Journal of Industrial and Management Optimization

JIMO, 22 (5): 2319–2346.
DOI: 10.3934/jimo.2026085
Received: 16 October 2025
Revised: 16 March 2026
Accepted: 16 March 2026
Published: 14 April 2026

https://www.aimspress.com/journal/jimo

Research article

Present value optimization of two-echelon supply chains with trade credit and lot-splitting under a discounted cash flow framework

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Appendix A. Proof of $\partial^2 HC(T,k)/\partial T^2 > 0$

Consider the holding-cost component,

$$HC(T,k)=\sum_{i=0}^{k-1}\int_{t_i}^{t_{i+1}} hI(t)e^{-Rt}dt,$$

where the delivery epochs are

$$t_i=\frac{iT}{k},\quad i=0,1,\ldots,k\ .$$

On each interval $[ti,\ ti{+}1)$, the inventory trajectory is

$$I(t)=D\left(\frac{(i+1)T}{k}-t\right).$$

Hence,

$$HC(T,k)=\sum_{i=0}^{k-1}\int_{t_i}^{t_{i+1}} hD\left(\frac{(i+1)T}{k}-t\right)e^{-Rt}dt.$$

Let

$$H_i(T)=\int_{t_i}^{t_{i+1}} hD\left(\frac{(i+1)T}{k}-t\right)e^{-Rt}dt,\quad i=0,1,\ldots,k-1,$$

so that

$$HC(T,k)=\sum_{i=0}^{k-1}H_i(T).$$

Applying Leibniz's rule,

$$\frac{dH_i(T)}{dT}=f(t_{i+1},T)\frac{dt_{i+1}}{dT}+\int_{t_i}^{t_{i+1}}\frac{\partial}{\partial T}f(t,T)dt,$$

where

$$f(t,T)=hD\left(\frac{(i+1)T}{k}-t\right)e^{-Rt}.$$

Since

$$t_i=\frac{iT}{k},\quad \frac{dt_i}{dT}=\frac{i}{k},\quad t_{i+1}=\frac{(i+1)T}{k},\quad \frac{dt_{i+1}}{dT}=\frac{i+1}{k},$$

and

$$I(t_{i+1})=D\left(\frac{(i+1)T}{k}-\frac{(i+1)T}{k}\right)=0,$$

the upper-boundary term vanishes. Moreover,

$$I(t_i)=D\left(\frac{(i+1)T}{k}-\frac{iT}{k}\right)=D\frac{T}{k},$$

and

$$\frac{\partial}{\partial T}\left(\frac{(i+1)T}{k}-t\right)=\frac{i+1}{k}.$$

Therefore,

$$\frac{dH_i(T)}{dT}=\frac{hD(i+1)}{k}\int_{t_i}^{t_{i+1}}e^{-RT}dt-\frac{hDiT}{k^2}e^{-Rt_i}.$$

Using

$$\int_{t_i}^{t_{i+1}}e^{-RT}dt=\frac{e^{-Rt_i}-e^{-Rt_{i+1}}}{R},$$

we obtain

$$\frac{dH_i(T)}{dT}=\frac{hD(i+1)}{kR}\left(e^{-Rt_i}-e^{-Rt_{i+1}}\right)-\frac{hDiT}{k^2}e^{-Rt_i}.$$

Differentiating once more gives

$$\frac{d^2H_i(T)}{dT^2}=\frac{hDR}{k^2}e^{-RiT/k}\left(e^{-RT/k}-1+\frac{RT}{k}\right).$$

Hence,

$$\frac{\partial^2 HC(T,k)}{\partial T^2}=\sum_{i=0}^{k-1}\frac{d^2H_i(T)}{dT^2}=\sum_{i=0}^{k-1}\frac{hDR}{k^2}e^{-RiT/k}\left(e^{-RT/k}-1+\frac{RT}{k}\right).$$

To establish positivity, let

$$x=\frac{RT}{k}>0.$$

Then,

$$e^x>1+x \qquad (x>0),$$

which implies

$$e^{-x}>1-x,$$

and hence

$$e^x-1+x>0.$$

Because

$$h>0,\quad D>0,\quad R>0,\quad k>0,$$

and

$$e^{-RiT/k}>0,$$

every term in the above summation is strictly positive. Therefore,

$$\frac{\partial^2 HC(T,k)}{\partial T^2}>0,\quad \forall T>0,$$

which establishes eq (21).