



Research article

Modeling the impact of uncertain demand on intracity express delivery network design by adjustable data-driven robust approach

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Appendix A. The omitted proofs

Appendix A.1. Proof of Theorem 4.2

Proof. Leveraging transformed form (4.1) of uncertainty set (3.7), $\max_{\mathbf{d} \in \mathcal{D}(\mathbf{d})} \sum_{c' \in [C]} \tilde{\mathbf{Z}}_{cc'}^\xi \mathbf{d}_{c'}^\xi$ can be presented as follows: ($\forall c \in [C], \forall \xi \in [\Xi]$),

$$\max_{\mathbf{d}, \boldsymbol{\varphi}} [\mathbf{d}^\xi]^\top \tilde{\mathbf{Z}}_c^\xi \tag{A1a}$$

$$\text{s.t. } \sum_{n \in \mathcal{O}} \beta_n [\boldsymbol{\varphi}_n]^\top \mathbf{1} \leq \theta, \tag{A1b}$$

$$-\boldsymbol{\varphi}_n \leq \bar{\mathbf{W}}(\mathbf{d} - \mathbf{d}^{(n)}) \quad \forall n \in \mathcal{O}, \tag{A1c}$$

$$\bar{\mathbf{W}}(\mathbf{d} - \mathbf{d}^{(n)}) \leq \boldsymbol{\varphi}_n \quad \forall n \in \mathcal{O}, \tag{A1d}$$

where $\mathbf{d}^\xi = (\mathbf{d}_{c'}^\xi)_{c' \in [C]}$ and $\tilde{\mathbf{Z}}_c^\xi = (\tilde{\mathbf{Z}}_{cc'})_{c' \in [C]}$.

Introducing dual variables $\hat{\mathcal{Q}}^{c\xi}, \hat{\Lambda}_n^{c\xi}, \hat{\Gamma}_n^{c\xi}$ for constraints (A1b)–(A1d), respectively, the dual of model (A1) can be presented as follows:

$$\min_{\hat{\mathcal{Q}}^{c\xi}, \hat{\Lambda}_n^{c\xi}, \hat{\Gamma}_n^{c\xi}} \sum_{n \in \mathcal{O}} (\hat{\Gamma}_n^{c\xi} - \hat{\Lambda}_n^{c\xi})^\top \bar{\mathbf{W}} \mathbf{d}^{(n)} + \hat{\mathcal{Q}}^{c\xi} \theta \tag{A2a}$$

$$\text{s.t. } \sum_{n \in \mathcal{O}} \bar{\mathbf{W}}(\dot{\mathbf{A}}_n^{c\xi} - \dot{\mathbf{I}}_n^{c\xi}) + \tilde{\mathbf{Z}}_c^\xi = \mathbf{0}, \quad (\text{A2b})$$

$$\dot{\mathbf{A}}_n^{c\xi} + \dot{\mathbf{I}}_n^{c\xi} = \dot{\mathbf{Q}}^{c\xi} \beta_n \mathbf{1} \quad \forall n \in \mathcal{O}, \quad (\text{A2c})$$

$$\dot{\mathbf{A}}_n^{c\xi}, \dot{\mathbf{I}}_n^{c\xi} \in \mathbf{R}_+^{|\mathcal{C}|}, \dot{\mathbf{Q}}^{c\xi} \in \mathbf{R}_+ \quad \forall n \in \mathcal{O}. \quad (\text{A2d})$$

Given that strong duality holds for the primal-dual pair (A1)–(A2) within the interval $0 < \theta < 1$, constraint (4.2c) admits a tractable linear representation as follows:

$$R(\mathbf{X}, \mathbf{d}^\xi) = \min \sum_{c \in [\mathcal{C}]} \left(\tilde{\mathbf{Z}}_c^\xi + \sum_{n \in \mathcal{O}} (\dot{\mathbf{I}}_n^{c\xi} - \dot{\mathbf{A}}_n^{c\xi})^T \bar{\mathbf{W}} \mathbf{d}^{(n)} + \dot{\mathbf{Q}}^{c\xi} \theta \right)$$

$$\text{s.t. } \sum_{n \in \mathcal{O}} \bar{\mathbf{W}}(\dot{\mathbf{A}}_n^{c\xi} - \dot{\mathbf{I}}_n^{c\xi}) + \tilde{\mathbf{Z}}_c^\xi = \mathbf{0},$$

$$\dot{\mathbf{A}}_n^{c\xi} + \dot{\mathbf{I}}_n^{c\xi} = \dot{\mathbf{Q}}^{c\xi} \beta_n \mathbf{1} \quad \forall n \in \mathcal{O},$$

$$\dot{\mathbf{A}}_n^{c\xi}, \dot{\mathbf{I}}_n^{c\xi} \in \mathbf{R}_+^{|\mathcal{C}|}, \dot{\mathbf{Q}}^{c\xi} \in \mathbf{R}_+ \quad \forall n \in \mathcal{O}.$$

□

Appendix A.2. Proof of Theorem 4.3

Proof. By the proof of Theorem (4.2), we can obtain the equivalent linear system of constraints (4.2d) as follows:

$$\sum_{c \in [\mathcal{C}]} \bar{Y}_{i'c}^\xi(t) + \sum_{c \in [\mathcal{C}]} \left(\sum_{n \in \mathcal{O}} (\ddot{\mathbf{I}}_n^{c\xi i't} - \ddot{\mathbf{A}}_n^{c\xi i't})^T \bar{\mathbf{W}} \mathbf{d}^{(n)} + \ddot{\mathbf{Q}}^{c\xi i't} \theta \right) \leq q \cdot X_{i'}(t)$$

$$\forall (i, i') \in [A], i \neq i', \forall t \in [T],$$

$$\sum_{n \in \mathcal{O}} \bar{\mathbf{W}}(\dot{\mathbf{A}}_n^{c\xi i't} - \dot{\mathbf{I}}_n^{c\xi i't}) + \bar{Y}_{i'c}^\xi(t) = \mathbf{0} \quad \forall (i, i') \in [A], i \neq i', \forall t \in [T],$$

$$\dot{\mathbf{A}}_n^{c\xi i't} + \dot{\mathbf{I}}_n^{c\xi i't} = \dot{\mathbf{Q}}^{c\xi i't} \beta_n \mathbf{1} \quad \forall n \in \mathcal{O}, \forall (i, i') \in [A], i \neq i', \forall t \in [T],$$

$$\dot{\mathbf{A}}_n^{c\xi i't}, \dot{\mathbf{I}}_n^{c\xi i't} \in \mathbf{R}_+^{|\mathcal{C}|}, \dot{\mathbf{Q}}^{c\xi i't} \in \mathbf{R}_+ \quad \forall n \in \mathcal{O}, \forall (i, i') \in [A], i \neq i', \forall t \in [T].$$

□

Appendix A.3. Proof of Theorem 4.4

Proof. Leveraging transformed form (4.1) of uncertainty set (3.7), $\min_{\mathbf{d} \in \mathcal{D}(\mathbf{d})} \sum_{c' \in [\mathcal{C}]} \tilde{\mathbf{Z}}_{cc'}^\xi \mathbf{d}_{c'}^\xi$ can be presented as follows: ($\forall c \in [\mathcal{C}], \forall \xi \in [\Xi]$),

$$\min_{\mathbf{d}, \boldsymbol{\varphi}} [\mathbf{d}^\xi]^\top \tilde{\mathbf{Z}}_c^\xi \quad (\text{A3a})$$

$$\text{s.t. } \sum_{n \in \mathcal{O}} \beta_n [\boldsymbol{\varphi}_n]^T \mathbf{1} \leq \theta, \quad (\text{A3b})$$

$$-\boldsymbol{\varphi}_n \leq \bar{\mathbf{W}}(\mathbf{d} - \mathbf{d}^{(n)}) \quad \forall n \in \mathcal{O}, \quad (\text{A3c})$$

$$\bar{\mathbf{W}}(\mathbf{d} - \mathbf{d}^{(n)}) \leq \boldsymbol{\varphi}_n \quad \forall n \in \mathcal{O}, \quad (\text{A3d})$$

where $\mathbf{d}^\xi = (\mathbf{d}_{c'}^\xi)_{c' \in [\mathcal{C}]}$ and $\tilde{\mathbf{Z}}_c^\xi = (\tilde{\mathbf{Z}}_{cc'}^\xi)_{c' \in [\mathcal{C}]}$.

Introducing dual variables $\bar{Q}^{c\xi}, \bar{A}_n^{c\xi}, \bar{I}_n^{c\xi}$ for constraints (A3b)–(A3d), respectively, the dual of model (A3) can be presented as follows:

$$\max_{\bar{Q}^{c\xi}, \bar{A}_n^{c\xi}, \bar{I}_n^{c\xi}} \sum_{n \in \mathcal{O}} (\bar{I}_n^{c\xi} - \bar{A}_n^{c\xi})^T \bar{W} \mathbf{d}^{(n)} + \bar{Q}^{c\xi} \theta \quad (\text{A4a})$$

$$\text{s.t.} \quad \sum_{n \in \mathcal{O}} \bar{W} (\bar{A}_n^{c\xi} - \bar{I}_n^{c\xi}) + \tilde{Z}_c^\xi = \mathbf{0}, \quad (\text{A4b})$$

$$\bar{A}_n^{c\xi} + \bar{I}_n^{c\xi} = \bar{Q}^{c\xi} \beta_n \mathbf{1} \quad \forall n \in \mathcal{O}, \quad (\text{A4c})$$

$$\bar{A}_n^{c\xi}, \bar{I}_n^{c\xi} \in R_-^{|\mathcal{C}|}, \bar{Q}^{c\xi} \in R_- \quad \forall n \in \mathcal{O}. \quad (\text{A4d})$$

In the case $0 < \theta < 1$, strong duality holds between primal problem (A3) and its dual problem (A4). Thus, constraints (4.2i) can be transformed into the following tractable linear system:

$$\tilde{Z}_c^\xi + \sum_{n \in \mathcal{O}} (\bar{I}_n^{c\xi} - \bar{A}_n^{c\xi})^T \bar{W} \mathbf{d}^{(n)} + \bar{Q}^{c\xi} \theta \geq 0 \quad \forall c \in [C],$$

$$\sum_{n \in \mathcal{O}} \bar{W} (\bar{A}_n^{c\xi} - \bar{I}_n^{c\xi}) + \tilde{Z}_c^\xi = \mathbf{0} \quad \forall c \in [C],$$

$$\bar{A}_n^{c\xi} + \bar{I}_n^{c\xi} = \bar{Q}^{c\xi} \beta_n \mathbf{1} \quad \forall n \in \mathcal{O}, \forall c \in [C],$$

$$\bar{A}_n^{c\xi}, \bar{I}_n^{c\xi} \in R_-^{|\mathcal{C}|}, \bar{Q}^{c\xi} \in R_- \quad \forall n \in \mathcal{O}, \forall c \in [C].$$

□

Appendix A.4. Proof of Theorem 4.5

Proof. By the proof of Theorem (4.4), we can obtain the equivalent linear system of constraints (4.2j) as follows:

$$\tilde{Y}_{ii'c}^\xi(t) + \sum_{n \in \mathcal{O}} (\tilde{I}_n^{c\xi ii't} - \tilde{A}_n^{c\xi ii't})^T \bar{W} \mathbf{d}^{(n)} + \tilde{Q}^{c\xi} \theta \geq 0$$

$$\forall (i, i') \in [A], i \neq i', \forall c \in [C], t \in [T],$$

$$\sum_{n \in \mathcal{O}} \bar{W} (\tilde{A}_n^{c\xi ii't} - \tilde{I}_n^{c\xi ii't}) + \tilde{Y}_{ii'cc'}^\xi(t) = \mathbf{0} \quad \forall (i, i') \in [A], i \neq i', \forall c \in [C], t \in [T],$$

$$\tilde{A}_n^{c\xi} + \tilde{I}_n^{c\xi} = \tilde{Q}^{c\xi} \beta_n \mathbf{1} \quad \forall n \in \mathcal{O}, \forall (i, i') \in [A], i \neq i', \forall c \in [C], t \in [T],$$

$$\tilde{A}_n^{c\xi ii't}, \tilde{I}_n^{c\xi ii't} \in R_-^{|\mathcal{C}|}, \tilde{Q}^{c\xi ii't} \in R_- \quad \forall n \in \mathcal{O}, \forall (i, i') \in [A], i \neq i', \forall c \in [C], \forall t \in [T].$$

□

Appendix B. Formulation of the budgeted robust optimization model

In this section, we formulate the budgeted robust optimization model for the intracity express delivery service problem, which is based on a budgeted uncertainty set.

Construction of budgeted uncertainty set. The fluctuations in demand \mathbf{d} are modeled via the following budgeted uncertainty set:

$$\mathcal{D}(\mathbf{d})_{\text{budgeted}} = \left\{ \mathbf{d} \mid d_c^\xi = \bar{d}_c^\xi + \sum_{j \in J} \eta_c^\xi(j) \Delta d_c^\xi(j), \forall c \in C, \forall \xi \in \Xi : \eta \in \mathcal{Z}^k \right\}, \quad (\text{B1})$$

where $\bar{\mathbf{d}}$, $\Delta \mathbf{d}$, and $\boldsymbol{\eta}$ denote the nominal value, the basic shift, and the perturbation vector of \mathbf{d} , respectively, and \mathcal{Z} is the following perturbation set:

$$\mathcal{Z} = \left\{ \boldsymbol{\eta} \in R^J \mid \|\boldsymbol{\eta}\|_\infty \leq 1, \|\boldsymbol{\eta}\|_1 \leq \sqrt{2 \times \log(1/\vartheta)} \right\}, \quad (\text{B2})$$

where ϑ serves as a regularization parameter that governs the size of uncertainty set (B2).

Formulation of the budgeted robust optimization model. Leveraging budgeted uncertainty set $\mathcal{D}(\mathbf{d})_{\text{budgeted}}$ (B1), we can formulate the following budgeted robust optimization model for the intracity express delivery service problem:

$$\begin{aligned} & \min \quad (3.8a) \\ & \text{s.t.} \quad \text{constraints (3.8b)–(3.8e), (3.8h)–(3.8k),} \\ & \quad \sum_{i' \in [I]} Y_{O_c i' c}^\xi(\gamma_c) - \sum_{i' \in [I]} Y_{i' O_c c}^\xi(\gamma_c) = d_c^\xi - Z_c^\xi \\ & \quad \forall \mathbf{d} \in \mathcal{D}(\mathbf{d})_{\text{budgeted}}, \forall c \in [C], \forall \xi \in [\Xi], \\ & \quad \sum_{i' \in [I]} Y_{D_c i' c}^\xi(\tau_c) - \sum_{i' \in [I]} Y_{i' D_c c}^\xi(\tau_c) = -d_c^\xi + Z_c^\xi \\ & \quad \forall \mathbf{d} \in \mathcal{D}(\mathbf{d})_{\text{budgeted}}, \forall c \in [C], \forall \xi \in [\Xi]. \end{aligned}$$



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