Journal of Industrial and Management Optimization

https://www.aimspress.com/journal/jimo

JIMO, 22(1): 754–775.
DOI:10.3934/jimo.2026027
Received: 15 July 2025
Revised: 25 September 2025
Accepted: 18 December 2025
Published: 25 December 2025

*Research article*

# To provide or not? The financing strategy in the automotive supply chain under asymmetric information

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## Supplementary

## A. Appendix

### *A.1. Proof of Lemma 1*

*Proof.* By applying backward induction method, the dealer maximizes his profit with the selling price

$$p_{ij}^{CF} = \frac{A_j + (1 + r_{ij}^{CF})w_{ij}^{CF}\alpha}{2\alpha}.$$

The manufacturer maximizes her profit by choosing

$$w_{ij}^{CF*} = \frac{A_i + c\alpha(1 + r_i^F)}{2(1 + r_i^F)\alpha},$$

Substituting $w_{ij}^{CF*}$ back into the dealer's best response function $p_{ij}^{CF}$, we derive the equilibrium selling price

$$p_{ij}^{CF*} = \frac{3A_j + c(1 + r_i^F)\alpha}{4\alpha}.$$

Suppose $r_{ij}^{CF}$ is an exogenous variable; the firms' profits are given by:

$$\Pi(w_{ij}^{CF*}) = \frac{[A_i - c(1 + r_{ij}^{CF})\alpha]^2}{8(1 + r_{ij}^{CF})\alpha},$$

$$\pi(p_{ij}^{CF*}) = Br_{ij}^{CF} + \frac{(A_j - c(1 + r_{ij}^{CF})\alpha)^2}{16}.$$

■

*A.2. Proof of Lemma 2*

*Proof.* Using the backward induction method, the dealer maximizes his profit with the selling price:

$$p_{ij}^{TF} = \frac{A_j + (1 + r_{ij}^{TF})w_{ij}^{TF}\alpha}{2\alpha}.$$

The manufacturer maximizes her profit by choosing

$$w_{ij}^{TF} = \frac{A_i + c\alpha}{2(1 + \bar{r}_i^F)\alpha}.$$

Substituting $w_{ij}^{TF}$ back to the profit formula of the manufacturer, we derive the first-order derivative concerning interest rate $r_{ij}^{TF}$, which is non-positive for all feasible interest rate $\bar{r}_i^F$. Therefore, the optimal interest rate is

$$r_{ij}^{TF*} = 0.$$

Accordingly, the manufacturer sets wholesale price at:

$$w_{ij}^{TF*} = \frac{A_i + c\alpha}{2\alpha},$$

and the dealer's selling price is:

$$p_{ij}^{TF*} = \frac{1}{4}\left(c + \frac{3A_j}{\alpha}\right).$$

The profits are:

$$\pi(p_{ij}^{TF*}) = \frac{(A_j - c\alpha)^2}{16\alpha}, \quad \Pi(w_{ij}^{TF}, r_{ij}^{TF*}) = \frac{(A_i - c\alpha)^2}{8\alpha}.$$

■

*A.3. Proof of Proposition 1*

*Proof.* The low-demand manufacturer's price under symmetric information varies depending on the type of contracts. In the wholesale price contract, the price is denoted as $w_{ij}^{CF*}$, while in the trade credit contract, the price is represented by $w_{ij}^{TF*}$. We have:

$$w_{ij}^{CF*} < w_{ij}^{TF*}.$$

The dealer's unit cost differs based on the financing method under symmetric information. When using bank financing, the unit cost is calculated as $w_{ij}^{CF*}(1 + r_{ij}^{CF})$. In contrast, under seller financing, the unit cost is $w_{ij}^{TF*}$. We have:

$$w_{ij}^{CF*}(1 + r_{ij}^{CF}) > w_{ij}^{TF*}.$$

The dealer's selling price differs based on the financing method under symmetric information. When using bank financing, the selling price is $p_{ij}^{CF*}$. In contrast, under seller financing, the selling price is $p_{ij}^{TF*}$. We have:

$$p_{ij}^{CF*} > p_{ij}^{TF*}.$$

■

### A.4. Proof of Proposition2

*Proof.* The low-demand manufacturer's profit under symmetric information varies depending on the type of contracts. In the wholesale price contract, the profit is denoted as $\Pi(w_{ij}^{CF*})$, while in the trade credit contract, the profit is represented by $\Pi(w_{ij}^{TF}, r_{ij}^{TF*})$. We have:

$$\Pi(w_{ij}^{TF}, r_{ij}^{TF*}) > \Pi(w_{ij}^{CF*}).$$

The dealer's profit differs based on the financing method under symmetric information. When using bank financing, the profit is $\pi_r(p_{ij}^{CF*})$. In contrast, under seller financing, the profit is $\pi_r(p_{ij}^{TF*})$. When $B > \frac{1}{16}c[2A_L - c(2 + r_{ij}^{CF})\alpha]$, we have:

$$\pi(p_{ij}^{CF*}) > \pi(p_{ij}^{TF*}).$$

■

### A.5. Proof of Proposition3

*Proof.* To satisfy constraint $\Pi(w_{LL}^{CS}) \geq \Pi_m(w_{LH}^{CF*})$, we need:

$$\frac{-A_H + 2A_L + c\alpha(1 + r_{ij}^{CF})}{2\alpha(1 + r_{ij}^{CF})} < w_{LL}^{CS} < \frac{A_H + c\alpha(1 + r_{ij}^{CF})}{2\alpha(1 + r_{ij}^{CF})}.$$

The Lagrangian function of the above nonlinear programming problem can be constructed as follows:

$$H(w_{LL}^{CS}, \lambda) = \max_{w_{LL}^{CS}} (w_{LL}^{CS} - c)(A_L - \alpha p_{LL}^{CS}) - \lambda\left[(w_{LL}^{CS} - c)(A_H - \alpha p_{LL}^{CS}) - \frac{(A_H - c(1 + r_{ij}^{CF})\alpha)^2}{8(1 + r_{ij}^{CF})\alpha}\right].$$

The KKT conditions of the Lagrange function are:

$$\frac{\partial H(w_{LL}^{CS}, \lambda)}{\partial w_L^S} \leq 0; \quad w_L^S \frac{\partial H(w_{LL}^{CS}, \lambda)}{\partial w_L^S} = 0; \quad \frac{\partial H(w_{LL}^{CS}, \lambda)}{\partial \lambda} \geq 0; \quad \lambda \frac{\partial H(w_{LL}^{CS}, \lambda)}{\partial \lambda} = 0.$$

where $w_{LL}^{CS} > 0$, $\lambda \geq 0$.

If $\lambda = 0$, we have: $\frac{\partial H(w_{LL}^{CS}, \lambda)}{\partial w_L^S} = 0$, the manufacturer sets wholesale price at $w_1^{CS} = \frac{A_L + c\alpha(1 + r_{ij}^{CF})}{2(1 + r_{ij}^{CF})\alpha}$, substituting $w_1^{CS}$ back to the constraint $\Pi_m(w_{LL}^{CS}) \geq \Pi_m(w_{LH}^{CF*})$, then we derive: $\phi \geq 3$.

If $\lambda > 0$, we have: $\frac{\partial H(w_{LL}^{CS}, \lambda)}{\partial \lambda} = 0$, the manufacturer's optimal wholesale price

$$w_{LL}^{CS*} = \frac{A_H^2 + 6A_H c(1 + r_{ij}^{CF})\alpha - c(1 + r_{ij}^{CF})\alpha[6A_L + c(1 + r_{ij}^{CF})\alpha]}{2(1 + r_{ij}^{CF})\alpha[4A_H - 3A_L - c(1 + r_{ij}^{CF})\alpha]}.$$

The manufacturer can intuitively set

$$w_{LL}^{CS*} = \frac{A_H^2 + 6A_H c(1 + r_{ij}^{CF})\alpha - c(1 + r_{ij}^{CF})\alpha[6A_L + c(1 + r_{ij}^{CF})\alpha]}{2(1 + r_{ij}^{CF})\alpha[4A_H - 3A_L - c(1 + r_{ij}^{CF})\alpha]},$$

and the constraint is obviously satisfied, where $1 < \phi < 3$, that is, the relative variation in demand is small.

Therefore, the dealer's optimal selling price is:

$$p_{LL}^{CS*} = \frac{-A_H^2 - 8A_H A_L + 6A_L^2 - 6A_H c(1 + r_{ij}^{CF})\alpha + 8A_L c(1 + r_{ij}^{CF})\alpha + c^2(1 + r_{ij}^{CF})^2\alpha^2}{4\alpha[-4A_H + 3A_L + c(1 + r_{ij}^{CF})\alpha]}.$$

■

## *A.6. Proof of Proposition4*

*Proof.* The Lagrangian function of the above nonlinear programming problem can be constructed as follows:

$$H(w_{LL}^{TS}, r_{LL}^{TS}, \lambda) = \max_{w_{LL}^{TS}, r_{LL}^{TS}} (w_{LL}^{TS} - c)q + r_{LL}^{TS}(q - B) - \lambda\left[(w_{LL}^{TS} - c)(A_H - \alpha p_{LL}^{TS}) + r_{LL}^{TS}((A_H - \alpha p_{LL}^{TS}) - B) - \frac{(A_H - c\alpha)^2}{8\alpha}\right].$$

The KKT conditions of the Lagrange function are:

$$\frac{\partial H(w_{LL}^{TS}, r_{LL}^{TS}, \lambda)}{\partial w_{LL}^{TS}} \leq 0; \quad w_L^S \frac{\partial H(w_{LL}^{TS}, r_{LL}^{TS}, \lambda)}{\partial w_{LL}^{TS}} = 0;$$
$$\frac{\partial H(w_{LL}^{TS}, r_{LL}^{TS}, \lambda)}{\partial r_{LL}^{TS}} \leq 0; \quad r_{LL}^{TS} \frac{\partial H(w_{LL}^{TS}, r_{LL}^{TS}, \lambda)}{\partial r_{LL}^{TS}} = 0;$$
$$\frac{\partial H(w_{LL}^{TS}, r_{LL}^{TS}, \lambda)}{\partial \lambda} \geq 0; \quad \lambda \frac{\partial H(w_{LL}^{TS}, r_{LL}^{TS}, \lambda)}{\partial \lambda} = 0.$$

where $\bar{w}_L^S > 0$, $r_{LL}^{TS} > 0$, $\lambda \geq 0$.

If $\lambda = 0$, we have:

$$\frac{\partial H(w_{LL}^{TS}, r_{LL}^{TS}, \lambda)}{\partial \bar{w}_{LL}^{TS}} = 0 \quad \text{and} \quad \frac{\partial H(w_{LL}^{TS}, r_{LL}^{TS}, \lambda)}{\partial r_{LL}^{TS}} = 0,$$

the manufacturer's optimal wholesale price $w_1^{TS} = \dfrac{A_L + c\alpha}{2\alpha}$ and $r_1^{TS} = 0$, substituting $w_{LL}^{TS}$ back to the constraint $\Pi_m(w_{LL}^{TS}, r_{LL}^{TS}) \geq \Pi_m(w_{LH}^{TF*}, r_{LH}^{TF*})$, then we derive: $\tau \geq 3$.

If $\lambda > 0$, we have:

$$\frac{\partial H(w_{LL}^{TS}, r_{LL}^{TS}, \lambda)}{\partial \lambda} = 0,$$

the manufacturer's optimal wholesale price

$$w_{LL}^{TS*} = \frac{4B(A_L + c\alpha)}{8B\alpha - (A_H - A_L)(A_H - 3A_L + 2c\alpha)},$$

and

$$r_{LL}^{TS*} = \frac{-A_H^2 + 4A_H A_L - 3A_L^2 - 2A_H c\alpha + 2A_L c\alpha}{8B\alpha}.$$

The manufacturer can intuitively set

$$w_{LL}^{TS*} = \frac{4B(A_L + c\alpha)}{8B\alpha - (A_H - A_L)(A_H - 3A_L + 2c\alpha)},$$

and

$$r_{LL}^{TS*} = \frac{-A_H^2 + 4A_HA_L - 3A_L^2 - 2A_Hc\alpha + 2A_Lc\alpha}{8B\alpha},$$

and the constraint is obviously satisfied, where $1 < \tau < 3$, that is, the relative variation in demand is small. ■

### A.7. Proof of Proposition5

*Proof.* The low-demand manufacturer's price under asymmetric information varies depending on the type of contracts. In the wholesale price contract, the price is denoted as $w_{LL}^{CS*}$, while in the trade credit contract, the price is represented by $w_{LL}^{TS*}$. We let $r^* = \frac{9(A_H - A_L)}{7A_L - A_H}$. When $r^* < r_{ij}^{CF} < 1$ and $0 < B < T$, where

$$T = \frac{(A_H - A_L)(A_H - 3A_L + 2c\alpha)\{A_H^2 + 6A_Hc(1 + r_i^F)\alpha - c(1 + r_i^F)\alpha[6A_L + c(1 + r)\alpha]\}}{8\alpha[A_H^2 - 2A_H(1 + r_i^F)(2A_L - c\alpha) + A_L(1 + r_i^F)(3A_L + c(-2 + r_i^F)\alpha)]},$$

we have:

$$w_{LL}^{CS*} > w_{LL}^{TS*}.$$

The dealer's unit cost differs based on the financing method under asymmetric information. When using bank financing, the unit cost is calculated as $w_{LL}^{CS*}(1 + r_{ij}^{CF})$. In contrast, under seller financing, the unit cost is $w_{LL}^{TS*}(1 + r_{LL}^{TS*})$. When $r^* < r_{ij}^{CF} < 1$, we have:

$$w_{LL}^{CS*}(1 + r_{ij}^{CF}) > w_{LL}^{TS*}(1 + r_{LL}^{TS*}).$$

The dealer's selling price differs based on the financing method under symmetric information. When using bank financing, the selling price is $p_{LL}^{CS*}$. In contrast, under seller financing, the selling price is $p_{LL}^{TS*}$. When $r^* < r_{ij}^{CF} < 1$, we have:

$$p_{LL}^{CS*} > p_{LL}^{TS*}.$$

■

### A.8. Proof of Proposition6

*Proof.* The low-demand manufacturer's profit under asymmetric information varies depending on the type of contracts. In the wholesale price contract, the profit is denoted as $\Pi(w_{LL}^{CS*}, r_{ij}^{CF})$, while in the trade credit contract, the profit is represented by $\Pi(w_{LL}^{TS*}, r_{LL}^{TS*})$. When $r^* < r_{ij}^{CF} < 1$, we have:

$$\Pi(w_{LL}^{TS*}, r_{LL}^{TS*}) > \Pi(w_{LL}^{CS*}, r_{ij}^{CF}).$$

The dealer's profit differs based on the financing method under symmetric information. When using bank financing, the profit is $\pi(p_{LL}^{CS*})$. In contrast, under seller financing, the profit is $\pi(p_{LL}^{TS*})$.

When $\frac{16}{9}r^* < r_{ij}^{CF} < 1$ and $B > G$, where

$$G = \frac{K}{16r_{ij}^{CF}\alpha[-4A_H + 3A_L + c(1 + r_{ij}^{CF})\alpha]^2} - 2A_H^2\Big[183A_L^2 - 2A_L c(39 + 7r_{ij}^{CF})\alpha + 2c^2(-3 + 10r_{ij}^{CF} + 9(r_{ij}^{CF})^2)\alpha^2\Big] + 4A_H Q - S,$$

where

$$K = -33A_H^4 + A_L^3(18c(2 + r_{ij}^{CF})\alpha - 81A_L) - \alpha^2\Big(3A_L^2c^2(-4 + 2r_{ij}^{CF} + 3(r_{ij}^{CF})^2) - 2A_L c^3 r_{ij}^{CF}(7 + 11r_{ij}^{CF} + 4(r_{ij}^{CF})^2)\alpha\Big);$$
$$S = c^4 r_{ij}^{CF}(1 + r_{ij}^{CF})^2(2 + r_{ij}^{CF})\alpha^4 + 4A_H^3(48A_L + c(-15 + r_{ij}^{CF})\alpha);$$
$$Q = 72A_L^3 - 3A_L^2c(11 + 4r_{ij}^{CF})\alpha + 2A_L c^2(-3 + 5r_{ij}^{CF} + 5(r_{ij}^{CF})^2)\alpha^2 + c^3 r_{ij}^{CF}(5 + 8r_{ij}^{CF} + 3(r_{ij}^{CF})^2)\alpha^3.$$

We have:

$$\pi(p_{LL}^{CS*}) > \pi(p_{LL}^{TS*}).$$

■

### A.9. Proof of Proposition7

*Proof.* In the trade credit contracts $(w_{LL}^{TS*}, r_{LL}^{TS*})$, the low-demand manufacturer's profit is $\Pi(w_{LL}^{TS*}, r_{LL}^{TS*})$ under separating equilibrium. When comparing this profit to the scenario under symmetric information, we observe that $\Pi(w_{LL}^{TS*}, r_{LL}^{TS*}) - \Pi(w_{LL}^{TF*}, r_{LL}^{TF*}) < 0$.

In the trade credit contracts $(w_{LL}^{TS*}, r_{LL}^{TS*})$, the dealer's profit is $\pi(p_{LL}^{TS*})$ under separating equilibrium. When comparing this profit to the scenario under symmetric information, we observe that $\pi(p_{LL}^{TF*}) - \pi(p_{LL}^{TS*}) > 0$.

■