-\rho) - \gamma\sigma]\pi_M}{(\alpha-\xi-\rho)k_M}, \tag{A.32}$$

$$s^{N^*} = \frac{[(\eta-\theta\varphi)(\alpha-\xi-\rho) - \delta\sigma]\pi_R}{(\alpha-\xi-\rho)k_R}. \tag{A.33}$$

Proof for Proposition 3 We use the backward induction to get equilibrium solutions of the Stackelberg game. From Equation (15), the cooperative's HJB equation is

$$\rho V_M^Y(z) = \max_g \left\{ \pi_M\left[\theta(g - \varphi s) + \eta s - \sigma z\right] - \frac{1}{2}(1-\psi)k_M g^2 + V_M^{Y'}(\alpha z - \gamma g - \delta s) \right\}. \tag{A.34}$$

Finding the first-order condition with respect to $g^Y(t)$ for Equation (A.35), we have

$$g^Y = \frac{\theta\pi - \gamma V_M^{Y'}}{(1-\psi)k_M}. \tag{A.35}$$

In order to maximize its profit, the retailer will make decisions based on the actions of the cooperative. From Equation (16), the retailer's HJB equation is

$$\rho V_R^Y(z)=\max_{s,\psi}\left\{\pi_M\left[\theta(g-\varphi s)+\eta s-\sigma z\right]-\frac{1}{2}\psi k_R s^2-\frac{1}{2}k_M g^2+V_M^{Y'}(\alpha z-\gamma g-\delta s)\right\}. \tag{A.36}$$

We substitute Equation (A.35) into Equation (A.36), and take the first-order condition with respect to $s^Y(t)$ and ψ for Equation (A.36), so we have

$$s=\frac{(\eta-\theta\varphi)\pi_R-\delta V_R^{Y'}}{k_R}, \tag{A.37}$$

$$\psi=\frac{\theta(2\pi_R-\pi_M)+\gamma(V_M^{Y'}-2V_R^{Y'})}{\theta(\pi_M+2\pi_R)-\gamma(V_M^{Y'}+2V_R^{Y'})}. \tag{A.38}$$

By substituting Equation (A.37) and Equation (A.38) into Equation (A.35), we have

$$g^Y=\frac{\theta(\pi_M+2\pi_R)-\gamma(V_M^{Y'}+2V_R^{Y'})}{2k_M}. \tag{A.39}$$

By substituting Equation (A.37), Equation (A.38), and Equation (A.39) into Equation (A.34) and Equation (A.36), we have

$$\rho V_M^Y(z)=[(\alpha-\xi)V_M^{Y'}-\sigma\pi_M]z+\frac{k_M[(\eta-\theta\varphi)(\pi_M+\frac{\pi_R}{2})+\delta(V_M^{Y'}+\frac{V_R^{Y'}}{2})]^2+k_R(\theta\pi_M-\gamma V_M^{Y'})^2}{2k_M k_R}, \tag{A.40}$$

$$\rho V_R^Y(z)=[(\alpha-\xi)V_R^{Y'}-\sigma\pi_R]z+\frac{[\theta(\frac{1}{2}\pi_M+\pi_R)-\gamma(\frac{1}{2}V_M^{Y'}+V_R^{Y'})]^2k_R+k_M[(\eta-\theta\varphi)\pi_R-\delta V_R^{Y'}]^2}{2k_M k_R}. \tag{A.41}$$

Based on the simplified structures of Equation (A.40) and Equation (A.41), the HJB equation can be expressed as a linear function of z. Thus, we set

$$V_M^Y=a_1z+b_1, \tag{A.42}$$

$$V_R^Y=a_2z+b_2. \tag{A.43}$$

By comparing the coefficients of Equation (A.40), Equation (A.41), Equation (A.42), and Equation (A.43), we can obtain

$$a_1^*=\frac{\sigma\pi_M}{\alpha-\xi-\rho},$$

$$b_1^*=\frac{k_M(\pi_M+\frac{\pi_R}{2})^2\left[(\eta-\theta\varphi)(\alpha-\xi-\rho)-\delta\sigma\right]^2+k_R\left[\pi_M(\theta(\alpha-\xi-\rho)-\gamma\sigma)\right]^2}{2(\alpha-\xi-\rho)^2\rho k_M k_R},$$

$$a_2^*=\frac{\sigma\pi_R}{\alpha-\xi-\rho},$$

$$b_2^*=\frac{\pi_R[k_M(\pi_M+\frac{1}{2}\pi_R)((\eta-\theta\varphi)(\alpha-\xi-\rho)-\delta\sigma)^2+2\pi_M k_R(\theta(\alpha-\xi-\rho)-\gamma\sigma)^2]}{2\rho(\alpha-\xi-\rho)^2k_M k_R}.$$

By substituting a_1^*, b_1^*, a_2^*, and b_2^* into Equation (A.42) and Equation (A.43), we can obtain the expressions of profit functions $V_M^Y(z)$ and $V_R^Y(z)$ as

$$V_M^Y=a_1^*z+b_1^*, \tag{A.44}$$

$$V_R^Y=a_2^*z+b_2^*. \tag{A.45}$$

By substituting the first derivative of Equation (A.44) and Equation (A.45) into Equation (A.37), Equation (A.38), and Equation (A.39), we can obtain the solutions of g, ψ, and s as

$$g^{Y^*}=\frac{(\pi_M+2\pi_R)(\theta(\alpha-\xi-\rho)-\gamma\sigma)}{2(\alpha-\xi-\rho)k_M}, \tag{A.46}$$

$$\psi^*=\frac{2\pi_R-\pi_M}{2\pi_R+\pi_M}, \tag{A.47}$$

$$s^{Y^*}=\frac{\pi_R[(\alpha-\xi-\rho)(\eta-\theta\varphi)-\delta\sigma]}{(\alpha-\xi-\rho)k_R}. \tag{A.48}$$

Proof for Proposition 4 The same as proposition 1, the HJB equations of Equation (25) and Equation (26) are

$$\begin{aligned}\rho V_M^A(z)=\max_g\{&\pi_M\left[\theta(g-\varphi s)+\eta s-\sigma z\right]-\frac{1}{2}k_M g^2+\phi_M(\pi_R\left[\theta(g-\varphi s)+\eta s-\sigma z\right]-\frac{1}{2}k_R s^2)\\&+V_M^{A'}[(\alpha-\xi)z-\gamma g-\delta s]\}\end{aligned}, \tag{A.49}$$

$$\begin{aligned}\rho V_R^A(z)=\max_s\{&\pi_R\left[\theta(g-\varphi s)+\eta s-\sigma z\right]-\frac{1}{2}k_R s^2+\phi_R[\pi_M(\theta(g-\varphi s)+\eta s-\sigma z)-\frac{1}{2}k_M g^2]\\&+V_R^{A'}[(\alpha-\xi)z-\gamma g-\delta s]\}\end{aligned}. \tag{A.50}$$

From the first derivative of Equation (A.49) and Equation (A.50) for s and g, we have

$$g^A=\frac{\theta(\pi_M+\phi_M\pi_R)-\gamma V_M^{A'}}{k_M}, \tag{A.51}$$

$$s^A=\frac{(\eta-\theta\varphi)(\phi_R\pi_M+\pi_R)-\delta V_R^{A'}}{k_R}. \tag{A.52}$$

By substituting Equation (A.51) and Equation (A.52) into Equation (A.49) and Equation (A.50), we have

$$\begin{aligned}\rho V_M^A=&[(\alpha-\xi)V_M^{A'}-\sigma(\pi_M+\phi_M\pi_R)]z+\frac{k_R[\gamma V_M^{A'}-\theta(\pi_M+\phi_M\pi_R)]^2}{2k_M k_R}\\&+\frac{k_M[((\eta-\theta\varphi)(\phi_R\pi_M+\pi_R)-\delta V_R^{A'})((\theta\varphi-\eta)((\phi_M\phi_R-2)\pi_M-\phi_M\pi_R)+\delta(2V_M^{A'}-\phi_M V_R^{A'}))]}{2k_M k_R}\end{aligned}, \tag{A.53}$$

$$\begin{aligned}\rho V_R^A=&[(\alpha-\xi)V_R^{A'}-\sigma(\pi_R+\phi_R\pi_M)]z+\\&\frac{k_M[(\theta\varphi-\eta)(\phi_R\pi_M+\pi_R)-\delta V_R^{A'}]^2-k_R(\gamma V_M^{A'}-\theta\pi_M)[\theta(\phi_R\pi_M+\pi_R)+\gamma(\phi_R V_M^{A'}-2V_R^{A'})]}{2k_M k_R}\end{aligned}. \tag{A.54}$$

Based on the simplified structures of Equation (A.53) and Equation (A.54), the HJB equation can be expressed as a linear function of z. Thus, we set

$$V_M^A(z)=c_1 z+d_1, \tag{A.55}$$

$$V_R^A(z)=c_2 z+d_2. \tag{A.56}$$

By comparing the coefficients of Equation (A.53), Equation (A.54), Equation (A.55), and Equation (A.56), we can obtain

$$c_1^*=\frac{\sigma(\pi_M+\phi_M\pi_R)}{\alpha-\xi-\rho},$$

$$\begin{aligned}d_1^*=&\frac{k_M(\phi_R\pi_M+\pi_R)((2-\phi_M\phi_R)\pi_M+\phi_M\pi_R)[(\eta-\theta\varphi)(\alpha-\xi-\rho)-\delta\sigma]^2}{2(\alpha-\xi-\rho)^2\rho k_M k_R}\\&+\frac{k_R[(\pi_M+\phi_M\pi_R)(\theta(\alpha-\xi-\rho)-\gamma\sigma)]^2}{2(\alpha-\xi-\rho)^2\rho k_M k_R}\end{aligned},$$

$$c_2^*=\frac{\sigma(\phi_R\pi_M+\pi_R)}{\alpha-\xi-\rho},$$

$$d_2^*=\frac{k_M\left[(\theta\varphi-\eta)(\phi_R\pi_M+\pi_R)-\delta c_2^*\right]^2-k_R(\gamma c_1^*-\theta\pi_M)\left[\theta(\phi_R\pi_M+\pi_R)+\gamma(\phi_R c_1^*-2c_2^*)\right]}{2\rho k_M k_R}.$$

By substituting c_1^*, c_2^*, d_1^*, and d_2^* into Equation (A.55) and Equation (A.56), we can obtain the expressions of profits functions $V_M^A(z)$ and $V_R^A(z)$ as

$$V_M^A(z)=c_1^* z+d_1^*, \tag{A.57}$$

$$V_R^A(z) = c_2^* z + d_2^* . \tag{A.58}$$

By substituting the first derivative of Equation (A.57) and Equation (A.58) into Equation (A.51) and Equation (A.52), we can obtain the solutions of s and g as

$$g^{A^*} = \frac{(\pi_M + \phi_M \pi_R)[\theta(\alpha - \xi - \rho) - \gamma\sigma]}{(\alpha - \xi - \rho)k_M}, \tag{A.59}$$

$$s^{A^*} = \frac{(\phi_R \pi_M + \pi_R)[(\eta - \theta\varphi)(\alpha - \xi - \rho) - \delta\sigma]}{(\alpha - \xi - \rho)k_R}. \tag{A.60}$$

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