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Research article

Referral coordination mechanism for the hierarchical system with Internet diagnosis platform

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Appendix

Proof of Proposition 1. From the individual rationality constraints, it can be seen that offline potential first-visit patients choose to enter the GH for consultation if and only if it meets $U \geq 0$. When $U = 0$, there will be no patients to enter the GH offline, thus, the effective arrival rate is $\lambda_{1e} = (\mu_1(v - p_1/2))/d - \lambda_2 + \varphi\lambda_2$. □

Proof of Proposition 2. Taking the first- and second-order partial derivatives of the total profit function of the system with respect to φ , we get $\partial\pi/\partial\varphi = (p_1 - c_1)\lambda_2/2 - \varphi(p_2 - c_2)\lambda_2 + (p_3 - c_3)\lambda_2 - \left(2c_3^w(\lambda_3 + \varphi\lambda_2)\lambda_2\right)/\mu_2, \partial^2\pi/\partial\boldsymbol{\varphi}^2 = -(p_1 - c_1)\lambda_2 - (2c_3^w\lambda_2^2)/\mu_2 < 0$. Based on the results of the derivation, we can conclude that the total profit of the systemis a concave function with respect to φ , according to $\partial\pi/\partial\varphi = 0$, is $\varphi^c = \left(\left((p_1 - c_1)/2 + (p_3 - c_3) - (2c_3^w\lambda_3)/\mu_2\right)\right) / \left(p_2 - c_2 + (2c_3^w\lambda_2)/\mu_2\right)$. □

Proof of Lemma 1. We take the second derivative of π_1 and π_2 with respect to φ, $\partial^2\pi_1/\partial\boldsymbol{\varphi}^2 = (c_2 - p_2)\lambda_2 < 0, \partial^2\pi_2/\partial\boldsymbol{\varphi}^2 = -(2c_2^w\lambda_2^2)/\mu_2 < 0$ yields the result in Lemma 1. □

Proof of Proposition 3. Based on $\partial\pi_1/\partial\varphi = 0$, and substituting the system optimal treatment threshold φ^c, the GH optimal outsourcing price can be obtained as

$$m^p = \frac{1}{2}(p_1 - c_1) - (p_2 - c_2)\varphi^c.$$

□

Proof of Proposition 4. By taking the first- and second-order partial derivatives of the objective function of the CH concerning φ , we can obtain $\partial\pi_2/\partial\varphi = (p_3 - c_3)\lambda_2 + m\lambda_2 - (2c_3^w(\lambda_3 + \varphi\lambda_2)\lambda_2)/\mu_2, \partial^2\pi_2/\partial\boldsymbol{\varphi}^2 = -(2c_3^w\lambda_2^2)/\mu_2 < 0$. According to $\partial\pi_2/\partial\varphi = (p_3 - c_3)\lambda_2 + m\lambda_2 - (2c_3^w(\lambda_3 + \varphi\lambda_2)\lambda_2)/\mu_2 = 0$, we have $\varphi^{NP} = (\mu_2(p_3 - c_3 + m))/(2c_3^w\lambda_2) - \lambda_3/\lambda_2$.

If $\varphi^{NP} < 0$, that is, that is, $m < (2c_3^w\lambda_3)/\mu_2 - (p_3 - c_3)$, we have $\varphi^{NP} = 0$; if $\varphi^{NP} > 1$, that is, $m > ((\lambda_3 + \lambda_2)2c_3^w)/(n\mu_2) - (p_3 - c_3)$, we have $\varphi^{NP} = 1$; otherwise, that is, $(2c_3^w\lambda_3)/\mu_2 - (p_3 - c_3) \le m \le ((\lambda_3 + \lambda_2)2c_3^w)/(n\mu_2) - (p_3 - c_3)$, which implies $\varphi^{NP} = (\mu_2(p_3 - c_3 + m))/(2c_3^w\lambda_2) - \lambda_3/\lambda_2$.

□

Proof of Proposition 5. By substituting φ^{NP} into the objective function of the GH and taking first- and second-order partial derivatives concerning m, we can obtain $\partial\pi_1/\partial m = (1/2)(p_1 - c_1)\lambda_2(\partial\varphi^{NP}/\partial m) - \varphi^{NP}(p_2 - c_2)\lambda_2(\partial\varphi^{NP}/\partial m) - \varphi^{NP}\lambda_2 - m(\partial\varphi^{NP}/\partial m)\lambda_2$, $\partial^2\pi_1/\partial m^2 = -(p_2 - c_2)(\partial\varphi^{NP}/\partial m)^2\lambda_2 - 2(\partial\varphi_D^{NP}/\partial m)\lambda_2 < 0$. According to $\partial\pi_1/\partial m = (1/2)(p_1 - c_1)\lambda_2(\partial\varphi^{NP}/\partial m) - \varphi^{NP}(p_2 - c_2)\lambda_2(\partial\varphi^{NP}/\partial m) - \varphi^{NP}\lambda_2 - m(\partial\varphi^{NP}/\partial m)\lambda_2 = 0$, we have $m^{NP} = \left((p_1 - c_1)c_3^w\lambda_2 - 2(p_3 - c_3)c_3^w\lambda_2 + 2(p_2 - c_2)c_3^w\lambda_3 - (p_3 - c_3)(p_2 - c_2)\mu_2 + (4c_3^{w^2}\lambda_2\lambda_3)/\mu_2\right)/(4c_3^w\lambda_2 + (p_2 - c_2)\mu_2)$.

When $m < (2c_2^w\lambda_3)/\mu_2 - (p_3 - c_3)$, that is, $c_3^w > ([(p_1 - c_1)\lambda_2 + 2(p_3 - c_3)\lambda_2 + 2(p_2 - c_2)(\lambda_3 - \lambda_2)]\mu_2)/(4\lambda_2(2\lambda_2 - \lambda_3))$, we have $\lambda_{1e} = (\mu_1(v - p_1/2))/d - \lambda_2$, $\varphi^{NP} = 0$, $m = ([(p_1 - c_1)\lambda_2 + 2(p_3 - c_3)\lambda_2 + 2(p_2 - c_2)(\lambda_3 - \lambda_2)]\mu_2)/(4\lambda_2(2\lambda_2 - \lambda_3))$; when $m > ((\lambda_3 + \lambda_2)2c_3^w)/(n\mu_2) - (p_3 - c_3)$, that is, $c_3^w < ([(p_1 - c_1)\lambda_2 + 2(p_3 - c_3)\lambda_2 - 2(p_2 - c_2)\lambda_2]\mu_2)/(4\lambda_2(\lambda_2 + \lambda_3))$, we have $\lambda_{1e} = (\mu_1(v - p_1/2))/d$, $\varphi^{NP} = 1$, $m = ([(p_1 - c_1)\lambda_2 + 2(p_3 - c_3)\lambda_2 - 2(p_2 - c_2)\lambda_2]\mu_2)/(4\lambda_2(\lambda_2 + \lambda_3))$; when $(2c_3^w\lambda_3)/\mu_2 - (p_3 - c_3) \le m \le ((\lambda_3 + \lambda_2)2c_3^w)/(n\mu_2) - (p_3 - c_3)$, that is, $((p_1 - c_1)\lambda_2 + 2(p_3 - c_3)\lambda_2 - 2(p_2 - c_2)\lambda_2)\mu_2/(4\lambda_2(\lambda_2 + \lambda_3)) \le c_3^w \le ((p_1 - c_1)\lambda_2 + 2(p_3 - c_3)\lambda_2 + 2(p_2 - c_2)(\lambda_3 - \lambda_2))\mu_2/(4\lambda_2(2\lambda_2 - \lambda_3))$, then we have $\lambda_{1e} = (\mu_1(v - p_1/2))/d - \lambda_2 + \varphi^{NP}\lambda_2$, $\varphi^{NP} = (\mu_2(p_3 - c_3 + m^{NP}))/(2c_3^w\lambda_2) - \lambda_3/\lambda_2$, $m^{NP} = \left((p_1 - c_1)c_3^w\lambda_2 - 2(p_3 - c_3)c_3^w\lambda_2 + 2(p_2 - c_2)c_3^w\lambda_3 - (p_3 - c_3)(p_2 - c_2)\mu_2 + (4c_3^{w^2}\lambda_2\lambda_3)/\mu_2\right)/(4c_3^w\lambda_2 + (p_2 - c_2)\mu_2)$. □

Proof of Proposition 6. As this proof is similar to that of Proposition 2, it is omitted here. □

Proof of Proposition 7. Based on $\partial\pi_1/\partial\varphi = 0$ and substituting the system-optimal referral threshold φ_D^C, the GH optimal outsourcing price can be obtained to satisfy $m^p = 1/2(p_1 - c_1) - (p_2 - c_2)\varphi_D^C$. □

Proof of Proposition 8. By taking the first- and second-order partial derivatives of the objective function of the CH concerning φ,we can obtain $\partial\pi_{2i}/\partial\varphi = (p_3 - c_3)\lambda_2/n + m_i\lambda_2/n - (2c_3^w(\lambda_{3i} + \varphi\lambda_2/n)\lambda_2/n)/\mu_2$, $\partial^2\pi_{2i}/\partial\varphi^2 = -(2c_3^w((\lambda_2/n)^2))/\mu_2 < 0$. According to $\partial\pi_{2i}/\partial\varphi = (p_3 - c_3)\lambda_2/n + m_i\lambda_2/n - (2c_3^w(\lambda_{3i} + \varphi\lambda_2/n)\lambda_2/n)/\mu_2 = 0$, we have $\varphi_D^{NP} = (\mu_2(p_3 - c_3 + m)n)/(2c_3^w\lambda_2) - (n\lambda_3)/\lambda_2$.

If $\varphi_D^{NP} < 0$, that is, $m < (2c_3^w\lambda_3)/\mu_2 - (p_3 - c_3)$, we have $\varphi_D^{NP} = 0$; if $\varphi_D^{NP} > 1$, that is, $m > ((n\lambda_3 + \lambda_2)2c_3^w)/(n\mu_2) - (p_3 - c_3)$,we have $\varphi_D^{NP} = 1$;if $0 \le \varphi_D^{NP} \le 1$, that is, $(2c_3^w\lambda_3)/\mu_2 - (p_3 - c_3) \le m \le ((n\lambda_3 + \lambda_2)2c_3^w)/(n\mu_2) - (p_3 - c_3)$, we have $\varphi_D^{NP} = (\mu_2(p_3 - c_3 + m)n)/(2c_3^w\lambda_2) - (n\lambda_3)/\lambda_2$. □

Proof of Proposition 9. By substituting φ_D^{NP} into the objective function of the GH and taking first- and second-order partial derivatives concerning m_i, we can obtain $\partial^2\pi_1'/\partial m_i^2 = -(p_2 - c_2)((\partial\varphi_D^{NP}/\partial m)^2)\lambda_2 - (1/n)m_i\lambda_2 - (1/n)(\partial\varphi_D^{NP}/\partial m)\lambda_2 < 0$. According to $\partial\pi_1'/\partial m_i = (1/2)(p_1 - c_1)\lambda_2(\partial\varphi_D^{NP}/\partial m) - \varphi_D^{NP}(p_2 - c_2)\lambda_2(\partial\varphi_D^{NP}/\partial m) - (1/n)\sum_{i=1}^{n} m_i(\partial\varphi_D^{NP}/\partial m)\lambda_2 - (1/n)\varphi_D^{NP}\lambda_2 = 0$, we have

$$m_i = \frac{\frac{1}{2}(p_1-c_1)-\frac{1}{n}(p_3-c_3)+\frac{(p_2-c_2)n\lambda_3}{\lambda_2}-\frac{(p_3-c_3)(p_2-c_2)\mu_2 n}{2c_3^w\lambda_2}+\frac{2c_3^w\lambda_3}{n\mu_2}}{\frac{n+1}{n}+\frac{n(p_2-c_2)\mu_2}{2c_3^w\lambda_2}}.$$

When $m_i < (2c_3^w\lambda_3)/\mu_2 - (p_3 - c_3)$, $c_2^w > \left(\left[(1/2)(p_1 - c_1) + (p_3 - c_3)\right]\mu_2\right)/(2\lambda_3)$, we have $\lambda_{1e} = (\mu_1(v - p_1/2))/d - \lambda_2$, $\varphi_D^{NP} = 0$, $m_i = (2c_3^w\lambda_3)/\mu_2 - (p_3 - c_3)$; when $m_i > ((n\lambda_3 + \lambda_2)2c_3^w)/(n\mu_2) - (p_3 - c_3)$, that is, $c_3^w < \left(\left[(1/2)(p_1 - c_1) + (p_3 - c_3) - (p_2 - c_2)\right]n^2\mu_2\right)/(2(n^2\lambda_3 + (n+1)\lambda_2))$, we have $\lambda_{1e} = (\mu_1(v - p_1/2))/d$, $\varphi_D^{NP} = 1$, $m_i = ((n\lambda_3 + \lambda_2)2c_3^w)/(n\mu_2) - (p_3 - c_3)$; when $(2c_3^w\lambda_3)/\mu_2 - (p_3 - c_3) \le m_i \le ((n\lambda_3 + \lambda_2)2c_3^w)/(n\mu_2) - (p_3 - c_3)$, that is, $\left(\left[(1/2)(p_1 - c_1) + (p_3 - c_3) - (p_2 - c_2)\right]n^2\mu_2\right)/(2(n^2\lambda_3 + (n+1)\lambda_2)) \le c_3^w \le \left(\left[(1/2)(p_1 - c_1) + (p_3 - c_3)\right]\mu_2\right)/(2\lambda_3)$, then we have $\lambda_{1e} = (\mu_1(v - p_1/2))/d - \lambda_2 + \varphi_D^{NP}\lambda_2$, $\varphi_D^{NP} = (\mu_2(p_3 - c_3 + m_i)n)/(2c_3^w\lambda_2) - (n\lambda_3)/\lambda_2$,

$$m_i = \frac{\frac{1}{2}(p_1-c_1)-\frac{1}{n}(p_3-c_3)+\frac{(p_2-c_2)n\lambda_3}{\lambda_2}-\frac{(p_3-c_3)(p_2-c_2)\mu_2 n}{2c_3^w\lambda_2}+\frac{2c_3^w\lambda_3}{n\mu_2}}{\frac{n+1}{n}+\frac{n(p_2-c_2)\mu_2}{2c_3^w\lambda_2}}.$$

□