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Research article

A general approximate computational framework for basket spread options pricing with and without default risk

Qifeng Zhong, Zheng Liu, Jing Yao* and Xingye Yue

Center for Financial Engineering and Department of Mathematics, Soochow University, Suzhou, 215006, China

* **Correspondence:** Email: j.yao@suda.edu.cn.

Appendix A Proof of Theorem 3.1

First, we perform a simplification on the LBS exercise region A in (2.2):

$$A = \{\omega : \sum_{i=1}^{n} b_i ln(S_i(T)) - \sum_{i=n+1}^{n+m} b_i ln(S_i(T)) \geq ln(L)\},$$

where $L = a \cdot f(\boldsymbol{b}_1)/f(\boldsymbol{b}_2)$, and $\boldsymbol{b}_1 = [1, 1, \cdots, 1, 0, \cdots, 0]$, $\boldsymbol{b}_2 = [0, 0, \cdots, 0, 1, \cdots, 1]$.

Then, we proceed to handle the following expectations: $G_0 = \mathbb{E}_{\mathbb{Q}}[I(A)]$ and $G_k = \mathbb{E}_{\mathbb{Q}}[S_k(T)I(A)]$, $k = 1, 2, \cdots, n+m$. $\mathbb{E}_{\mathbb{Q}}[I(A)]$ is naturally interpreted as a probability $\mathbb{Q}(A)$. For expectations like $\mathbb{E}_{\mathbb{Q}}[S_i(T)I(A)]$, we apply the measure change technique to handle them. We introduce a list new probability measure $\mathbb{Q}_k$ which is defined by

$$\mathbb{Q}_k(B) = \frac{\mathbb{E}_{\mathbb{Q}}[I(B)S_k(T)]}{\mathbb{E}_{\mathbb{Q}}[S_k(T)]}$$

for any event $B \in \mathcal{F}_T$. Then, the moment-generating function of underlying asset returns under $\mathbb{Q}_k$:

$$f_k(\phi) = \mathbb{E}_{\mathbb{Q}_k}[e^{\sum_{i=1}^{n+m} \phi_i lnS_i(T)}] = \frac{\mathbb{E}_{\mathbb{Q}}[S_k(T)e^{\sum_{i=1}^{n+m} \phi_i lnS_i(T)}]}{\mathbb{E}_{\mathbb{Q}}[S_k(T)]} = \frac{f(\phi + \boldsymbol{e_k})}{f(\boldsymbol{e_k})},$$

where $\boldsymbol{e_k} = [0, 0, \cdots, 1, 0, \cdots, 0]$. Then, with Proposition 2.1,we have

$$G_0 = \mathbb{Q}(A) = \Psi_{f_0}(\hat{L}), \quad f_0(u) = f(u\boldsymbol{b})e^{-u\boldsymbol{b}\cdot\ln(\boldsymbol{S}(0))},$$
$$G_k = \mathbb{E}_{\mathbb{Q}}[S_k(T)]\mathbb{Q}_k(A) = S_k(0)e^{rT}\Psi_{f_k}(\hat{L}), \quad f_k(u) = f(u\boldsymbol{b} + \boldsymbol{e}_k)e^{-(u\boldsymbol{b}+\boldsymbol{e}_k)\cdot\ln(\boldsymbol{S}(0))-rT}.$$

Appendix B Proof of Theorem 4.1

We need to calculate the following expectations: $G_0^r = \mathbb{E}_{\mathbb{Q}}[I(A, \tau > T)]$ and $G_k^r = \mathbb{E}_{\mathbb{Q}}[S_k(T)I(A, \tau > T)]$, $k = 1, 2, \cdots, n+m$. We introduce new probability measures $\mathbb{Q}_0^r$ and $\mathbb{Q}_k$, which are defined by

$$\mathbb{Q}_0^r(B) = \frac{\mathbb{E}_{\mathbb{Q}}[I(B)I(\tau > T)]}{\mathbb{E}_{\mathbb{Q}}[I(\tau > T)]}, \quad \mathbb{Q}_k^r(B) = \frac{\mathbb{E}_{\mathbb{Q}}[I(B)I(\tau > T)S_k(T)]}{\mathbb{E}_{\mathbb{Q}}[I(\tau > T)S_k(T)]}.$$

Then, in conjunction with Proposition 2.1,

$$G_0^r = \mathbb{E}_{\mathbb{Q}_0^r}[I(\tau > T)] \cdot \mathbb{Q}_0^r(A) = f^r(\mathbf{0}, -1)\Psi_{f_0^r}(\hat{L}), \quad G_k^r = \mathbb{E}_{\mathbb{Q}_k^r}[I(\tau > T)S_k(T)] \cdot \mathbb{Q}_k^r(A) = f^r(\boldsymbol{e}_k, -1)\Psi_{f_k^r}(\hat{L}).$$

Appendix C Proof of Theorem 4.2

We need to calculate the following expectations:

$$\begin{aligned}
G_0^s &= \mathbb{E}_{\mathbb{Q}}[I(A, V(T) > D^*)], & G_k^s &= \mathbb{E}_{\mathbb{Q}}[S_k(T)I(A, V(T) > D^*)],\\
G_0^{sc} &= \mathbb{E}_{\mathbb{Q}}[I(A^c, V(T) > D^*)], & G_k^{sc} &= \mathbb{E}_{\mathbb{Q}}[S_k(T)I(A^c, V(T) > D^*)],\\
G_0^{sv} &= \mathbb{E}_{\mathbb{Q}}[V(T)I(A, V(T) \le D^*)], & G_k^{sv} &= \mathbb{E}_{\mathbb{Q}}[S_k(T)V(T)I(A, V(T) \le D^*)],\\
G_0^{svc} &= \mathbb{E}_{\mathbb{Q}}[V(T)I(A^c, V(T) \le D^*)], & G_k^{svc} &= \mathbb{E}_{\mathbb{Q}}[S_k(T)V(T)I(A^c, V(T) \le D^*)].
\end{aligned}$$

We present the following proposition to compute the probability for a two-dimensional random variate, which is a variant of standard probability theory (see, e.g., [1]).

Proposition C.1. Let $g(\cdot, \cdot)$ be the joint moment-generating function of X and Y, and let there exist constants x and y; then based on the standard probability theory, we can calculate the joint probability of X exceed x and Y exceed y as follows:

$$\mathbb{Q}(X > x, Y > y) = \frac{1}{2}\mathbb{Q}(X > x) + \frac{1}{2}\mathbb{Q}(Y > y) - H_g(x, y),$$

where

$$H_g(x, y) = \frac{1}{2\pi^2} \int_0^\infty \int_0^\infty Re(\frac{e^{-it_1x - it_2y} g(it_1, it_2)}{t_1t_2}) - Re(\frac{e^{-it_1x + it_2y} g(it_1, -it_2)}{t_1t_2}) dt_1 dt_2.$$

We introduce a list of new probability measures as follows for $k = 1, 2, \cdots, n+m$:

$$\mathbb{Q}_k^s(B) = \frac{\mathbb{E}_{\mathbb{Q}}[I(B)S_k(T)]}{\mathbb{E}_{\mathbb{Q}}[S_k(T)]}, \quad \mathbb{Q}_v^s(B) = \frac{\mathbb{E}_{\mathbb{Q}}[V(T)I(B)]}{\mathbb{E}_{\mathbb{Q}}[V(T)]}, \quad \mathbb{Q}_{v,k}^s(B) = \frac{\mathbb{E}_{\mathbb{Q}}[S_k(T)V(T)I(B)]}{\mathbb{E}_{\mathbb{Q}}[S_k(T)V(T)]}.$$

For convenience, we introduce the following auxiliary functions:

$$\begin{aligned}
f_0^s(t_1, t_2) &= f^s(it_1\boldsymbol{b}, it_2)e^{-it_1\boldsymbol{b}\ln(\boldsymbol{S}(0)) - it_2\ln(V(0))},\\
f_k^s(t_1, t_2) &= f^s(it_1\boldsymbol{b} + \boldsymbol{e}_k, it_2)e^{-(it_1\boldsymbol{b}+\boldsymbol{e}_k)\ln(\boldsymbol{S}(0)) - rT - it_2\ln(V(0))},\\
f_0^v(it_1, it_2) &= f^s(it_1\boldsymbol{b}, it_2 + 1)e^{-it_1\boldsymbol{b}\ln(\boldsymbol{S}(0)) - (it_2+1)\ln(V(0)) - rT},\\
f_k^v(t_1, t_2) &= f^s(it_1\boldsymbol{b} + \boldsymbol{e}_k, it_2 + 1)e^{-it_1\boldsymbol{b}\ln(\boldsymbol{S}(0)) - it_2\ln(V(0))}/f^s(\boldsymbol{e}_k, 1).
\end{aligned}$$

Then, with the measure change technique, we have

$$
\begin{aligned}
G_0^s &= \mathbb{Q}(A, V(T) > D^*) = \frac{1}{2}G_0 + \frac{1}{2}\Psi_{f_0^s(0,\cdot)}(\hat{D}) - H_{f_0^s}(\hat{L}, \hat{D}),\\
G_k^s &= \mathbb{Q}_k^s(A, V(T) > D^*) = \frac{1}{2}G_k + \frac{1}{2}S_k(0)e^{rT}\Psi_{f_k^s(0,\cdot)}(\hat{D}) - H_{f_k^s}(\hat{L}, \hat{D}),\\
G_0^{sc} &= \mathbb{Q}(A^c, V(T) > D^*) = \frac{1}{2}(1 - G_0) + \frac{1}{2}\Psi_{f_0^s(0,\cdot)}(\hat{D}) - H_{f_0^s(-\cdot,\cdot)}(-\hat{L}, \hat{D}),\\
G_k^{sc} &= \mathbb{Q}_k^s(A^c, V(T) > D^*) = \frac{1}{2}(S_k(0)e^{rT} - G_k) + \frac{1}{2}S_k(0)e^{rT}\Psi_{f_k^s(0,\cdot)}(\hat{D}) - H_{f_k^s(-\cdot,\cdot)}(-\hat{L}, \hat{D}),\\
G_0^{sv} &= \mathbb{Q}_v^s(A, V(T) \le D^*) = \frac{1}{2}V(0)e^{rT}\Psi_{f_0^v(\cdot,0)}(\hat{L}) + \frac{1}{2}V(0)e^{rT}\Psi_{f_0^v(0,-\cdot)}(-\hat{D}) - H_{f_0^v(\cdot,-\cdot)}(\hat{L}, -\hat{D}),\\
G_k^{sv} &= \mathbb{Q}_{v,k}^s(A, V(T) \le D^*) = \frac{1}{2}f^s(e_k, 1)\Psi_{f_k^v(\cdot,0)}(\hat{L}) + \frac{1}{2}f^s(e_k, 1)\Psi_{f_k^v(0,-\cdot)}(-\hat{D}) - H_{f_k^v(\cdot,-\cdot)}(\hat{L}, -\hat{D}),\\
G_0^{svc} &= \mathbb{Q}_v^s(A^c, V(T) \le D^*) = \frac{1}{2}(V(0)e^{rT} - \Psi_{f_0^v(\cdot,0)}(\hat{L})) + \frac{1}{2}(V(0)e^{rT} - \Psi_{f_0^v(0,-\cdot)}(-\hat{D})) - H_{f_0^v(-\cdot,-\cdot)}(-\hat{L}, -\hat{D}),\\
G_k^{svc} &= \mathbb{Q}_{v,k}^s(A^c, V(T) \le D^*) = \frac{1}{2}(f^s(e_k, 1) - \Psi_{f_k^v(\cdot,0)}(\hat{L})) + \frac{1}{2}(f^s(e_k, 1) - \Psi_{f_k^v(0,-\cdot)}(-\hat{D})) - H_{f_k^v(-\cdot,-\cdot)}(-\hat{L}, -\hat{D}),
\end{aligned}
\tag{C.1}
$$

where $\hat{D} = \ln(D^*)/\ln(V(0))$.

Appendix D Some functions

$$
\begin{aligned}
h(\psi) &= \left(\frac{e^{T\frac{\sqrt{\alpha^2-2\sigma^2\psi}+\alpha}{2}}}{1 - \frac{\sqrt{\alpha^2-2\sigma^2\psi}+\alpha}{\sqrt{\alpha^2-2\sigma^2\psi}}(1 - e^{T\sqrt{\alpha^2-2\sigma^2\psi}})}\right)^{\frac{2\gamma}{\sigma^2}} exp(X(0)[\frac{\sqrt{\alpha^2 - 2\sigma^2\psi} + \alpha}{\sigma^2} - \frac{\frac{\sqrt{\alpha^2-2\sigma^2\psi}+\alpha}{\sigma^2}e^{T\sqrt{\alpha^2-2\sigma^2\psi}}}{1 - \frac{\sqrt{\alpha^2-2\sigma^2\psi}+\alpha}{\sqrt{\alpha^2-2\sigma^2\psi}}(1 - e^{T\sqrt{\alpha^2-2\sigma^2\psi}})}]),\\
g_k(u, v) &= exp(-\frac{u\rho_k}{\sigma_k}(Y_k(0) - \gamma_k T) + \phi_k(\frac{u\rho_k}{\sigma_k}, v + \frac{1}{2}u^2(1 - \rho_k^2) + \frac{u\rho_k\alpha_k}{\sigma_k}) + \psi_k(\frac{u\rho_k}{\sigma_k}, v + \frac{1}{2}u^2(1 - \rho_k^2) + \frac{u\rho_k\alpha_k}{\sigma_k})Y_k(0)),\\
\phi_k(u, v) &= \frac{(\sqrt{\alpha_k^2 - 2\sigma_k^2 v} + \alpha_k)\gamma_k T}{\sigma_k^2} - \frac{2\gamma_k}{\sigma_k^2} ln(1 + \frac{\sigma_k^2}{2}(u - \frac{\sqrt{\alpha_k^2 - 2\sigma_k^2 v} + \alpha_k}{\sigma_k^2})\frac{1 - e^{T\sqrt{\alpha_k^2-2\sigma_k^2 v}}}{\sqrt{\alpha_k^2 - 2\sigma_k^2 v}}),\\
\psi_k(u, v) &= \frac{\sqrt{\alpha_k^2 - 2\sigma_k^2 v} + \alpha_k}{\sigma_k^2} + \frac{(u - \frac{\sqrt{\alpha_k^2-2\sigma_k^2 v}+\alpha_k}{\sigma_k^2})e^{T\sqrt{\alpha_k^2-2\sigma_k^2 v}}}{1 + \frac{\sigma_k^2}{2}(u - \frac{\sqrt{\alpha_k^2-2\sigma_k^2 v}+\alpha_k}{\sigma_k^2})\frac{1-e^{T\sqrt{\alpha_k^2-2\sigma_k^2 v}}}{\sqrt{\alpha_k^2-2\sigma_k^2 v}}}, \quad k = 0, 1, \cdots, n + m.
\end{aligned}
\tag{D.1}
$$

$$
\begin{aligned}
H_0(\phi,\psi) &= \sum_{k=1}^{n+m}\phi_k[lnS_k(0)+rT-\frac{1}{2}\vartheta_k^2\sigma_k^2T]+\psi[lnV(0)+rT-\frac{1}{2}\vartheta_0^2\sigma_0^2T],\\
H_1(\phi,\psi) &= e^{\frac{1}{2}[\sum_{k=1}^{n+m}\phi_k^2\vartheta_k^2\sigma_k^2T+2\sum_{j,k=1}^{n+m}\phi_j\phi_k\vartheta_j\vartheta_k\sigma_j\sigma_k\rho_{jk}T]+\frac{1}{2}\psi^2\vartheta_0^2\beta_0^2\sigma_0^2T+\psi\vartheta_0\beta_0\sigma_0\sum_{k=1}^{n+m}\phi_k\vartheta_k\sigma_k\rho_kT},\\
H_2(\phi,\psi) &= e^{-\frac{1}{2\sigma_a}\rho(\sum_{k=1}^{n+m}\phi_k\beta_k+\psi\beta_0)[a^2(0)+\sigma_a^2T]+A(\omega_1,\omega_2,\omega_3)},\\
\omega_1 &= -[\frac{\tilde{\kappa}_a}{\sigma_a}\rho(\sum_{k=1}^{n+m}\phi_k\beta_k+\psi\beta_0)-\frac{1}{2}(\sum_{k=1}^{n+m}\phi_k\beta_k^2+\psi\beta_0^2)+\frac{1}{2}(1-\rho^2)(\sum_{k=1}^{n+m}\phi_k\beta_k+\psi\beta_0)^2],\\
\omega_2 &= \frac{\tilde{\kappa}_a\tilde{\theta}_a}{\sigma_a}\rho(\sum_{k=1}^{n+m}\phi_k\beta_k+\psi\beta_0),\quad \omega_3=\frac{1}{2\sigma_a}\rho(\sum_{k=1}^{n+m}\phi_k\beta_k+\psi\beta_0),\\
A(\omega_1,\omega_2,\omega_3) &= \mathbb{E}_{\mathbb{Q}}[e^{-\omega_1\int_0^T a^2(t)dt-\omega_2\int_0^T a(t)dt+\omega_3a^2(T)}]=e^{\frac{1}{2}A_1a(0)^2+A_2a(0)+A_3}\\
A_1 &= \frac{1}{\sigma_a^2}(\tilde{\kappa}_a-\delta_1\frac{\sinh(\delta_1T)+\delta_2\cosh(\delta_1T)}{\cosh(\delta_1T)+\delta_2\sinh(\delta_1T)})\\
A_2 &= \frac{1}{\sigma_a^2\delta_1}(\frac{(\tilde{\kappa}_a\tilde{\theta}_a\delta_1-\delta_2\delta_3)}{\cosh(\delta_1T)+\delta_2\sinh(\delta_1T)}-\tilde{\kappa}_a\tilde{\theta}_a\delta_1)+\frac{\delta_3}{\sigma_a^2\delta_1}(\frac{\sinh(\delta_1T)+\delta_2\cosh(\delta_1T)}{\cosh(\delta_1T)+\delta_2\sinh(\delta_1T)}),\\
A_3 &= -\frac{1}{2}\ln(\cosh(\delta_1T)+\delta_2\sinh(\delta_1T))+\frac{1}{2}\tilde{\kappa}_aT+\frac{(\tilde{\kappa}_a^2\tilde{\theta}_a^2\delta_1^2-\delta_3^2)}{2\sigma_a^2\delta_1^3}(\frac{\sinh(\delta_1T)}{\cosh(\delta_1T)+\delta_2\sinh(\delta_1T)}-\delta_1T)\\
&\quad+\frac{(\tilde{\kappa}_a\tilde{\theta}_a\delta_1-\delta_2\delta_3)\delta_3}{\sigma_a^2\delta_1^3}(\frac{\cosh(\delta_1T)-1}{\cosh(\delta_1T)+\delta_2\sinh(\delta_1T)}),\\
\delta_1 &= \sqrt{2\sigma_a^2\omega_1+\tilde{\kappa}_a^2},\quad \delta_2=\frac{1}{\delta_1}(\tilde{\kappa}_a-2\sigma_a^2\omega_3),\quad \delta_3=\tilde{\kappa}_a^2\tilde{\theta}_a-\sigma_a^2\omega_2.
\end{aligned}
\tag{D.2}
$$

References

1 N. G. Shephard, From characteristic function to distribution function: a simple framework for the theory, *Econom. Theory*, **7** (1991), 519–529. https://doi.org/10.1017/S0266466600004746