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Research article

Which is better for remanufacturing: carbon asset pledge financing vs equity financing?

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Appendix

Proof of Lemma 1 and Lemma 2

From Eq.(1), the Hessian matrix is

$$H = \begin{bmatrix} \frac{\partial^2 \pi_m}{\partial q_n^2} & \frac{\partial^2 \pi_m}{\partial q_n \partial q_r} \\ \frac{\partial^2 \pi_m}{\partial q_r \partial q_n} & \frac{\partial^2 \pi_m}{\partial q_r^2} \end{bmatrix} = \begin{bmatrix} -2 & -\delta \\ -\delta & -2\delta \end{bmatrix}$$

and we obtain

$$|H_1| = -2, \qquad |H| = 4(2-\delta) > 0$$

Thus, we can see that π_m is concave function on (q_n, q_r). The Lagrange function is below

$$L(\lambda) = (p_n - c_n)q_n + (p_r - c_r)q_r + (1+t)p_e(E - L) + \lambda(B - c_n q_n - c_r q_r)$$

$$\begin{cases} 1 - 2q_n - \delta q_r - c_n - (1+t)p_e e_n - \lambda c_n &= 0 \\ \delta - 2\delta q_n - 2\delta q_r - c_r - (1+t)(1-\varepsilon)p_e e_r - \lambda c_r &= 0 \\ \lambda(B - c_n q_n - c_r q_r) &= 0 \\ \lambda &\geq 0 \end{cases}$$

We can then obtain Lemma 1 and Lemma 2 in terms of the value of λ.

Case 1: $\lambda_1 = 0$. By solving $L(\lambda)$, we have

$$q_n^N = \frac{\gamma_1 - \gamma_2}{2(1-\delta)}, \quad q_r^N = \frac{\gamma_2 - \delta\gamma_1}{2\delta(1-\delta)},$$

$$p_n^N = 1 - \frac{\gamma_1}{2}, \quad p_r^N = \delta - \frac{\gamma_2}{2},$$

$$\gamma_1 = 1 - c_n - e_n p_e(1-\epsilon)(1+t), \quad \gamma_2 = \delta - c_r - e_n p_e \beta(1-\epsilon)(1+t),$$

and the remanufacturer's initial capital satisfies $B > B^N = c_n q_n^N + c_r q_r^N = \dfrac{c_n\delta(\gamma_1-\gamma_2) + c_r(\gamma_2 - \delta\gamma_1)}{2\delta(1-\delta)}$.

Case 2: $\lambda_1 > 0$. By solving $L(\lambda)$, we have

$$q_n^s = \frac{(\gamma_1 - \gamma_2) - \lambda(c_n - c_r)}{2(1-\delta)}, \quad q_r^s = \frac{(\gamma_2 - \delta\gamma_1) + \lambda(c_n\delta - c_r)}{2\delta(1-\delta)},$$

$$p_n^s = 1 - \frac{\gamma_1}{2} + \frac{\lambda c_n}{2}, \quad p_r^s = \delta - \frac{\gamma_1}{2} + \frac{\lambda c_r}{2}.$$

Moreover, λ_1 is the shadow price of the remanufacturer's initial capital, and

$$\lambda_1 = \frac{\delta c_n(\gamma_1 - \gamma_2) + c_r(\gamma_2 - \delta\gamma_1) - 2\delta(1-\delta)B}{\delta(c_n - c_r)^2 + c_r^2(1-\delta)},$$

the remanufacturer's initial capital satisfies $B < B^N$.

Proof of Proposition 1

From Lemma 2, we have

$$\frac{\partial q_n^s}{\partial B} = \frac{\delta(c_n - c_r)}{\delta(c_n - c_r)^2 + c_r^2(1-\delta)} > 0, \quad \frac{\partial p_n^s}{\partial B} = \frac{-\delta c_n(1-\delta)}{\delta(c_n - c_r)^2 + c_r^2(1-\delta)} < 0,$$

$$\frac{\partial q_r^s}{\partial B} = \frac{-(c_n\delta - c_r)}{\delta(c_n - c_r)^2 + c_r^2(1-\delta)} < 0, \quad \frac{\partial p_r^s}{\partial B} = \frac{-\delta c_r(1-\delta)}{\delta(c_n - c_r)^2 + c_r^2(1-\delta)} < 0,$$

$$\frac{\partial E^s}{\partial B} = \frac{e_n[c_n\delta(1-\beta) + c_r(1-\delta)]}{\delta(c_n - c_r)^2 + c_r^2(1-\delta)} > 0.$$

Proof of Proposition 2

From the proof of Lemma 2, λ is the shadow price of the remanufacturer's initial capital. Thus, if $r < \lambda_1$, the remanufacturer can profit from CAPF loans, and we obtain

$$B < \overline{B} = \frac{(\gamma_1 - \gamma_2)\delta c_n + (\gamma_2 - \delta\gamma_1)c_r - r[\delta c_n(c_n - 2c_r) + c_r^2]}{2\delta(1-\delta)}.$$

Proof of Lemma 3

From Eq. (3), the Hessian matrix is

$$H=\begin{bmatrix}\frac{\partial^2\pi_m^{CF}}{\partial q_n^2} & \frac{\partial^2\pi_m^{CF}}{\partial q_n\partial q_r}\\ \frac{\partial^2\pi_m^{CF}}{\partial q_r\partial q_n} & \frac{\partial^2\pi_m^{CF}}{\partial q_r^2}\end{bmatrix}=\begin{bmatrix}-2 & -2\delta\\ -2\delta & -2\delta\end{bmatrix},$$

and $|H_1|=-2$, $|H_2|=4(2-\delta)>0$. Thus we can see that π_m^{CF} is concave function on q_n, q_r. We can then use backward induction. Let $\partial\pi_m^{CF}/\partial q_n=0$ and $\partial\pi_m^{CF}/\partial q_r=0$ simultaneously. We then obtain

$$q_n=\frac{\gamma_1-\gamma_2-e^{rT}p_e\theta e_n(1-\beta)}{2(1-\delta)},\qquad q_r=\frac{\gamma_2-\delta\gamma_1+e^{rT}p_e\theta e_n(\delta-\beta)}{2\delta(1-\delta)}.$$

Substituting q_n, q_r into Eq. (4), we obtain

$$\frac{\partial^2\pi_b^{CF}}{\partial\theta^2}=-e^{rT}p_e e_n\left(\frac{e^{rT}p_e e_n(1-\beta)}{(1-\delta)}+\beta\frac{e^{rT}p_e e_n(\beta-\delta)}{\delta(1-\delta)}\right)<0,$$

Thus, we can see that π_b^{CF} is concave function on θ. Then we let $\partial\pi_b^{CF}/\partial\theta=0$ and obtain

$$\theta^{CF}=\frac{(\gamma_1-\gamma_2)\delta+\beta(\gamma_2-\delta\gamma_1)}{2e^{rT}p_e e_n\xi_1}.$$

Substituting θ^{CF} into

$$q_n=\frac{\gamma_1-\gamma_2-e^{rT}p_e\theta e_n(1-\beta)}{2(1-\delta)}$$

and

$$q_r=\frac{\gamma_2-\delta\gamma_1+e^{rT}p_e\theta e_n(\delta-\beta)}{2\delta(1-\delta)},$$

we see that the optimal production quantity is

$$q_n^{CF}=\frac{(\gamma_1-\gamma_2)(\xi_1-\beta(\delta-\beta))-\beta(1-\beta)(\gamma_2-\delta\gamma_1)}{4(1-\delta)\xi_1},$$

$$q_r^{CF}=\frac{(\gamma_2-\delta\gamma_1)[\delta(1-\beta)+\xi_1]+\delta(\delta-\beta)(\gamma_1-\gamma_2)}{4\delta(1-\delta)\xi_1},$$

$\xi_1=\delta(1-\beta)-\beta(\delta-\beta)$ and $\xi_2=\delta c_n(1-\beta)-c_r(\delta-\beta)$.

Proof of Lemma 4

Under the EF strategy, remanufacturers are not required to repay the principal or interest, nor are they burdened with debt-related risks. Instead, they distribute the profit dividends to investors according to the predetermined percentage. Therefore, remanufacturers cannot only obtain sufficient funds but also formulate operational decisions as if they were in a financially unconstrained scenario. Thus, the optimal production decision of remanufacturers is identical in Model N and Model EF, and

$$q_n^{EF}=q_n^N=\frac{\gamma_1-\gamma_2}{2(1-\delta)},\qquad q_r^{EF}=q_r^N=\frac{\gamma_2-\delta\gamma_1}{2\delta(1-\delta)}.$$

Proof of Proposition 3

From Lemma 3, we have

$$\frac{\partial\theta}{\partial c_n} = \frac{-\delta(1-\beta)}{2e^{rT}\epsilon e_n\xi_1 p_e} < 0, \quad \frac{\partial\theta}{\partial c_r} = \frac{\delta-\beta}{2e^{rT}\epsilon e_n\xi_1 p_e} > 0,$$

$$\frac{\partial\theta}{\partial r} = \frac{-Te^{rT}[(\gamma_1-\gamma_2)\delta+\beta(\gamma_2-\delta\gamma_1)]}{2e^{rT}e_n\xi_1 p_e} < 0,$$

$$\frac{\partial\theta}{\partial\epsilon} = \frac{-[\delta\epsilon[1-\delta-(c_n-c_r)]+\beta[\delta e_n-c_r]]}{2e^{rT}e_n\xi_1 p_e} < 0,$$

$$\frac{\partial\theta}{\partial p_e} = \frac{-[\delta p_e[1-\delta-(c_n-c_r)]+\beta[\delta c_n-c_r]]}{2e^{rT}\epsilon e_n\xi_1 p_e^2} < 0.$$

Proof of Proposition 4

From Lemma 3, if r is given, we have

$$\frac{\partial q_n^{CF}}{\partial\theta} = \frac{-p_e e^{rT}\epsilon e_n(1-\beta)}{2(1-\delta)} < 0, \qquad \frac{\partial q_r^{CF}}{\partial\theta} = \frac{e^{rT}p_e\epsilon e_n(1-\beta)}{2\delta(1-\delta)} > 0.$$

If θ is given, we have

$$\frac{\partial q_n^{CF}}{\partial r} = \frac{-Te^{rT}p_e\theta\epsilon e_n(1-\beta)}{2(1-\delta)} < 0, \qquad \frac{\partial q_r^{CF}}{\partial r} = \frac{Te^{rT}p_e\epsilon e_n\theta(1-\beta)}{2\delta(1-\delta)} > 0.$$

Proof of Corollary 2

When both the remanufacturing enterprise and the investor accept the EF strategy, the profits of both parties must be satisfied: $\pi_m^{EF} > 0$ and $\pi_e^{EF} > 0$. From Lemma 3, we have $q_n^{EF} = q_n^N$, $q_r^{EF} = q_r^N$, and we can obtain

$$\pi_m^{EF} = (1-\alpha)(\pi_m^N + B^*) - B, \qquad \pi_e^{EF} = \alpha(\pi_m^N + B^*) - B^*.$$

Remanufacturers and investors can accept EF when the condition $\pi_m^{EF} > 0$, $\pi_e^{EF} > 0$ is satisfied, and we can obtain $\alpha_1 < \alpha < \alpha_2$ as follows:

$$\alpha_1 = \frac{4\delta(1-\delta)B^*}{\gamma_1(\gamma_1-\gamma_2)+\gamma_2(\gamma_2-\delta\gamma_1)+2\delta(1-\delta)B^*},$$

$$\alpha_2 = 1 - \frac{4\delta(1-\delta)B}{\gamma_1(\gamma_1-\gamma_2)+\gamma_2(\gamma_2-\delta\gamma_1)+2\delta(1-\delta)B^*}.$$

Proof of Proposition 5

From Lemma 3 and Lemma 4, we have

$$\frac{\partial q_n^{CF}}{\partial\epsilon} = \frac{e_n p_e(1-\epsilon)(1-\beta)[\xi_1+(1-\beta)-(\delta-\beta)]}{4(1-\delta)\xi_1} > 0,$$

$$\frac{\partial q_n^{CF}}{\partial \epsilon} = \frac{e_n p_e(1-\beta)(1-\epsilon)[2\delta(1-\beta)+(\delta-\beta)^2]}{4\delta(1-\delta)\xi_1} > 0,$$
$$\frac{\partial q_n^{CF}}{\partial \epsilon} = \frac{e_n p_e(1-\beta)(1+t)}{2(1-\delta)} > 0,$$
$$\frac{\partial q_n^{CF}}{\partial p_e} = \frac{-e_n(1-\epsilon)(1-\beta)[\xi_1+1-\beta(\delta-\beta)]}{4(1-\delta)\xi_1} < 0,$$
$$\frac{\partial q_n^{CF}}{\partial p_e} = \frac{-e_n(1-\epsilon)(1-\beta)[2\delta(1-\beta)-(\delta+\beta)(\delta-\beta)]}{4\delta(1-\delta)\xi_1} < 0,$$
$$\frac{\partial q_n^{CF}}{\partial p_e} = \frac{-e_n(1-\epsilon)(1-\beta)(1+t)}{2\delta(1-\delta)} < 0,$$
$$\frac{\partial q_n^{CF}}{\partial p_e} = \frac{-e_n(1-\epsilon)(1-\beta)(1+t)}{2(1-\delta)} < 0,$$
$$\frac{\partial q_n^{CF}}{\partial t} = \frac{e_n p_e(1-\beta)(1-\epsilon)[\xi_1+(1-\beta)-(\delta-\beta)]}{4(1-\delta)\xi_1} > 0,$$
$$\frac{\partial q_n^{CF}}{\partial t} = \frac{e_n p_e(1-\beta)(1-\epsilon)[2\delta(1-\beta)+(\delta-\beta)^2]}{4\delta(1-\delta)\xi_1} > 0,$$
$$\frac{\partial q_n^{CF}}{\partial t} = \frac{-e_n p_e(1-\beta)(1-\epsilon)}{2(1-\delta)} < 0,$$
$$\frac{\partial q_n^{CF}}{\partial t} = \frac{e_n p_e(1-\beta)(1-\epsilon)}{2(1-\delta)} < 0.$$

Proof of Proposition 6

From Lemma 3 and Lemma 4, we have

$$q_n^{EF} - q_n^{CF} = \frac{\delta(1-\beta)(\gamma_1-\gamma_2)+\beta(1-\beta)(\gamma_2-\delta\gamma_1)}{4(1-\delta)\xi_1} > 0,$$

$$q_r^{EF} - q_r^{CF} = \frac{-(\gamma_2-\gamma_1\delta)\beta(\delta-\beta)-\delta(\gamma_1-\gamma_2)}{4\delta(1-\delta)\xi_1} < 0,$$

$$E^{EF} - E^{CF} = e_n[q_n^{EF} - q_n^{CF} + \beta(q_r^{EF} - q_r^{CF})] = \frac{e_n[\delta(\gamma_1-\gamma_2)+\beta(\gamma_2-\delta\gamma_1)]}{4\delta(1-\delta)} > 0.$$