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Research article

Decision-making of third-party remanufacturing supply chain with risk aversion and retail service

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Appendix

Proof of propositions.

Proof of Proposition 1.

According to Eq. (5), since $\frac{\partial^2 E\left(\pi_t^A\right)}{\partial q_r^{\ 2}} = -\left(2\delta + k\right) < 0$, $E\left(\pi_t^A\right)$ is concave in q_r. By solving $\frac{\partial E\left(\pi_t^A\right)}{\partial q_r} = 0$, we can obtain the following equation.

$$q_r = \frac{\delta\phi - c_r - f - \delta q_n}{2\delta + k} \qquad \text{(A1)}$$

From Eq. (6), the Hessian matrix of U_r can be derived: $H = \begin{vmatrix} \frac{\partial^2 U_r^A}{\partial q_n^2} & \frac{\partial^2 U_r^A}{\partial q_n \partial e} \\ \frac{\partial^2 U_r^A}{\partial e \partial q_n} & \frac{\partial^2 U_r^A}{\partial e^2} \end{vmatrix} = \begin{vmatrix} -2 & \theta \\ \theta & -\beta \end{vmatrix}$. When $2\beta - \theta^2 > 0$, the Hessian matrix is negative definite. By solving $\frac{\partial U_r^A}{\partial q_n} = 0$ and $\frac{\partial U_r^A}{\partial e} = 0$ simultaneously, we can obtain:

$$q_n^A = \frac{\beta(\phi - \eta\sigma - \delta q_r - w)}{2\beta - \theta^2} \qquad \text{(A2)}$$

$$e^A = \frac{\theta(\phi - \eta\sigma - \delta q_r - w)}{2\beta - \theta^2} \qquad \text{(A3)}$$

From Eqs. (A1) to (A3), the Eqs. (7) to (9) can be obtained.

Proof of Proposition 2.

According to Eq. (10), the Hessian matrix of $E\left(\pi_m^A\right)$ is:

$$H = \begin{vmatrix} \frac{\partial^2 E\left(\pi_m^A\right)}{\partial w^2} & \frac{\partial^2 E\left(\pi_m^A\right)}{\partial w \partial f} \\ \frac{\partial^2 E\left(\pi_m^A\right)}{\partial f \partial w} & \frac{\partial^2 E\left(\pi_m^A\right)}{\partial f^2} \end{vmatrix} = \begin{vmatrix} \frac{-2(2\delta + k)\beta}{(2\beta - \theta^2)(2\delta + k) - \beta\delta^2} & \frac{2\beta\delta}{(2\beta - \theta^2)(2\delta + k) - \beta\delta^2} \\ \frac{2\beta\delta}{(2\beta - \theta^2)(2\delta + k) - \beta\delta^2} & \frac{-2(2\beta - \theta^2)}{(2\beta - \theta^2)(2\delta + k) - \beta\delta^2} \end{vmatrix}$$

. The matrix is negative definite when $(2\delta + k)(2\beta - \theta^2) - \beta\delta^2 > 0$. By solving $\frac{\partial E\left(\pi_m^A\right)}{\partial w} = 0$ and $\frac{\partial E\left(\pi_m^A\right)}{\partial f} = 0$ simultaneously, we can obtain the Eqs. (11) and (12).

Proof of Proposition 3.

According to Eq. (14), since $\frac{\partial^2 E\left(\pi_t^O\right)}{\partial q_r^2} = -k < 0$, $E\left(\pi_t^O\right)$ is concave in q_r. By solving $\frac{\partial E\left(\pi_t^O\right)}{\partial q_r} = 0$, we can obtain Eq. (15). Substituting Eq. (15) to retailer's response, the Eqs. (16) and (17) can be obtained.

Proof of Proposition 4.

From Eq. (18), Hessian matrix of $E\left(\pi_m^O\right)$ is:

$H=\begin{vmatrix}\frac{\partial^2 E\left(\pi_m^O\right)}{\partial w^2} & \frac{\partial^2 E\left(\pi_m^O\right)}{\partial w\partial p_o}\\ \frac{\partial^2 E\left(\pi_m^O\right)}{\partial p_o\partial w} & \frac{\partial^2 E\left(\pi_m^O\right)}{\partial p_o^2}\end{vmatrix}=\begin{vmatrix}-\frac{2\beta}{2\beta-\theta^2} & 0\\ 0 & -\frac{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\beta\delta^2\right)}{k^2\left(2\beta-\theta^2\right)}\end{vmatrix}$. It can be seen that when $\left(\delta+k\right)\left(2\beta-\theta^2\right)-\beta\delta^2>0$, the matrix is negative definite and $E\left(\pi_m^O\right)$ is jointly concave in w and p_o. By solving $\frac{\partial E\left(\pi_m^O\right)}{\partial w}=0$ and $\frac{\partial E\left(\pi_m^O\right)}{\partial p_o}=0$ simultaneously, Proposition 4 is proved.

Proof of Corollary 1.

(i) $\frac{\partial w^{A*}}{\partial\eta}=\frac{\partial w^{O*}}{\partial\eta}=-\frac{\sigma}{2}<0$. $\frac{\partial q_n^{A*}}{\partial\eta}=-\frac{\beta\left(2\delta+k\right)\sigma}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)}<0$.

$\frac{\partial e^{A*}}{\partial\eta}=-\frac{\theta\left(2\delta+k\right)\sigma}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)}<0$. $\frac{\partial q_r^{A*}}{\partial\eta}=\frac{\beta\delta\sigma}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)}>0$.

$\frac{\partial p_n^{A*}}{\partial\eta}=\frac{\left(\left(\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)\sigma}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)}$, it can be seen that when $\left(\beta-\theta^2\right)\left(2\delta+k\right)>\beta\delta^2$, $\frac{\partial p_n^{A*}}{\partial\eta}>0$; otherwise, $\frac{\partial p_n^{A*}}{\partial\eta}<0$. $\frac{\partial p_r^{A*}}{\partial\eta}=\frac{\delta\left(\delta+k\right)\beta\sigma}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)}>0$.

$\frac{\partial E\left(\pi_m^{A*}\right)}{\partial\eta}-\frac{\beta\sigma\Delta_1}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)}<0$. $\frac{\partial E\left(\pi_r^{A}\right)}{\partial\eta}=\frac{\beta\sigma\left(\begin{array}{l}\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-2\beta\delta^2\right)\Delta_1\\ -2\eta\sigma\left(2\delta+k\right)\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)\end{array}\right)}{4\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^2}$, it can be obtained that when $\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-2\beta\delta^2\right)\Delta_1>2\eta\sigma\left(2\delta+k\right)\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)$, $\frac{\partial E\left(\pi_r^{A}\right)}{\partial\eta}>0$; otherwise, $\frac{\partial E\left(\pi_r^{A}\right)}{\partial\eta}<0$. $\frac{\partial U_r^{A*}}{\partial\eta}=-\frac{\beta\sigma\left(2\beta-\theta^2\right)\left(2\delta+k\right)\Delta_1}{4\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^2}<0$.

$\frac{\partial E\left(\pi_t^{A*}\right)}{\partial\eta}=\frac{\left(2\delta+k\right)\beta\delta\sigma\Delta_2}{4\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^2}>0$.

$\frac{\partial E\left(I\right)^{A*}}{\partial\eta}=-\frac{\left(\left(1+\lambda_u\right)\left(2\delta+k\right)-\left(\lambda_r-\lambda_u\right)\delta\right)\beta\sigma}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)}<0$.

$\frac{\partial p_o^{O*}}{\partial\eta}=\frac{k\delta\beta\sigma}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)}>0$. $\frac{\partial q_n^{O*}}{\partial\eta}=-\frac{\beta\sigma\left(\delta+k\right)}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)}<0$.

$\frac{\partial q_r^{O*}}{\partial\eta}=\frac{\delta\beta\sigma}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)}>0$. $\frac{\partial e^{O*}}{\partial\eta}=-\frac{\sigma\theta\left(\delta+k\right)}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)}<0$.

$\frac{\partial p_n^{O*}}{\partial \eta}=\frac{\left((\delta+k)\left(\beta-\theta^2\right)-\delta^2\beta\right)\sigma}{2\left((\delta+k)\left(2\beta-\theta^2\right)-\delta^2\beta\right)}>0$, it can be seen that when $(\delta+k)\left(\beta-\theta^2\right)>\delta^2\beta$, $\frac{\partial p_n^{O*}}{\partial \eta}>0$;otherwise, $\frac{\partial p_n^{O*}}{\partial \eta}<0$. $\frac{\partial p_r^{O*}}{\partial \eta}=\frac{\beta\sigma k}{2\left((\delta+k)\left(2\beta-\theta^2\right)-\delta^2\beta\right)}>0$.

$\frac{\partial E\left(\pi_m^{O*}\right)}{\partial \eta}=-\frac{\beta\sigma\Delta_6}{2\left((\delta+k)\left(2\beta-\theta^2\right)-\delta^2\beta\right)}<0$.

$\frac{\partial E\left(\pi_r{}^{O*}\right)}{\partial \eta}=\frac{\sigma\beta\begin{pmatrix}\left((\delta+k)\left(2\beta-\theta^2\right)-2\delta^2\beta\right)\Delta_6\\ -2\eta\sigma(\delta+k)\left((\delta+k)\left(2\beta-\theta^2\right)-\delta^2\beta\right)\end{pmatrix}}{4\left((\delta+k)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}$, it can be seen that when

$\left((\delta+k)\left(2\beta-\theta^2\right)-2\delta^2\beta\right)\Delta_6>2\eta\sigma(\delta+k)\left((\delta+k)\left(2\beta-\theta^2\right)-\delta^2\beta\right)$, $\frac{\partial E\left(\pi_r{}^{O*}\right)}{\partial \eta}>0$; otherwise, $\frac{\partial E\left(\pi_r{}^{O*}\right)}{\partial \eta}<0$. $\frac{\partial U_r^{O*}}{\partial \eta}=-\frac{\beta\sigma(\delta+k)\left(2\beta-\theta^2\right)\Delta_6}{4\left((\delta+k)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}<0$.

$\frac{\partial\left(\pi_t^{O*}\right)}{\partial \eta}=\frac{\sigma\delta\beta k\Delta_2}{4\left((\delta+k)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}>0$.

$\frac{\partial E(I)^{O*}}{\partial \eta}=-\frac{\left((\delta+k)(1+\lambda_u)-(\lambda_r-\lambda_u)\delta\right)\beta\sigma}{2\left((\delta+k)\left(2\beta-\theta^2\right)-\delta^2\beta\right)}<0$.

Proof of Corollary 2.

$\frac{\partial q_n^{A*}}{\partial \beta}=-\frac{\theta^2(2\delta+k)\left((2\delta+k)(\phi-\eta\sigma-c_n)-\delta(\delta\phi-c_r)\right)}{2\left(\left(2\beta-\theta^2\right)(2\delta+k)-\beta\delta^2\right)^2}<0$.

$\frac{\partial e^{A*}}{\partial \beta}=-\frac{\theta\left((2\delta+k)(\phi-\eta\sigma-c_n)-\delta(\delta\phi-c_r)\right)\left(2(2\delta+k)-\delta^2\right)}{2\left(\left(2\beta-\theta^2\right)(2\delta+k)-\beta\delta^2\right)^2}<0$.

$\frac{\partial q_r^{A*}}{\partial \beta}=\frac{\theta^2\delta\Delta_1}{2\left(\left(2\beta-\theta^2\right)(2\delta+k)-\beta\delta^2\right)^2}>0$. $\frac{\partial p_n^{A*}}{\partial \beta}=-\frac{(2\delta+k)\theta^2\Delta_1}{2\left(\left(2\beta-\theta^2\right)(2\delta+k)-\beta\delta^2\right)^2}<0$.

$\frac{\partial p_r^{A*}}{\partial \beta}=\frac{\theta^2\delta(\delta+k)\Delta_1}{2\left(\left(2\beta-\theta^2\right)(2\delta+k)-\beta\delta^2\right)^2}>0$. $\frac{\partial E\left(\pi_m^{A*}\right)}{\partial \beta}=-\frac{\theta^2\Delta_1{}^2}{4\left(\left(2\beta-\theta^2\right)(2\delta+k)-\beta\delta^2\right)^2}<0$.

Since $E\left(\pi_r^{A*}\right)$ is greater than zero, it can be known that

$2\left(\left(2\beta-\theta^2\right)(2\delta+k)-\beta\delta^2\right)(\phi+\eta\sigma-c_n)-\left(2\beta-\theta^2\right)\Delta_1-2\delta\Delta_2>0$, then wo can obtain

$\frac{\partial E\left(\pi_r^{A*}\right)}{\partial \beta}<-\frac{\theta^2\Delta_1{}^2\delta^2\beta}{8\left(\left(2\beta-\theta^2\right)(2\delta+k)-\beta\delta^2\right)^3}<0$.

$$\frac{\partial U_r^{A*}}{\partial \beta} = -\frac{\Delta_1{}^2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)+\beta\delta^2\right)\theta^2}{8\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^3} < 0 .$$

$$\frac{\partial E\left(\pi_t^{A*}\right)}{\partial \beta} = \frac{\left(2\delta+k\right)\theta^2\delta\Delta_1\Delta_2}{4\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^3} > 0 .$$

$$\frac{\partial E\left(I\right)^{A*}}{\partial \beta} = -\frac{\theta^2\Delta_1\left(\left(2\delta+k\right)\left(1+\lambda_u\right)-\left(\lambda_r-\lambda_u\right)\delta\right)}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^2} < 0 .$$

$$\frac{\partial p_o^{O*}}{\partial \beta} = \frac{\theta^2 k\delta\Delta_6}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2} > 0 .$$

$$\frac{\partial q_n^{O*}}{\partial \beta} = -\frac{\theta^2\left(\delta+k\right)\Delta_6}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2} < 0 .$$

$$\frac{\partial q_r^{O*}}{\partial \beta} = \frac{\theta^2\delta\Delta_6}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2} > 0 .$$

$$\frac{\partial e^{O*}}{\partial \beta} = -\frac{\theta\Delta_6\left(2\left(\delta+k\right)-\delta^2\right)}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2} < 0 .$$

$$\frac{\partial p_n^{O*}}{\partial \beta} = -\frac{\theta^2\left(\delta+k\right)\Delta_6}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2} < 0 .$$

$$\frac{\partial p_r^{O*}}{\partial \beta} = \frac{k\theta^2\Delta_6}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2} > 0 .$$

$$\frac{\partial E\left(\pi_m^{O*}\right)}{\partial \beta} = -\frac{\theta^2\Delta_6{}^2}{4\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2} < 0 .$$

$$\frac{\partial E\left(\pi_r{}^{O*}\right)}{\partial \beta} < -\frac{\theta^2\delta^2\beta\Delta_6{}^2}{8\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^3} < 0 .$$

$$\frac{\partial U_r^{O*}}{\partial \beta} = -\frac{\left(\left(2\beta-\theta^2\right)\left(\delta+k\right)+\delta^2\beta\right)\Delta_6{}^2\theta^2}{8\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^3} < 0 .$$

$$\frac{\partial\left(\pi_t^{O*}\right)}{\partial \beta} = \frac{k\theta^2\delta\Delta_2\Delta_6}{4\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^3} > 0 .$$

$$\frac{\partial E\left(I\right)^{O*}}{\partial \beta} = -\frac{\theta^2\Delta_6\left(\left(1+\lambda_u\right)\left(\delta+k\right)-\left(\lambda_r-\lambda_u\right)\delta\right)}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2} < 0 .$$

Proof of Corollary 3.

$\frac{\partial q_n^{A*}}{\partial k}=\frac{\delta\beta\Delta_2}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^2}>0$.

$\frac{\partial e^{A*}}{\partial k}=\frac{\theta\delta\Delta_2}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^2}>0$.

$\frac{\partial q_r^{A*}}{\partial k}=-\frac{\Delta_2\left(2\beta-\theta^2\right)}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^2}<0$.

$\frac{\partial p_n^{A*}}{\partial k}=\frac{\beta\delta\Delta_2}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^2}>0$.

$\frac{\partial p_r^{A*}}{\partial k}=\frac{\delta\left(2\beta-\theta^2-\beta\delta\right)\Delta_2}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^2}$, we can know when $2\beta-\theta^2-\beta\delta>0$, $\frac{\partial p_r^{A*}}{\partial k}>0$; otherwise, $\frac{\partial p_r^{A*}}{\partial k}<0$.

Since $2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)\left(\phi+\eta\sigma-c_n\right)-\left(2\beta-\theta^2\right)\Delta_1-2\delta\Delta_2>0$, we can obtain that $\frac{\partial E\left(\pi_r^A\right)}{\partial k}>\frac{\delta\beta\Delta_1\Delta_2\left(2\beta-\theta^2\right)}{8\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^3}>0$.

$\frac{\partial U_r^{A*}}{\partial k}=\frac{\delta\beta\left(2\beta-\theta^2\right)\Delta_1\Delta_2}{4\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\delta^2\beta\right)^3}>0$.

$\frac{\partial E\left(\pi_m^{A*}\right)}{\partial k}=-\frac{\Delta_2^{\,2}}{4\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^2}<0$.

$\frac{\partial E\left(\pi_t^{A*}\right)}{\partial k}=-\frac{\Delta_2^{\,2}\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)+\beta\delta^2\right)}{8\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^3}<0$.

$\frac{\partial p_o^{O*}}{\partial k}=\frac{\delta\Delta_2\left(2\beta-\theta^2-\delta\beta\right)}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}$, it can be seen that $\frac{\partial p_o^{O*}}{\partial k}>0$ when $2\beta-\theta^2-\delta\beta>0$; otherwise, $\frac{\partial p_o^{O*}}{\partial k}<0$.

$\frac{\partial q_n^{O*}}{\partial k}=\frac{\beta\delta\Delta_2}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}>0$.

$\frac{\partial q_r^{O*}}{\partial k}=-\frac{\Delta_2\left(2\beta-\theta^2\right)}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}<0$.

$\frac{\partial e^{O*}}{\partial k}=\frac{\theta\delta\Delta_2}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}>0$.

$\frac{\partial p_n^{O*}}{\partial k}=\frac{\beta\delta\Delta_2}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}>0$.

$\frac{\partial p_r^{O*}}{\partial k}=\frac{\Delta_2\left(2\beta-\theta^2-\beta\delta\right)}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}$, it can be known that $\frac{\partial p_r^{O*}}{\partial k}>0$ when $2\beta-\theta^2-\beta\delta>0$; otherwise $\frac{\partial p_r^{O*}}{\partial k}<0$.

$$\frac{\partial E\left(\pi_m^{O*}\right)}{\partial k}=-\frac{{\Delta_2}^2}{4\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}<0.$$

Since $E\left(\pi_r^{O*}\right)>0$, we have $2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)\left(\phi+\eta\sigma-c_n\right)>\Delta_2\delta+\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)\left(\phi-\eta\sigma-c_n\right)$, and it can be known that $\frac{\partial E\left(\pi_r^{O*}\right)}{\partial k}>\frac{\left(2\beta-\theta^2\right)\beta\delta\Delta_2\Delta_6}{8\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^3}>0$.

$$\frac{\partial U_r^{O*}}{\partial k}=\frac{\delta\beta\left(2\beta-\theta^2\right)\Delta_2\Delta_6}{4\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^3}>0.$$

$\frac{\partial\left(\pi_t^{O*}\right)}{\partial k}=\frac{\left(\left(\delta-k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right){\Delta_2}^2}{8\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^3}$, it can be seen that when $\left(\delta-k\right)\left(2\beta-\theta^2\right)>\delta^2\beta$, $\frac{\partial\left(\pi_t^{O*}\right)}{\partial k}>0$; otherwise, $\frac{\partial\left(\pi_t^{O*}\right)}{\partial k}<0$.

From $\frac{\partial E\left(I\right)^{A*}}{\partial k}=\frac{\Delta_2\left(\left(1+\lambda_u\right)\beta\delta-\left(\lambda_r-\lambda_u\right)\left(2\beta-\theta^2\right)\right)}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^2}$ and $\frac{\partial E\left(I\right)^{O*}}{\partial k}=\frac{\Delta_2\left(\left(1+\lambda_u\right)\beta\delta-\left(\lambda_r-\lambda_u\right)\left(2\beta-\theta^2\right)\right)}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}$, we can obtain that $\frac{\partial E\left(I\right)^{N*}}{\partial k}>0$ when $\frac{\left(1+\lambda_u\right)\beta\delta}{2\beta-\theta^2}+\lambda_u>\lambda_r$; otherwise, $\frac{\partial E\left(I\right)^{N*}}{\partial k}<0$.

Proof of Corollary 4.

$$q_n^{A*}-q_n^{O*}=\frac{\delta^2\beta\Delta_2}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)}>0.$$

$$q_r^{A*}-q_r^{O*}=-\frac{\delta\left(2\beta-\theta^2\right)\Delta_2}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)}<0.$$

$$e^{A*}-e^{O*}=\frac{\delta^2\theta\Delta_2}{2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)}>0.$$

$$E\left(\pi_m^{A*}\right)-E\left(\pi_m^{O*}\right)=-\frac{\begin{array}{c}\delta\left(\left(2\beta-\theta^2\right)\left(\delta+k\right)-\delta^2\beta\right){\Delta_2}^2\\+\delta\phi\left(\delta\phi-c_r\right)\left(2\beta-\theta^2\right)^2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)\left(1-\delta\right)k\end{array}}{4\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}<0.$$

Since $E\left(\pi_r^{A*}\right)$>0, thus $2\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)\left(\phi+\eta\sigma-c_n\right)-\left(2\beta-\theta^2\right)\Delta_1-2\delta\Delta_2>0$, we can obtain that $E\left(\pi_r^{A*}\right)-E\left(\pi_r^{O*}\right)>\dfrac{\Delta_2\Delta_6\delta^2\left(2\beta-\theta^2\right)\beta}{8\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}>0$.

$$U_r^{A*}-U_r^{O*}=\frac{\beta\delta^2\left(2\beta-\theta^2\right)\Delta_2\begin{pmatrix}\Delta_6\left(2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)+\delta\left(2\beta-\theta^2\right)\right)\\+\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)\delta\left(\phi-\eta\sigma-c_n\right)\end{pmatrix}}{8\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2\left(\left(2\delta+k\right)\left(2\beta-\theta^2\right)-\beta\delta^2\right)^2}>0.$$

$E\left(\pi_t^{A*}\right)-E\left(\pi_t^{O*}\right)=\dfrac{\delta^2\Delta_2^{\,2}\left(\left(2\beta-\theta^2-2\delta\beta\right)\left(2\delta+k\right)\left(2\beta-\theta^2\right)+2\delta^3\beta^2\right)}{8\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)^2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)^2}$, it can be seen that when $\left(2\beta-\theta^2-2\delta\beta\right)\left(2\delta+k\right)\left(2\beta-\theta^2\right)+2\delta^3\beta^2>0$, $E\left(\pi_t^{A*}\right)>E\left(\pi_t^{O*}\right)$;otherwise, $E\left(\pi_t^{A*}\right)<E\left(\pi_t^{O*}\right)$.

$E\left(I\right)^{A*}-E\left(I\right)^{O*}=\dfrac{\begin{pmatrix}\left(2\beta-\theta^2\right)\left(\delta\phi-c_r\right)\\-\beta\delta\left(\phi-\eta\sigma-c_n\right)\end{pmatrix}\left(\left(1+\lambda_u\right)\beta\delta^2-\left(\lambda_r-\lambda_u\right)\delta\left(2\beta-\theta^2\right)\right)}{2\left(\left(\delta+k\right)\left(2\beta-\theta^2\right)-\delta^2\beta\right)\left(\left(2\beta-\theta^2\right)\left(2\delta+k\right)-\beta\delta^2\right)}$, it can be known that when $\dfrac{\left(1+\lambda_u\right)\beta\delta}{2\beta-\theta^2}+\lambda_u>\lambda_r$, $E\left(I\right)^{A*}>E\left(I\right)^{O*}$; otherwise $E\left(I\right)^{A*}<E\left(I\right)^{O*}$.