Journal of Industrial and Management Optimization
https://www.aimspress.com/journal/jimo

JIMO, 22(3): 1244–1283.
DOI:10.3934/jimo.2026046
Received: 25 July 2025
Revised: 15 January 2026
Accepted: 30 January 2026
Published: 09 February 2026

Research article

Quality upgrade strategies in a supply chain with advanced disclosure under AI technology

Qi Zheng and Keke Xie*

School of Management, Shanghai University of Engineering Science, Shanghai 201620, China

* **Correspondence:** Email: 15538897124@163.com.

Appendix A. Equilibrium solution results

According to the decision sequence outlined in the main text, the optimal equilibrium solution is derived through backward induction. We take the LN scenario as an example, while the computational processes for other scenarios can be obtained similarly.

First, by taking the second-order partial derivative of the retailer's second-period profit with respect to p_2, we obtain $\partial^2\pi_{r2}/\partial {p_2}^2 = -\dfrac{2(1+\gamma\delta)}{\delta+(-1+\gamma)\delta^2} < 0$. This confirms the existence of an optimal solution. Setting the first-order condition $\partial\pi_{r2}/\partial p_2 = 0$, the optimal second-period selling price is derived as $p_2 = \dfrac{w_2 + w_2\gamma\delta + \delta\left(p_1 - (\overline{m} - m_\iota)\beta(-1+\gamma)\delta\right)}{2+2\gamma\delta}$.

Substituting the selling price into the supplier's second-period profit function and taking the second-order derivative with respect to the wholesale price w_2, we obtain:

$$\partial^2\pi_{m_2}/\partial {w_2}^2 - \frac{1+\gamma\delta}{\delta+(-1+\gamma)\delta^2} < 0.$$

This confirms the existence of an optimal solution. Setting the first-order condition $\partial\pi_{m2}/\partial w_2 = 0$. The optimal wholesale price in the second period is derived as

$$w_2 = \frac{\delta\left(p_1 - (\overline{m} - m_L)\beta(-1+\gamma)\delta\right)}{2+2\gamma\delta}.$$

Following this, we derive the second-order derivative of the retailer's total two-period profit with respect to the first-period selling price p_1, yielding $\partial^2\pi_r/\partial {p_1}^2 = -\dfrac{3}{8+8(-1+\gamma)\delta} - \dfrac{13}{8+8\gamma\delta} < 0$.

This confirms the existence of an optimal solution. Setting the first-order condition $\partial\pi_r/\partial p_1 = 0$, the optimal first-period selling price p_1^{LN} is

$$p_1 = \frac{\begin{array}{c} 8\left(1 + w_1 + m_L\beta\right) + 2\left(-4 - 3w_1 - 4m\beta + 4\left(2 + w_1 + (m + m_L)\beta\right)\gamma\right)\delta \\ + \left(-1 + \gamma\right)\left(7m_L\beta + 8\gamma + m\beta(-7 + 8\gamma)\right)\delta^2 \end{array}}{16 + (-13 + 16\gamma)\delta}.$$

Next, we derive the second-order derivative of the supplier's total two-period profit with respect to the first-period wholesale price w_1, yielding

$$\partial^2\pi_m/\partial w_1^2 = -\frac{2(4 + (-3 + 4\gamma)\delta)^2(8 + (-7 + 8\gamma)\delta)}{(1 + (-1 + \gamma)\delta)(1 + \gamma\delta)(16 + (-13 + 16\gamma)\delta)^2} < 0.$$

This confirms the existence of an optimal solution. Setting the first-order condition $\partial\pi_m/\partial w_1 = 0$, the optimal first-period wholesale price w_1^{LN} is

$$w_1 = \frac{\begin{array}{l} \delta\Big(-656 + 1024\gamma + 558\delta + m\beta(-1 + \gamma)(4 + (-3 + 4\gamma)\delta)^2(16 + (-15 + 16\gamma)\delta) \\ +2\delta(-79\delta + \gamma)) + a_{64} \end{array}}{(4(4 + (-3 + 4\gamma)\delta)^2(8 + (-7 + 8\gamma)\delta))}.$$

Substituting the optimal solutions into the demand function and the profit function, we obtain the equilibrium outcomes for the benchmark model. The results serve as a foundation for comparing the effects of alternative strategies, such as quality upgrades and information disclosure, in other models.

Table A1

LN	
w_1	$\frac{\frac{1}{2}m_L\beta\left(512 + \chi_3 + \chi_4\delta^3 + \chi_5\delta^4 + k\chi_1\chi_5\right) + \chi_9 + \chi_6\delta^2 + \chi_7\delta^3 + \chi_8\delta^4}{4\chi_2^2\chi_{31}}$
p_1	$\frac{4\left(1 + \chi_1\delta\right)\chi_{10} + m_L\beta\left(96 + \delta\left(\chi_{11} + \chi_1\left(\chi_{12} + k\chi_1\chi_2\chi_{13}\right)\right)\right)}{4\chi_2\chi_{31}}$
p_2	$\frac{3\delta\left(4\left(1 + \chi_1\delta\right)\chi_{10} + m_L\beta\left(96 + \delta\left(\chi_{14} + \chi_1\chi_{15}\delta^2 - k\chi_1{\chi_2}^2\right)\right)\right)}{16(1 + \gamma\delta)\chi_2\chi_{31}}$
D_1	$\frac{4\left(1 + \chi_1\delta\right)\chi_{16} + m_L\beta\left(32 + \delta\left(-52 + 80\gamma + 4\chi_{18}\delta + \chi_{17}\delta^2 + k\chi_1\chi_2^2\right)\right)}{16\left(1 + \chi_1\delta\right)\left(1 + \gamma\delta\right)\chi_{31}}$
D_2	$\frac{4\left(1 + \chi_1\delta\right)\chi_{10} + m_L\beta\left(96 + \delta\left(\chi_{14} + \chi_1\chi_{15}\delta^2 - k\chi_1{\chi_2}^2\right)\right)}{16\left(1 + \chi_1\delta\right)\chi_2\chi_{31}}$
π_r	$\frac{\delta\chi^2 + 2\chi_{31}\left(\chi + m_L\beta\chi_{35}\left(4\left(1 + \chi_1\delta\right)\chi + m_L\beta\left(32 + \delta\chi_2\right)\right)\right)}{256\left(1 + \chi_1\delta\right)\left(1 + \gamma\delta\right)^2\chi_2{\chi_{31}}^2}$
π_m	$\frac{\delta{\chi_{40}}^2 + 2\left[\chi_{30} + m_L\beta\left(32 + \delta\left(\chi_{33} + k\chi_1{\chi_2}^2\right)\right)\right)\left(\frac{1}{2}m_L\beta\chi_{36} + \chi_{37}\right)}{128\left(1 + \chi_1\delta\right)\left(1 + \gamma\delta\right){\chi_2}^2{\chi_{31}}^2}$

$\chi_1 = \gamma - 1$
$\chi_2 = 4 - 3\delta + 4\gamma\delta$
$\chi_3 = (1792\gamma - 1056)\delta + \left(492 - 2544\gamma + 2304\gamma^2\right)\delta^2$
$\chi_4 = 188 + 480\gamma - 1920\gamma^2 + 1280\gamma^3$
$\chi_5 = (4 + (4\gamma - 3)\delta)^2(16 + (16\gamma - 15)\delta)$
$\chi_6 = 1536\gamma^2 - 1968\gamma + 558$
$\chi_7 = 1024\gamma^3 - 1968\gamma 2 + 1116\gamma - 158$
$\chi_8 = 256\gamma^4 - 656\gamma^3 + 558\gamma^2 - 158\gamma$
$\chi_9 = 256 + (1024\gamma - 656)\delta$
$\chi_{10} = (1 + \gamma\delta)(24 + (24\gamma - 19)\delta)$
$\chi_{11} = 240\gamma + 24\gamma(8\gamma - 7)\delta$
$\chi_{12} = -33 + 4\gamma + 48\gamma^2\delta^2 - 4(31 + \delta)$
$\chi_{13} = 12 - 11\delta + 12\gamma\delta$
$\chi_{14} = 4(76\gamma - 47) + 20(5 + 4\gamma(4\gamma - 5))\delta$
$\chi_{15} = 9 + 4\gamma(28\gamma - 25)$
$\chi_{16} = (1 + \gamma\delta)(8 + (8\gamma - 9\delta))$
$\chi_{17} = 9 + \gamma(3 + 4\gamma(4\gamma - 7))$
$\chi_{18} = 3 + 4\gamma(4\gamma - 5)$
$\chi_{19} = (4 - 3\delta)^2(15\delta - 16)$
$\chi_{20} = (54 - 23\delta)\delta - 32$
$\chi_{21} = 656 + 2\delta(79\delta - 279)$
$\chi_{22} = 32 + \delta(23\delta - 54)$
$\chi_{23} = 48 - 86\delta + 38\delta^2$
$\chi_{24} = 48 + \delta(33\delta + k(5\delta - 6) - 80)$
$\chi_{25} = (\delta - 2)(27\delta - 32)$
$\chi_{26} = 2 + (1 + k)m_L\beta$
$\chi_{27} = 80\gamma + 16\gamma(4\gamma - 3)\delta - 4(9 + \delta) + \chi_1\chi_{32} + k\chi_1{\chi_2}^2$
$\chi_{28} = (1 + \gamma\delta)(8 + (8\gamma - 5)\delta)$
$\chi_{29} = 3 + 4\gamma(4\gamma - 5))\,\delta + (9 + \gamma(3 + 4\gamma(4\gamma - 7)))\delta^2$
$\chi_{30} = 4\,(1 + \chi_1\delta)\,(1 + \gamma\delta)(8 + (8\gamma - 9)\delta)$

Table A2

LD	
w_1	$\frac{(1+m_L\beta)\,(\delta\lambda-1)(128+\delta\lambda(79\delta\lambda-200))}{2(4-3\delta\lambda)^2(7\delta\lambda-8)}$
w_2	$\frac{(1+m_L\beta)\,\delta\lambda(\delta\lambda-1)(19\delta\lambda-24)}{2(3\delta\lambda-4)(7\delta\lambda-8)}$
p_1	$\frac{(1+m_L\beta)\,(\delta\lambda-1)(19\delta\lambda-24)}{(3\delta\lambda-4)(7\delta\lambda-8)}$
p_2	$\frac{3\,(1+m_L\beta)\,\delta\lambda(\delta\lambda-1)(19\delta\lambda-24)}{4(3\delta\lambda-4)(7\delta\lambda-8)}$
D_1	$1-\frac{-m_L\beta+m_L\beta\delta\lambda+\chi_{42}-\chi_{43}}{1-\delta\lambda}$
D_2	$\frac{-m_L\beta+m_L\beta\delta\lambda+\chi_{42}-\chi_{43}}{1-\delta\lambda}-\frac{-m_L\beta\delta\lambda+\chi_{43}}{\delta\lambda}$
π_r	$\frac{(1+m_L\beta)^2\,(\delta\lambda-1)(\delta\lambda(2112+\delta\lambda(269\delta\lambda-1376))-1024)}{16(8-7\delta\lambda)^2(4-3\delta\lambda)^2}-c$
π_m	$\frac{(1+m_L\beta)^2\,(8-5\delta\lambda)^2(\delta\lambda-1)}{4(4-3\delta\lambda)^2(7\delta\lambda-8)}$

$\chi_{31}=8+(8\gamma-7)\delta$

$\chi_{32}=(-9+4\gamma(1+4\gamma))\delta^2$

$\chi_{33}=-52+80\gamma+4\chi_{18}\delta+(9+\gamma(3+4\gamma(-7+4\gamma)))\delta^2$

$\chi_{34}=4(-47+76\gamma)+20(5+4\gamma(-5+4\gamma))\delta+\chi_1(9+4\gamma(28\gamma-25))\delta^2-k\chi_1{\chi_2}^2$

$\chi_{35}=32+\delta\left(-52+80\gamma+4\left(\chi_{29}+k\chi_1{\chi_2}^2\right)\right)$

$\chi_{36}=512+\delta\big(32(56\gamma-33)+12(4\gamma-1)(48\gamma-41)\delta+\chi_{39}+\chi_1\chi_{38}+k\chi_1{\chi_2}^2(16-15\delta+$

$\chi_{37}=2\,(1+\chi_1\delta)\,(1+\gamma\delta)(128+\delta(79\delta+8\gamma(32+(16\gamma-25)\delta)-200))$

$\chi_{38}=\left(135+4\gamma\left(64\gamma^2-47-44\gamma\right)\right)\delta^3$

$\chi_{39}=4(47+40\gamma(3+4\gamma(2\gamma-3)))\delta^2$

$\chi_{40}=4\,(1+\chi_1\delta)\,\chi_{10}+m_L\beta\,(96+\delta\chi_{34})$

$\chi_{41}=m_L\beta{\chi_2}^2(16+(16\gamma-15)\delta)+c_k(1+\gamma\delta)(32+\delta(-54+23\delta+2\gamma(32+(16\gamma-27)$

$\chi_{42}=\frac{(1+m_L\beta)\,(\delta\lambda-1)(19\delta\lambda-24)}{(3\delta\lambda-4)(7\delta\lambda-8)}$

$\chi_{43}=\frac{3\,(1+m_L\beta)\,\delta\lambda(\delta\lambda-1)(19\delta\lambda-24)}{4(3\delta\lambda-4)(7\delta\lambda-8)}$

Table A3

HN	
w_1	$\frac{256-656\delta+\chi_{41}+\delta\left(\chi_{45}+2\delta\chi_{44}+km_L\beta\chi_{46}\right)}{4\chi_2^2\chi_{31}}$
w_2	$\frac{tk^2(1+\gamma\delta)\chi_{48}+\delta\left(4\left(1+\chi_1\delta\right)\chi_{10}+m_L\beta\left(-\left((2+\chi_1\delta)\chi_2{}^2\right)+k\chi_{47}\right)\right)}{8(1+\gamma\delta)\chi_2\chi_{31}}$
p_1	$\frac{4\chi_{51}+tk^2(1+\gamma\delta)(6+(6\gamma-5)\delta)+m_L\beta\chi_{50}}{4\chi_2\chi_{31}}$
p_2	$\frac{tk^2\chi_{52}+3\delta\left(4\chi_{51}+m_L\beta\left(k\chi_{47}-\left((2+\chi_1\delta)\chi_2^2\right)\right)\right)}{16(1+\gamma\delta)\chi_2\chi_{31}}$
D_1	$\frac{\chi_{30}+tk^2(1+\gamma\delta)(10+(10\gamma-9)\delta)+m_L\beta\chi_{54}}{16\left(1+\chi_1\delta\right)(1+\gamma\delta)\chi_{31}}$
D_2	$\frac{-tk^2(1+\gamma\delta)\chi_{55}+\delta\left(4\chi_{51}+m_L\beta\left(k\chi_{47}-\left((2+\chi_1\delta)\chi_2{}^2\right)\right)\right)}{16\delta\left(1+\chi_1\delta\right)\chi_2\chi_{31}}$
π_r	$\frac{2\delta\chi_{31}\left(4\left(1+\chi_1\delta\right)\chi_{28}+tk^2\chi_{71}+m_L\beta\chi_{70}\right)+\chi_{73}{}^2}{256\delta\left(1+\chi_1\delta\right)(1+\gamma\delta)\chi_2{}^2\chi_{31}}$
π_m	$\frac{2\delta\chi_{78}\chi_{79}+\left(tk^2\chi_{76}+\delta\chi_{75}\right)\left(tk^2\chi_{76}+\delta\chi_{77}\right)}{128\delta\left(1+\chi_1\delta\right)(1+\gamma\delta)\chi_2{}^2\chi_{31}{}^2}$

$$\chi_{44}=-79\delta+\gamma\left(-984+768\gamma+558\delta+8\gamma(64\gamma-123)\delta+\chi_1(79+8\gamma(16\gamma-25))\delta^2\right)$$
$$\chi_{45}=1024\gamma+558\delta+\frac{1}{2}(1+k)m_L\beta\chi_1\chi_2^2(16+(16\gamma-15)\delta)$$
$$\chi_{46}=\left(1+\chi_1\delta\right)(224+\delta(-346+135\delta+2\gamma(224+(112\gamma-173)\delta)))$$
$$\chi_{47}=128+\delta\left(368\gamma+118(\delta-2)+32\gamma(11\gamma-14)\delta+\chi_1(9+4\gamma(28\gamma-25))\delta^2\right)$$
$$\chi_{48}=64+\delta(-98+37\delta+2\gamma(64+(-49+32\gamma)\delta))$$
$$\chi_{49}=2\left(73-35k-4(42+k)\gamma+48(2+k)\gamma^2\right)\delta^2+\chi_1\left(-33(k-1)+4(k-20)\gamma+48(1+k)^2\phi^3\right.$$
$$\chi_{50}=96+4(-52+9k+12(5+k)\gamma)\delta+\chi_{49}$$
$$\chi_{51}=\left(1+\chi_1\delta\right)(1+\gamma\delta)(24-19\delta+24\gamma\delta)$$
$$\chi_{52}=(1+\gamma\delta)(2+(2\gamma-1)\delta)(32+(32\gamma-27)\delta)$$
$$\chi_{53}=-4(16+k)+16(5+k)\gamma+2\left(-3(k-7)-4(13+4k)\gamma+16(2+k)\gamma^2\right)\delta$$
$$\chi_{54}=32+\delta\left(\chi_{53}+\chi_1\left(9-9k-12(2+k)\gamma+16(1+k)\gamma^2\right)\delta^2\right)$$
$$\chi_{55}=64+\delta(-110+47\delta+2\gamma(64+(-55+32\gamma)\delta))$$
$$\chi_{56}=2(\delta-1)(5\delta(400+\delta(-350+103\delta))-768)$$

Table A4

HD	
w_1	$\dfrac{-256 + m_L\beta\chi_{19} + \frac{1}{2}tk^2\chi_{20} + \delta\left(\chi_{21} + km_L\beta\chi_{22}\right)}{4(4-3\delta)^2(7\delta-8)}$
w_2	$\dfrac{1}{2}\left(\dfrac{1}{2}tk^2 + (k-1)m_L\beta\delta + \dfrac{\delta\left(\chi_{23} + \frac{1}{2}tk^2(6-5\delta) + m_L\beta\chi_{24}\right)}{2(3\delta-4)(7\delta-8)}\right)$
p_1	$\dfrac{\chi_{23} + \frac{1}{2}tk^2(6-5\delta) + m_L\beta\chi_{24}}{2(3\delta-4)(7\delta-8)}$
p_2	$\dfrac{\frac{1}{2}tk^2\chi_{25} + 3\delta\left(\chi_{23} + m_L\beta\left(k(64+\delta(47\delta-110)) - (4-3\delta)^2\right)\right)}{8(3\delta-4)(7\delta-8)}$
D_1	$\dfrac{2\left(8 + \frac{5}{2}tk^2 + 8m_L\beta\right) - \left(34 + \frac{9}{2}tk^2 + 2(12+5k)m_L\beta\right)\delta + 9\chi_{26}\delta^2}{8(\delta-1)(7\delta-8)}$
D_2	$\dfrac{1}{16}\left(\dfrac{4 - \frac{3}{2}tk^2 + 4km_L\beta}{4-3\delta} + \dfrac{2\left(\frac{1}{2}tk^2 - (k-1)m_L\beta\right)}{\delta-1} - \dfrac{2tk^2}{\delta} + \dfrac{\frac{7}{2}tk^2 - 8\chi_{26}}{7\delta-8}\right)$
π_r	$\dfrac{\frac{1}{2}t^2k^4\chi_{60} + tk^2\delta\chi_{57} + \delta\left(\chi_{61} - 4m_L\beta\chi_{62} + m_L^2\beta^2\chi_{59}\right)}{64(8-7\delta)^2(4-3\delta)^2(\delta-1)\delta}$
π_m	$\dfrac{\frac{1}{2}t^2k^4\chi_{65} + tk^2\delta\chi_{66} + \delta\left(4(8-5\delta)^2(-1+\delta)^2 - 4m_L\beta\chi_{63} + m_L^2\beta^2\chi_{64}\right)}{16(4-3\delta)^2(7\delta-8)(\delta-1)\delta}$

$$\chi_{57} = \chi_{56} + m_L\beta\left(-(4-3\delta)^2(9\delta-10)(13\delta-16) + k(4096 + \delta(\delta 17388 + \delta(2083\delta - 9804)))\right)$$
$$\chi_{58} = k^2(-4096 + \delta(13760 + \delta(-17388 + (9804 - 2083\delta)\delta)))$$
$$\chi_{59} = -4096 + \delta\left(15616 + 2k(4-3\delta)^2(9\delta-10)(13\delta-16) + 3\delta(9\delta(672 + \delta(39\delta - 256))) + \chi_{58}\right)$$
$$\chi_{60} = -4096 + \delta(13760 + \delta(-17388 + (9804 - 2083\delta)\delta))$$
$$\chi_{61} = -64c(8-7\delta)^2(4-3\delta)^2(\delta-1) + 4(\delta-1)^2(-1024 + \delta(2112 + \delta(-1376 + 269\delta)))$$
$$\chi_{62} = (\delta-1)(-2048 + \delta(7040 - 3\delta(2992 + 3\delta(117\delta - 560)) + k(5\delta(400 + 103\delta))))$$
$$\chi_{63} = (\delta-1)(128 + \delta(-336 + 9(32 - 9\delta)\delta + k(48 + \delta(-78 + 31\delta))))$$
$$\chi_{64} = 256 + \delta\left(2k(4-3\delta)^2(9\delta-10) + 3(3\delta-8)(32 + 3\delta(3\delta-8)) + k^2(256 + \delta((524 - 143\delta)\delta))\right)$$
$$\chi_{65} = 256 + \delta(-636 + (524 - 143\delta)\delta)$$
$$\chi_{66} = 2(\delta-1)(48 + \delta(31\delta - 78)) + mL\beta\left(-(4-3\delta)^2(9\delta-10) + k(\delta(636 + \delta(143\delta - 524)))\right)$$
$$\chi_{67} = 2\left(21 - 11k - 52\gamma + 16(2+k)\gamma^2\right)\delta^2 + \chi_1\left(9 - 9k + 4(k-6)\gamma + 16(1+k)\gamma^2\right)\delta^3)$$

Table A5

HND	
w_1	$\dfrac{b_{11}64(1+\delta(-1+\lambda))^3}{4b_{14}(4-3\delta)(20+\delta(-20+7\lambda)+2\delta(1-\lambda))^2}$
w_{2H}	$\dfrac{32(1+\delta(-1+\lambda))^2 96\left(m_L\beta(-2+\delta)+2(-1+\delta)\right)(-1+\delta)^2}{8(8+\delta(-8+\lambda))(1+\delta(-1+\lambda))}$
w_{2L}	$\dfrac{tk^2+tk^2\delta(-1+\lambda)+\delta\lambda\left(b_2+m_L\beta b_3\right)}{2(2+2\delta(-1+\lambda)+\delta(1-\lambda))}$
p_1	$\dfrac{\delta\lambda\left(b_2+m_L\beta\delta(\lambda-1)\right)-2\left(tk^2+4(-8+\delta(8+\lambda))+m_L\beta\left(4+\delta b_8\right)\right.}{2(2+2\delta(-1+\lambda)+\delta(1-\lambda))}$
p_{2H}	$\dfrac{256(-1+\delta)(1+\delta(-1+\lambda))^3-2(-12+\delta(12+\lambda))}{8(8+\delta(-8+\lambda))(1+\delta(-1+\lambda))}$
p_{2L}	$\dfrac{3\delta\lambda\delta(1-\lambda)^5-2\left(tk^2+4b_{14}+mL\beta(4+\delta(-2-2k+\lambda))\right)\delta(1-\lambda)^6)}{8(4+\delta(-4+\lambda))(1+\delta(-1+\lambda)}$
D_1	$\dfrac{32(1+\delta(-1+\lambda))^2 b_9+2(1+\delta(-1+\lambda))\left(8+29m_L\beta\right)\delta^3\lambda^3+b_{10}}{(16(-1+\delta)(1+\delta(-1+\lambda))+(-8+\delta(8+\lambda))\delta[1-\lambda])}$
D_{2H}	$\dfrac{-1024(-1+\delta)b_9(4+\delta(-4+\lambda))(1+\delta(-1+\lambda))^3\left(1/2tk^2+m_L\beta\delta\right)+b_{12}}{8\delta(1-\lambda)(4(8+\delta(-8+\lambda))(1+\delta(-1+\lambda))}$
D_{2L}	$\dfrac{16(-1+\delta)^2\left(83/2tk^2+336(-1+\delta)+m_L\beta b_7\right)+8(-1+\delta)\delta}{8\delta[1-\lambda](4(8+\delta(-8+\lambda))(1+\delta(-1+\lambda)}$
π_r	$\dfrac{1}{256}\left(\dfrac{2\left(tk^2+4b_{10}\right)}{b_{14}(7\delta-8)(\delta-1)}-\dfrac{b_{13}\left(-7tk^2-64\delta\right)}{8(4+\delta\lambda)(\lambda-1)}-\dfrac{b_{12}(-6+26\delta-17k\delta)}{b_{14}{}^2(1+\delta(-1+\lambda))}\right.$
π_m	$\dfrac{(1-\delta)b_7b_{12}(8-\lambda)+\delta b_{13}b_{14}(-1+\delta)^3-b_9m_L\beta(4+\delta(-2-2k+\lambda))}{16\delta(8-7\delta)^2(4-3\delta)^2(20+(20-7\lambda))}$

$\chi_{68} = 32+4(-16+3k+4(5+k)\gamma)\delta+\chi_{67}$
$\chi_{69} = \left(9-9k-12(2+k)\gamma+16(1+k)\gamma^2\right)\delta^2$
$\chi_{70} = 32+\delta\left(-4(16+k)+16(5+k)\gamma+2\left(-3(k-7)-4(13+4k)\gamma+16(2+k)\gamma^2\right)\delta+\chi_1\chi_{69}\right)$
$\chi_{71} = (1+\gamma\delta)(2+(2\gamma-1)\delta)+m_L\beta\chi_{68})\left(\chi_{30}+2c_k(1+\gamma\delta)(10+(10\gamma-9)\delta)\right.$
$b_1 = 656+2\delta(79\delta-279)+km_L\beta(32+\delta(23\delta-54))$
$b_2 = 48+c_k(6-5\delta)-86\delta+38\delta^2$
$b_3 = 48+\delta(-80+33\delta+k(5\delta-6))$
$b_4 = \delta\left(48-86\delta+38\delta^2+m_L\beta k(64+\delta(-110+47\delta))-(4-3\delta)^2\right)$
$b_5 = \delta(110+58\lambda+\delta((42\delta-99)\lambda-47))-64$

Table A6

HDD	
w_1	$\dfrac{b_{11}}{4b_{14}(4-3\delta)}$
w_{2H}	$\dfrac{1}{2}\left(\dfrac{1}{2}tk^2+(k-1)m_L\beta\delta+\dfrac{\delta\left(b_2+m_L\beta b_3\right)}{2b_{14}}\right)$
w_{2L}	$\dfrac{\delta\lambda\left(b_2+m_L\beta b_3\right)}{4b_{14}}$
p_1	$\dfrac{b_2+m_L\beta b_3}{2b_{14}}$
p_{2H}	$\dfrac{\frac{1}{2}tk^2(\delta-2)(27\delta-32)+3b_4}{8b_{14}}$
p_{2L}	$\dfrac{3\delta\lambda\left(b_2+m_L\beta b_3\right)}{8b_{14}}$
D_1	$\dfrac{b_9}{8(\delta-1)(7\delta-8)}$
D_{2H}	$\dfrac{b_8-\frac{1}{2}tk^2b_5-m_L\beta\delta\left(b_7+k\left(64+b_6\right)\right)}{8\delta b_{14}(\delta-1)(\lambda-1)}$
D_{2L}	$\dfrac{\frac{1}{2}tk^2+m_L\beta\delta-km_L\beta\delta}{4\delta-4\delta\lambda}$
π_r	$\dfrac{1}{64}\left(-64c+\dfrac{2b_9b_{10}}{b_{14}(3\delta-4)(\delta-1)}-\dfrac{b_{13}}{b_{14}(\lambda-1)}-\dfrac{b_{12}}{b_{14}{}^2(\delta-1)\delta(\lambda-1)}\right.$
π_m	$\dfrac{\delta b_9b_{11}(1-\lambda)+\delta b_{13}b_{14}(1-\delta)-b_{12}}{32\delta(8-7\delta)^2(4-3\delta)^2(\delta-1)(\lambda-1)}$

$b_6=\delta(-2(55+29\lambda)+\delta(47+(99-42\delta)\lambda))$; $b_7=(-4+3\delta)(4-3\delta+(12+\delta(14\delta-27))\lambda)$
$b_8=2(\delta-1)\delta(19\delta-24)(\lambda-1)$
$b_9=2\left(8+5c_k+8m_L\beta\right)-\left(34+9c_k+2(12+5k)m_L\beta\right)\delta+9\left(2+(1+k)m_L\beta\right)\delta^2$
$b_{10}=\left(2\left(8+c_k+8m_L\beta\right)-\left(26+c_k+2(12+k)m_L\beta\right)\delta+\left(10+(9+k)m_L\beta\right)\delta^2\right)$
$b_{11}=256-m_L\beta(4-3\delta)^2(15\delta-16)+c_k(32+\delta(-54+23\delta))-\delta b_1$
$b_{12}=\left(c_k(-64+(110-47\delta)\delta)+b_4\right)\left(64c_k+b_8+c_kb_6+m_L\beta\delta\left(kb_5-b_7\right)\right)$
$b_{13}=2\lambda\left(c_k-(k-1)m_L\beta\delta\right)\left(b_2+m_L\beta b_3\right)$; $b_{14}=(7\delta-8)(3\delta-4)$

Considering the AI technology investment level

Table A7

HNL	
k^*	$\frac{2\beta(1+m_L\beta)(1-\delta)\delta(48+\delta(31\delta-78))\eta\mu}{8t(4-3\delta)^2(1-\delta)(8-7\delta)-\beta^2\delta(256-\delta(636+\delta(-524+143\delta)))\eta^2\mu^2}$
w_1	$\frac{2(1+m_L\beta)(\delta-1)(128+\delta(79\delta-200))+k^*\beta\delta(32+\delta(23\delta-54))\eta\mu}{4(4-3\delta)^2(7\delta-8)}$
w_2	$\frac{2(1+m_L\beta)(1-\delta)\delta(24-19\delta)+k^*\beta\delta(64-\delta(110-47\delta))\eta\mu}{4(4-3\delta)(8-7\delta)}$
p_1	$\frac{2(1+m_L\beta)(1-\delta)(24-19\delta)-k^*\beta\delta(6-5\delta)\eta\mu}{2(4-3\delta)(8-7\delta)}$
p_2	$\frac{6(1+m_L\beta)(1-\delta)\delta(24-19\delta)+3k^*\beta\delta(64-\delta(110-47\delta))\eta\mu}{8(4-3\delta)(8-7\delta)}$
D_1	$\frac{2(1+m_L\beta)(1-\delta)(8-9\delta)-k^*\beta\delta(10-9\delta)\eta\mu}{8(1-\delta)(8-7\delta)}$
D_2	$\frac{2(1+m_L\beta)(1-\delta)(24-19\delta)+k^*\beta(64-(110-47\delta)\delta)\eta\mu}{8(1-\delta)(4-3\delta)(8-7\delta)}$
π_r	$\frac{4(1+m_L\beta)^2(8-5\delta)^2(1-\delta)^2-4k\beta(1+m_L\beta)M_1\eta\mu-k^{*2}\left(8t(4-3\delta)^2(1-\delta)(8-7\delta)-\beta^2M_2\eta^2\mu^2\right)}{(16(4-3\delta)^2(1-\delta)(8-7\delta))}$
π_m	$\frac{4(1+m_L\beta)^2M_3-4k^*\beta(1+mL\beta)M_4\eta\mu+k^{*2}\beta^2\delta(-4096+\delta(13760+\delta(-17388+(9804-2083\delta)\delta)))\eta^2\mu^2}{64(8-7\delta)^2(4-3\delta)^2(-1+\delta)}$

$M_1=(-1+\delta)\delta(48+\delta(-78+31\delta))$ $\quad M_3=(-1+\delta)^2(-1024+\delta(2112+\delta(-1376+269\delta)))$
$M_2=\delta(256+\delta(-636+(524-143\delta)\delta))M_4=(-1+\delta)\delta(-768+5\delta(400+\delta(-350+103\delta)))$

Table A8

HDL	
k^*	$\dfrac{2\beta\tau_0(1+m_L\beta)(1-\delta)\delta(47+\delta(6\delta-5))\eta^2\mu}{(4-3\delta)^2(8-7\delta)-\beta^2(256-\delta(\tau_0+\delta(-24+5\delta)))\eta^2\mu^2}$
w_1	$\dfrac{\left(-256+2m_L\beta M_5+\delta\left(656+32M_6+18\delta(-31+3M_6)+\delta^2(158+23M_6)\right)\right)}{4(4-3\delta)^2(-8+7\delta)}$
w_2	$\dfrac{\frac{1}{2}\delta(M_6+(48+2m_L\beta(-1+\delta)(-24+19\delta)+\delta(-86-6M_6+\delta(38+5M_6)))}{(2(-4+3\delta)(-8+7\delta))}$
p_1	$\dfrac{(48+2m_L\beta(-1+\delta)(-24+19\delta)+\delta(-86-6M_6+\delta(38+5M_6)))}{(2(-4+3\delta)(-8+7\delta))}$
p_2	$\dfrac{\left(3\delta\left(48+2m_L\beta(-1+\delta)(-24+19\delta)+64M_6+2\delta(-43-55M_6)+\delta^2(38+47M_6)\right)\right)}{(8(-4+3\delta)(-8+7\delta))}$
D_1	$\dfrac{(16+2m_L\beta(-1+\delta)(-8+9\delta)+\delta(-34-10M_6+9\delta(2+M_6)))}{8(-1+\delta)(-8+7\delta)}$
D_2	$\dfrac{\left(-48-2m_L\beta(-1+\delta)(-24+19\delta)-64M_6+\delta^2(-38+47M_6)+2\delta(43+55M_6)\right)}{(8(-1+\delta)(-4+3\delta)(-8+7\delta))}$
π_r	$\dfrac{\delta(M_7+M_6M_{15})^2+2(8-7\delta)(M_9+\delta((9\delta+10)M_6+18\delta-34)) \times(M_8+\delta(10\delta-26+(2+\delta)M_6))}{64(8-7\delta)^2(4-3\delta)^2(1-\delta)}-c$
π_m	$\dfrac{M_{11}+k^{*4}\alpha^2\beta^2\delta M_{10}\eta^2(\mu-1)^2+\left(2k^{*3}\alpha\beta^2\delta M_{10}\eta^2(\mu-1)-4k^*M_{12}\beta\eta\right)M_{13} +k^{*2}\left(M_{14}+\beta\delta\eta\left(M_{12}+\beta M_{10}\eta M_{13}{}^2\right)\right)}{16(8-7\delta)^2(4-3\delta)^2(1-\delta)}$

$M_5=(-1+\delta)(128+\delta(-200+79\delta))$
$M_6=k^*\beta\eta(\mu+k^*(\alpha-\alpha\mu)+\tau_0-\mu\tau_0)$
$M_7=48+2m_L\beta(-1+\delta)(-24+19\delta)-86\delta+38\delta^2$
$M_8=16+2m_L\beta(-1+\delta)(-8+5\delta)$
$M_9=16+2m_L\beta(-1+\delta)(-8+9\delta)$
$M_{10}=(-256+\delta(636+\delta(-524+143\delta)))$
$M_{11}=(-8+7\delta)\left(-4(1+m_L\beta)^2(8-5\delta)^2(-1+\delta)^2\right.$
$M_{12}=-4\alpha(1+m_L\beta)(-1+\delta)\delta(48+\delta(-78+31\delta))(\mu-1)$
$M_{13}=(\mu(-1+\tau_0)-\tau_0)$
$M_{14}=8t(4-3\delta)^2(-1+\delta)(-8+7\delta)$
$M_{15}=\left(64+110\delta+47\delta^2\right)$

Appendix B. Proofs of the corollary and propositions.

Proof of **Proposition 1.** Define k_1, as the solution to $w_2^{LN}(k_1)=w_2^{LD}(k_1)$, which exists because both functions are continuous in k, $\lim k\to 0, w_2^{LN}>w_2^{LD}$, $\lim k\to\infty$, $w_2^{LN}<w_2^{LD}$, and $\exists$ k_1 ,

where equality holds for $k < k_1 : w_2^{LN}(k) > w_2^{LN}(k_1) = w_2^{LD}(k_1) < w_2^{LD}(k)$.

$P = -32(1+\gamma) + 4(13 + (3 - 8\gamma)\gamma)\delta, Q = \delta(128 + \delta(-208 + 8\gamma(16 - 13\delta) + 85\delta)$

$D = (-4+3\delta)^3(-8+7\delta)+16\delta^2(32+\delta(-76+41\delta))\gamma, k_1 = (-4+3\delta)(4+(-3+4\gamma)\delta)P+4\gamma(1+\gamma\delta)Q/m_L\beta D$

Proof of **Corollary 1.** Under the parameter constraints $0 < \beta < 1, 0 < \gamma < 10 < \delta < 1$, and $k > 1$, the first-order derivative of w_1^{HD} with respect to k is strictly negative, i.e., $\frac{\partial w_1^{HD}}{\partial k} < 0$, The function w_1^{HD} is given by $w_1^{HD} = \frac{\text{Numerator}}{4(4-3\delta)^2(-8+7\delta)}$. The portion of the numerator dependent on k is Numerator $\_k = km_L\beta\delta(32 + \delta(-54 + 23\delta))$. Since the denominator $4(4-3\delta)^2(-8+7\delta)$ is independent of k, the derivative simplifies to $\frac{\partial w_1^{HD}}{\partial k} = \frac{m_L\beta\delta(32 + \delta(-54+23\delta))}{4(4-3\delta)^2(-8+7\delta)}; 4(4-3\delta)^2 > 0, -8+7\delta < 0$ for $\delta \in (0,1)$ (as $\delta < 8/7$). The numerator is strictly positive while the denominator is strictly negative; therefore, $\frac{\partial w_1^{HD}}{\partial k} = \frac{\text{Positive}}{\text{Negative}} < 0$.

Under the given parameter constraints, we have established that $\frac{\partial w_2^{LN}}{\partial k} > 0$, Under the parameter constraints $0 < \beta < 1, 0 < \gamma < 10 < \delta < 1$, and $k > 1$, the first-order derivative of w_2^{LN} with respect to k is strictly negative, i.e., $\frac{\partial w_2^{LN}}{\partial k} > 0$, The function w_1^{HD} is given by $w_2^{LN} = \frac{\delta.\text{Numerator}}{8(1+\gamma\delta)(4+(-3+4\gamma)\delta)(8+(-7+8\gamma)\delta)}$ where the numerator contains both k-independent terms and the k-dependent term: $-km_L\beta(-1+\gamma)(4-3\delta+4\gamma\delta)^2$.

Differentiating with respect to k yields $\frac{\partial w_1^{HD}}{\partial k} = \frac{\delta(-m_L\beta(-1+\gamma)(4-3\delta+4\gamma\delta)2)}{8(1+\gamma\delta)(4+(-3+4\gamma)\delta)(8+(-7+8\gamma)\delta)}, -m_L\beta(-1+\gamma) > 0(4-3\delta+4\gamma\delta)^2 \geq 0$. Thus, the numerator is strictly positive. Since both the numerator and denominator are strictly positive, $\frac{\partial w_2^{LN}}{\partial k} = \frac{\text{Positive}}{\text{Positive}} < 0$.

We analyze the derivative of w_2^{HD} with respect to k. The explicit k-dependent terms yield $\frac{\partial w_2^{HD}}{\partial k} = \frac{m_L\beta\delta}{2}\left(1 + \frac{-6+5\delta}{(-4+3\delta)(-8+7\delta)}\right)$. Since $m_L\beta\delta > 0$ and the denominator $(-4+3\delta)(-8+7\delta) > 0$ for $\delta \in (0,1)$, we focus on the bracketed term. Let $f(\delta) = (-4+3\delta)(-8+7\delta) - (6-5\delta)$. For $\delta \in (0,1)$, $f(\delta)$ is minimized as $\delta \to 1$, where $f(1) = 0$. Monotonicity (via $f'(\delta) = -47 + 42\delta < 0$) ensures that $f(\delta) > 0$ on $(0,1)$. Thus, $1 + -6 + 5\delta(-4+3\delta)(-8+7\delta) = (-4+3\delta)(-8+7\delta)$, and $f(\delta) > 0$, proving $\frac{\partial w_2^{HD}}{\partial k} > 0$.

Proof of **Proposition 2.** For the proof of $\pi_m^{HN} > \pi_m^{LN}$, when $k > k_2$, we begin by analyzing the profit functions of the supplier (π_m^{LN}) and the retailer (π_m^{HN}). Both expressions share a common denominator: $128(1+(\gamma-1)\delta)(1+\gamma\delta)(4+(4\gamma-3)\delta)^2(8+(8\gamma-7)\delta)^2$. Thus, comparing π_m^{HN} and π_m^{LN} reduces to comparing their numerators. Let k_2 be the threshold value satisfying $\pi_m^{HN} = \pi_m^{LN}$.

By solving: $2\delta\cdot$ Numerator $_c$+ Term $_c$ = $\delta\cdot$ Numerator $_a$+2$\cdot$ Term $_a$, we obtain $k = k_2$. The coefficient of k in the numerator is positive, implying $\frac{\partial \pi_m^{HN}}{\partial k} > 0$. Thus, π_m^{HN} increases with k. The coefficient of k is negative, giving $\frac{\partial \pi_m^{LN}}{\partial k} < 0$. Hence, π_m^{LN} decreases with k. For $k > k_2$, the monotonicity ensures that

$\pi_m^{HN}(k) > \pi_m^{HN}(k_2) = \pi_m^{LN}(k_2) > \pi_m^{LN}(k)$. Therefore, $\pi_m^{HN} > \pi_m^{LN}$ when k exceeds k_2.

Similarly, we can find the proof of $\pi_m^{LD} > \pi_m^{HD}$.

Proof of **Proposition 3.** We analyze the profit functions of the supplier under different regimes: π_r^{LN} and π_r^{LD}. Here π_r^{LN} incorporates terms for coordination benefits (through γ and $m_L\beta$) and competitive effects (through k), whereas π_r^{lD} is simpler, with a fixed cost c and a term dependent on δ and λ. For any $\delta\ \lambda$ that is sufficiently small, the term $(-1024+\delta\lambda(2112+\delta\lambda(-1376+269\delta\lambda)))$ in π_r^{LD} is negative, making π_r^{LD} negative when $c > 0$. However, π_r^{LN} is strictly positive for $\gamma \in (0,1)$ and $\delta \in (0,1)$, as all terms in the numerator and denominator are positive under these conditions.

Proof of $\pi_r^{HN} > \pi_r^{HD}$, when $\beta > \beta_1$. We compare the retailer's profit under two regimes: π_r^{HN} and π_r^{HD}. Solve $\pi_r^{HN} = \pi_r^{HD}$ for β to obtain the critical threshold β_1. This exists because both profits are continuous in β. π_r^{HN} : The terms involving $m_L\beta$ are positive and linear in β, and thus, π_r^{HN} increases with β. π_r^{HD}: The terms involving $m_L\beta$ are negative (verified by coefficient inspection). Thus, π_r^{HD} decreases with β. For $\beta = \beta_1, \pi_r^{HN} = \pi_r^{HD}$. For $\beta = \beta_1$, the monotonicity implies that $\pi_r^{HN}(\beta) > \pi_r^{HN}(\beta_1) = \pi_r^{HD}(\beta_1) > \pi_r^{HD}(\beta)$.

Proof of $\pi_r^{LD} > \pi_r^{LN}$, when $\gamma < \gamma_1$. We analyze the retailer's profit under two regimes: π_r^{LD} and π_r^{LN}. Solve $\pi_r^{LD} = \pi_r^{LN}$ for γ to obtain the critical threshold γ_1. This exists because both profits are continuous in γ. π_r^{LN}: The terms involving γ are positive and increasing (verified by inspecting the coefficient). Thus, π_r^{LN} increases with γ. π_r^{LD}: The terms involving γ through λ are negative (since $(-1+\delta\lambda)$ dominates for $\delta\lambda < 1$). Thus, π_r^{LD} decreases with γ. For $\gamma = \gamma_1, \pi_r^{LD} = \pi_r^{LN}$. For $\gamma < \gamma_1$, the monotonicity implies that $\pi_r^{LD}(\gamma) > \pi_r^{LD}(\gamma_1) = \pi_r^{LN}(\gamma_1) > \pi_r^{LN}(\gamma)$.

Similarly, We can find the proof of $\pi_r^{HD} > \pi_r^{HN}$.

The proofs of Propositions 4 through 7 follow the same logical structure and use identical analytical techniques as presented in the preceding demonstration.