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Research article

Research on digital diffusion of firms under the dual disturbance of carbon trading price volatility and yield uncertainty

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Appendix A

Proof of Hypothesis 3

Carbon trading price p_e has a correlation coefficient of ρ_{p_eX} with the yield uncertainty variable X, and a correlation coefficient of $\rho_{p_eQ_i}$ with firms' yields Q_i.

$$\rho_{p_eQ_D}=\frac{Cov(p_e,Q_D)}{\sqrt{D(p_e)D(Q_D)}}=\frac{Cov(p_e,q_D-(1-d_D)X)}{\sqrt{D(p_e)D[q_D-(1-d_D)X]}}=\frac{-Cov[p_e,(1-d_D)X]}{(1-d_A)\sqrt{D(p_e)D(X)}}=-\frac{Cov[p_e,(1-d_D)X]}{(1-d_D)\sqrt{D(p_e)D(X)}}$$
$$=-\frac{(1-d_A)Cov(p_e,X)}{(1-d_A)\sqrt{D(p_e)D(X)}}=-\frac{Cov(p_e,X)}{\sqrt{D(p_e)D(X)}}=-\rho_{p_eX} \quad (A1)$$

$$\rho_{p_eQ_L}=\frac{Cov(p_e,Q_L)}{\sqrt{D(p_e)D(Q_L)}}=\frac{Cov(p_e,q_L-X)}{\sqrt{D(p_e)D(q_L-X)}}=\frac{Cov(p_e,-X)}{\sqrt{D(p_e)D(-X)}}=\frac{-Cov(p_e,X)}{\sqrt{D(p_e)D(X)}}$$
$$=-\frac{Cov(p_e,X)}{\sqrt{D(p_e)D(X)}}=-\rho_{p_eX} \quad (A2)$$

So, we have $\rho_{p_eQ_A}=\rho_{p_eQ_B}=-\rho_{p_eX}$, $\rho=\rho_{p_eQ_A}=\rho_{p_eQ_B}=-\rho_{p_eX}$.

Appendix B

Proof of Proposition 1

$$\frac{\partial q_D}{\partial b}=\frac{-agY(bY-A)^2+eg\{p_0Y(bY-A)^2-2[(bY)^2+2b(-1+d)YA+(-A)^2]\rho\sigma\sigma_e\}}{[(bY)^2-(-A)^2]^2}<0 \tag{B1}$$

$$\frac{\partial \eta_D}{\partial b}=\frac{Y\{-aY(bY-A)^2+ep0Y(bY-A)^2-2e[(bY)^2+2b(-1+d)YA+(-A)^2]\rho\sigma\sigma e\}}{(bY+A)^2(bY-A)^2}<0 \tag{B2}$$

$$\frac{\partial q_L}{\partial b}=\frac{\begin{array}{c}[(bY)^2-(-A)^2]\{Y[g(a-ep_0)+2bY]-2e(b+(-1+d)g+bep_0]\rho\sigma\sigma_e\}\\ -2YbY\{bg[B-2(-1+d)e\rho\sigma\sigma e]+b^2H+A[-g(2+a-ep_0)+H]\}\end{array}}{[(bY)^2-(-A)^2]^2} \tag{B3}$$

$$\frac{\partial \eta_L}{\partial b}=\frac{Y\{B(bY-A)^2+2e[(-1+d)(bY)^2+2bYA+(-1+d)A^2]\rho\sigma\sigma e\}}{(bY+A)^2(bY-A)^2} \tag{B4}$$

From the Hessen matrix, $\frac{\partial q_D}{\partial b}<0$, $\frac{\partial \eta_D}{\partial b}<0$. At $0<d<d_2$, $\frac{\partial q_L}{\partial b}<0$, $\frac{\partial \eta_L}{\partial b}<0$, and at $d_2<d<1$, $\frac{\partial q_L}{\partial b}>0$, $\frac{\partial \eta_L}{\partial b}>0$.

Here, $d_2=\frac{(bY-A)^2}{(bY)^2+(-A)^2}$.

Proof of Proposition 2

$$\frac{\partial q_D}{\partial \sigma}=\frac{e[2bg+b^2(d-1)Y-(d-1)Y(-A)]\rho\sigma_e}{(bY)^2-(-A)^2} \tag{$B5$}$$

$$\frac{\partial q_D}{\partial \sigma_e}=\frac{e[2bg+b^2(d-1)Y-(d-1)Y(-A)]\rho\sigma}{(bY)^2-(-A)^2} \tag{$B6$}$$

$$\frac{\partial \eta_D}{\partial \sigma}=\frac{2e[bY+(d-1)A]\rho\sigma_e}{(bY-A)(bY+A)} \tag{$B8$}$$

$$\frac{\partial \eta_D}{\partial \sigma_e}=\frac{2e[bY+(d-1)A]\rho\sigma}{(bY-A)(bY+A)} \tag{$B9$}$$

$$\frac{\partial q_L}{\partial \sigma}=\frac{e[-2b(-1+d)g-b^2Y+Y(-A)]\rho\sigma_e}{(bY)^2-(-A)^2}>0 \tag{$B10$}$$

$$\frac{\partial q_L}{\partial \sigma_e}=\frac{e[-2b(-1+d)g-b^2Y+Y(-A)]\rho\sigma}{(bY)^2-(-A)^2}>0 \tag{$B11$}$$

$$\frac{\partial \eta_L}{\partial \sigma}=\frac{2e[-2g-b(d-1)Y+Y^2]\rho\sigma_e}{(bY-A)(bY+A)}<0 \tag{$B12$}$$

$$\frac{\partial \eta_L}{\partial \sigma_e}=\frac{2e[-2g-b(d-1)Y+Y^2]\rho\sigma}{(bY-A)(bY+A)}<0 \tag{$B13$}$$

From the Hessen matrix, $\frac{\partial q_L}{\partial \sigma}>0$, $\frac{\partial q_L}{\partial \sigma_e}>0$, $\frac{\partial \eta_L}{\partial \sigma}<0$, $\frac{\partial \eta_L}{\partial \sigma_e}<0$. At $0<d<d_3$, $\frac{\partial q_D}{\partial \sigma}>0$, $\frac{\partial q_D}{\partial \sigma_e}>0$, $\frac{\partial \eta_D}{\partial \sigma}>0$, $\frac{\partial \eta_D}{\partial \sigma_e}>0$. At $d_3<d<d_1$, $\frac{\partial q_D}{\partial \sigma}<0$, $\frac{\partial q_D}{\partial \sigma_e}<0$, $\frac{\partial \eta_D}{\partial \sigma}>0$, $\frac{\partial \eta_D}{\partial \sigma_e}>0$. At $d_1<d<$

1, $\frac{\partial q_D}{\partial \sigma} < 0$, $\frac{\partial q_D}{\partial \sigma_e} < 0$, $\frac{\partial \eta_D}{\partial \sigma} < 0$, $\frac{\partial \eta_D}{\partial \sigma_e} < 0$.

Here, $d_3 = -\frac{(-1+b-ep_0)(b-2g+bep_0+(1+ep_0)^2)}{(1+ep_0)(-b^2-2g+(1+ep_0)^2)}$, $d_1 = 1 + \frac{b+bep_0}{-2g+(1+ep_0)^2}$.

Proof of Proposition 3

$$q_D - q_L = d[-1 + \frac{e(b+Y)\rho\sigma\sigma e}{bY-A}] < 0 \qquad (B14)$$

$$\eta_D - \eta_L = \frac{2ed\rho\sigma\sigma e}{bY-A} < 0 \qquad (B15)$$