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Research article

How strategic consumers shape manufacturer refurbishment strategies under collection target

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Appendix A

Proof of Lemma 1

Because the manufacturer (*M*) needs to decides her/his collection and refurbishment strategies, nine scenarios may happen: (1) Collect all products and refurbish some of them; (2) collect all products but do not refurbish any; (3) collect all products and refurbish all of them; (4) collect enough products to meet the collection target and refurbish some of them; (5) collect enough products to meet the collection target but do not refurbish any; (6) collect enough products to meet the collection target and refurbish all of them; (7) collect as many products as possible and refurbish some of them; (8) collect as many products as possible but do not refurbish any; (9) collect as many products as possible and refurbish all of them. For simplicity, we use *C* to represent the manufacturer's collection strategy and *R* to denote the manufacturer's refurbishment strategy. Moreover, we use C_iR_j, where $i, j \in \{0,+,=\}$ denote the manufacturer's collection and refurbishment strategies, as shown in Table A1.

Table A1. The manufacturer's collection and refurbishment strategies.

Manufacturer's strategy		Collection		
		0	+	=
Refurbishment	0	C_0R_0	C_+R_0	$C_=R_0$
	+	C_0R_+	C_+R_+	$C_=R_+$
	=	$C_0R_=$	$C_+R_=$	$C_=R_=$

$$M_2 = (p_{n2} - c_n)q_{n2} + (p_{r2} - c_r)q_{r2} - k_c q_c^2 / 2$$
$$s.t. \begin{cases} \beta_c q_{n1} \le q_c \le q_{n1} \\ q_{r2} \le q_c \\ q_c \ge 0, q_{r2} \ge 0 \end{cases} \tag{1}$$

$$M_1 = (p_{n1} - c_n)q_{n1} + M_2 \tag{2}$$

In order to analyze the problem in each scenario, we first consider the Lagrangian for the manufacturer's problem (10) shown below:

$$L_2 = M_2 + \eta_1(q_{n1} - q_c) + \eta_2(q_c - \beta_c q_{n1}) + \eta_3(q_c - q_{r2}) + \eta_4 q_c + \eta_5 q_{r2}, \tag{3}$$

where $\eta_i (i = 1, 2 \ldots 5)$ are Lagrangian multipliers. The first-order conditions are then as follows: $\partial L_2 / \partial q_{n2} = \bar{v} - c_n - 2q_{n2} - 2q_{r2}\alpha$, $\partial L_2 / \partial q_{r2} = -c_r - q_{n2}\alpha - q_{r2}\alpha - \eta_3 + \eta_5 - \alpha(q_{n2} + q_{r2} - \bar{v})$, $\partial L_2 / \partial q_c = -k_c q_c - \eta_1 + \eta_2 + \eta_3 + \eta_4$, $\partial L_2 / \partial \eta_1 = q_1 - q_c$, $\partial L_2 / \partial \eta_2 = q_c - \beta_c q_{n1}$, $\partial L_2 / \partial \eta_3 = q_c - q_{r2}$, $\partial L_2 / \partial \eta_4 = q_c$, $\partial L_2 / \partial \eta_5 = q_{r2}$.
In addition, the complimentary slackness conditions $\eta_1(q_{n1} - q_c) = 0$, $\eta_2(q_c - \beta_c q_{n1}) = 0$, $\eta_3(q_c - q_{r2}) = 0$, $\eta_4 q_c = 0$, and $\eta_5 q_{r2} = 0$, as well as the feasibility conditions $q_{n2} \ge 0$, $q_c \ge 0$, $q_{r2} \ge 0$, $\eta_1 \ge 0$, $\eta_2 \ge 0$, $\eta_3 \ge 0$, $\eta_4 \ge 0$, and $\eta_5 \ge 0$ should hold.

Proof of Proposition 1

Scenario *C₀R₀* ($q_c = \beta_c q_{n1}$, $q_{r2} = 0$)

In this scenario, the manufacturer sells the new products in the first period and then collects enough products to meet the collection target in the second period without refurbishment. Therefore, we use the first-order conditions $\partial L_2 / \partial q_{n2} = \bar{v} - c_n - 2q_{n2} - 2q_{r2}\alpha$, $\partial L_2 / \partial q_c = -k_c q_c - \eta_1 + \eta_2 + \eta_3 + \eta_4$, and $\partial L_2 / \partial \eta_2 = q_c - \beta_c q_{n1}$.

Solving the optimal solutions in the second period, we get $q_{n2} = (\bar{v} - c_n)/2$, $q_c = \beta_c q_{n1}$, and $\eta_2 = k_c q_{n1} \beta_c$. The optimal pricing decisions in the second period can be written as $p_{n2}^* = (c_n - \bar{v})/2$. We put p_{n2}^* back into $\bar{v} = (p_{n1} - \delta p_{n2}^*)/(1-\delta)$ and we have $\bar{v} = (2p_{n1} - c_n\delta)/(2-\delta)$.

Table A2. The equilibrium for the C_0R_0 scenario.

Parameters	Value	Parameters	Value
q_{n1}	$(1-c_n)/2$	M_1	$(1-c_n)^2(5-2k_c\beta_c^2+2\delta)/16$
q_{n2}	$(1-c_n)/4$	M_2	$(1-c_n)^2(1-2k_c\beta_c^2)/16$
q_c	$\beta_c(1-c_n)/2$	η_1	0
q_{r2}	0	η_2	$k_c\beta_c(1-c_n)/2$
p_{n1}	$(2-\delta+c_n(2+\delta))/4$	η_3	0
p_{n2}	$(1+3c_n)/4$	η_4	0
p_{r2}	0	η_5	-

When the demand in the first period is $q_{n1}=1-\overline{v}$, the optimal price of the new products in the first period should be $p_{n1}=1-q_{n1}+\delta(c_n+q_{n1}-1)/2$. We put p_{n1} back into the manufacturer's problem (P1) and set $\partial M_1/\partial q_{n1}=0$. The optimal decision for the new product in the first period is $q_{n1}^*=(1-c_n)/2$. We present the equilibrium for the *C0R0* scenario in Table A2.

However, $q_{n1}>0$, $q_{n2}>0$, $q_c>0$, $\eta_2>0$, $M_1>0$, and $M_2>0$ should hold. To sum up these conditions, we have $\beta_c \le 1/\sqrt{2k_c}$.

Scenario *C0R+* ($q_c=\beta_c q_{n1}$, $0<q_{r2}<q_c$)

In this scenario, the manufacturer collects enough products to meet the collection target in the second period with some refurbishment. Therefore, we use the first-order conditions $\partial L_2/\partial q_{n2}=\overline{v}-c_n-2q_{n2}-2q_{r2}\alpha$, $\partial L_2/\partial q_c=-k_c q_c-\eta_1+\eta_2+\eta_3+\eta_4$, $\partial L_2/\partial \eta_2=q_c-\beta_c q_{n1}$, and $\partial L_2/\partial q_{r2}=-c_r-q_{n2}\alpha-q_{r2}\alpha-\eta_3+\eta_5-\alpha(q_{n2}+q_{r2}-\overline{v})$.

Solving the optimal solutions in the second period, we get $q_{n2}=(c_r+(1-\alpha)\overline{v}-c_n)/2(1-\alpha)$, $q_c=\beta_c q_{n1}$, $q_{r2}=(c_n\alpha-c_r)/(2\alpha(1-\alpha))$, and $\eta_2=k_c q_{n1}\beta_c$. The optimal pricing decisions in the second period can be written as $p_{n2}^*=(c_n+\overline{v})/2$ and $p_{r2}^*=(c_r+\alpha\overline{v})/2$. We put p_{n2}^* back into $\overline{v}=(p_{n1}-\delta p_{n2}^*)/(1-\delta)$ and we have $\overline{v}=(2p_{n1}-c_n\delta)/(2-\delta)$.

When the demand in the first period is $q_{n1}=1-\overline{v}$, the optimal price of the new products in the first period should be $p_{n1}=1-q_{n1}+\delta(c_n+q_{n1}-1)/2$. We put p_{n1} back into the manufacturer's problem (P1) and set $\partial M_1/\partial q_{n1}=0$. The optimal decision for the new product in the first period is $q_{n1}^*=(1-c_n)(1-\delta)/(3+2k_c\beta_c^2-2\delta)$. We present the equilibrium for the *C0R+* scenario in Table A3.

Table A3. The equilibrium for the *C0R+* scenario.

Parameters	Value	Parameters	Value
q_{n1}	$\frac{(1-c_n)(1-\delta)}{F_1}$	M_1	$\frac{(1-c_n)^2(2-\delta)(1-\delta)(F_1-1+\delta)}{2F_1^2}$
q_{n2}	$\frac{1}{4}\left(F_2+\frac{(1-c_n)(1+2k_c\beta_c^2)}{F_1}\right)$	M_2	$\frac{c_r^2F_1^2-2c_nc_r\alpha F_1^2+\alpha(F_4+c_n^2F_5)}{4\alpha(1-\alpha)F_1^2}$
q_c	$\frac{(1-c_n)(1-\delta)\beta_c}{F_1}$	η_1	0
q_{r2}	$\frac{c_n\alpha-c_r}{2\alpha(1-\alpha)}$	η_2	$\frac{(1-c_n)(1-\delta)\beta_c k_c}{F_1}$
p_{n1}	$\frac{(F_1-1-\delta)(2-\delta)+c_nF_3}{2F_1}$	η_3	0
p_{n2}	$\frac{F_1+c_n(F_1+1-\delta)-1-\delta}{2F_1}$	η_4	0
p_{r2}	$\frac{c_rF_1+\alpha(c_n+F_1-1-(c_n-1)\delta)}{2F_1}$	η_5	0

Here, $F_1 = 3 + 2k_c\beta_c^2 - 2\delta$, $F_2 = 1 + c_n - 2(c_n - c_r)/(1-\alpha)$, $F_3 = 2 + 2k_c\beta_c^2\delta - \delta^2$, $F_4 = (1-\alpha)(1-2c_n)((2-\delta)^2 + 2k_c\beta_c^2(3+2k_c\beta_c^2 - \delta^2))$, $F_5 = (2-\delta)^2 + \alpha(1-\delta)(5+2k_c\beta_c^2(3-\delta)+3\delta) + 2k_c\beta_c^2(3+2k_c\beta_c^2-\delta^2)$.

However, $q_{n1} > 0$, $q_{n2} > 0$, $q_c > 0$, $q_{r2} > 0$, $\eta_2 > 0$, $M_1 > 0$, and $M_2 > 0$ should hold. To sum up these conditions, we have (*i*) $\frac{c_r}{c_n} < \alpha \le \frac{2+3c_r-2c_n}{2+c_n}$; (*ii*) $\frac{2+3c_r-2c_n}{2+c_n} < \alpha < 1 + c_r - c_n$ and $\sqrt{\frac{2+3c_r-2\alpha-c_n(2+\alpha)}{2k_c(\alpha+c_n-1-c_r)}} \le \beta_c \le 1$; and (*iii*) $\frac{2+3c_r-2c_n}{2+c_n} < \alpha < 1 + c_r - c_n$, $0 < \beta_c < \sqrt{\frac{2+3c_r-2\alpha-c_n(2+\alpha)}{2k_c(\alpha+c_n-1-c_r)}}$, and $\delta > \delta_1$, where $\delta_1 = \frac{2-2c_n+3c_r-2\alpha-c_n\alpha+2k_c\left(1-c_n+c_r-\alpha\right)\beta_c^2}{1-c_n+2c_r-\alpha-c_n\alpha}$.

Scenario *C₀R₌* **(**$q_c = \beta_c q_{n1}$**,** $q_{r2} = q_c$**)**

In this scenario, the manufacturer collects enough products to meet the collection target in the second period with refurbishment of all. Therefore, we use the first-order conditions $\partial L_2/\partial q_{n2} = \bar{v} - c_n - 2q_{n2} - 2q_{r2}\alpha$, $\partial L_2/\partial q_c = -k_c q_c - \eta_1 + \eta_2 + \eta_3 + \eta_4$, $\partial L_2/\partial \eta_2 = q_c - \beta_c q_{n1}$, $\partial L_2/\partial q_{r2} = -c_r - q_{n2}\alpha - q_{r2}\alpha - \eta_3 + \eta_5 - \alpha(q_{n2} + q_{r2} - \bar{v})$, and $\partial L_2/\partial \eta_3 = q_c - q_{r2}$.

Solving the optimal solutions in the second period, we get $q_{n2} = (\bar{v} - c_n - 2q_{n1}\alpha\beta_c)/2$, $q_c = \beta_c q_{n1}$, $q_{r2} = q_{n1}\beta_c$, $\eta_2 = c_r - c_n\alpha + q_{n1}\beta_c(k_c + 2\alpha(1-\alpha))$, and $\eta_3 = \alpha(c_n - 2q_{n1}\beta_c(1-\alpha)) - c_r$. The optimal pricing decisions in the second period can be written as $p_{n2}^* = (c_n + \bar{v})/2$ and $p_{r2}^* = \alpha(c_n - 2q_{n1}\beta_c(1-\alpha) + \bar{v})/2$. We put p_{n2}^* back into $\bar{v} = (p_{n1} - \delta p_{n2}^*)/(1-\delta)$ and we have $\bar{v} = (2p_{n1} - c_n\delta)/(2-\delta)$.

when the demand in the first period is $q_{n1} = 1 - \bar{v}$, the optimal price of the new products in the first period should be $p_{n1} = 1 - q_{n1} + \delta(c_n + q_{n1} - 1)/2$. We put p_{n1} back into the manufacturer's problem (P1) and set $\partial M_1/\partial q_{n1} = 0$. The optimal decision for the new product in the first period is $q_{n1}^* = (1-c_n)/2$. We present the equilibrium for the *C₀R₌* scenario in Table A4.

Table A4. The equilibrium for the *C₀R₌* scenario.

Parameters	Value	Parameters	Value
q_{n1}	$(1-c_n)/2$	M_1	$\frac{(1-c_n)(F_6 - c_n(5-2\beta_c F_7 - 2\delta) + 2\delta)}{16}$
q_{n2}	$(1-c_n)(1-2\alpha\beta_c)/4$	M_2	$\frac{(1-c_n)(F_8 - 2(1-c_n)F_9)}{16}$
q_c	$(1-c_n)\beta_c/2$	η_1	0
q_{r2}	$(1-c_n)\beta_c/2$	η_2	$c_r - c_n\alpha + \beta_c(1-c_n)(k_c + 2\alpha(1-\alpha))/2$
p_{n1}	$(2-\delta+c_n(2+\delta))/4$	η_3	$\alpha(c_n - (1-c_n)(1-\alpha)\beta_c) - c_r$
p_{n2}	$(1+3c_n)/4$	η_4	0
p_{r2}	$\alpha(1+3c_n - 2\beta_c(1-c_n)(1-\alpha))/4$	η_5	0

Here, $F_6 = 5 - 8c_r\beta_c - 2\beta_c^2(k_c + 2\alpha(1-\alpha))$, $F_7 = k_c\beta_c + 2\alpha(2 + \beta_c(1-\alpha))$, and $F_8 = 1 - c_n + 8\beta_c(c_n\alpha - c_r)$, $F_9 = k_c + 2\alpha\beta_c^2(1-\alpha)$.

However, $q_{n1} > 0$, $q_{n2} > 0$, $q_c > 0$, $q_{r2} > 0$, $\eta_2 > 0$, $M_1 > 0$, and $M_2 > 0$ should hold. To sum up these conditions, we have $\beta_c > \dfrac{2(c_n\alpha - c_r)}{(1-c_n)(k_c + 2\alpha(1-\alpha))}$, $0 < c_r < \alpha$, and either (*i*) $0 < \alpha \le 1/2$ and $\beta_c < \dfrac{\alpha - c_r - \alpha(1-c_n)}{\alpha(1-c_n)(1-\alpha)}$, or (*ii*) $1/2 < \alpha < 1$ and $\beta_c \le 1/(2\alpha)$.

Proof of Proposition 2

Scenario *C=R0* ($q_c = q_{n1}$, $q_{r2} = 0$)

In this scenario, the manufacturer collects all end-of-life products in the second period without refurbishment. Therefore, we use the first-order conditions $\partial L_2 / \partial q_{n2} = \overline{v} - c_n - 2q_{n2} - 2q_{r2}\alpha$, $\partial L_2 / \partial q_c = -k_c q_c - \eta_1 + \eta_2 + \eta_3 + \eta_4$, $\partial L_2 / \partial q_{r2} = -c_r - q_{n2}\alpha - q_{r2}\alpha - \eta_3 + \eta_5 - \alpha(q_{n2} + q_{r2} - \overline{v})$, and $\partial L_2 / \partial \eta_1 = q_{n1} - q_c$, $\partial L_2 / \partial \eta_5 = q_{r2}$.

Solving the optimal solutions in the second period, we get $q_{n2} = (\overline{v} - c_n)/2$, $q_c = q_{n1}$, $q_{r2} = 0$, $\eta_1 = -k_c q_{n1}$, $\eta_5 = c_r - c_n\alpha$. We find that $\eta_1 = -k_c q_{n1} < 0$, which does not satisfy the former condition $\eta_1 > 0$. Therefore, the scenario *C=R0* will not exist.

Scenario *C=R+* ($q_c = q_{n1}$, $0 < q_{r2} < q_c$)

In this scenario, the manufacturer collects all end-of-life products in the second period with some refurbishment. Therefore, we use the first-order conditions $\partial L_2 / \partial q_{n2} = \overline{v} - c_n - 2q_{n2} - 2q_{r2}\alpha$, $\partial L_2 / \partial q_c = -k_c q_c - \eta_1 + \eta_2 + \eta_3 + \eta_4$, $\partial L_2 / \partial q_{r2} = -c_r - q_{n2}\alpha - q_{r2}\alpha - \eta_3 + \eta_5 - \alpha(q_{n2} + q_{r2} - \overline{v})$, and $\partial L_2 / \partial \eta_1 = q_{n1} - q_c$.

Solving the optimal solutions in the second period, we get $q_{n2} = (c_n - c_r + (1-\alpha)\overline{v})/(2(\alpha - 1))$, $q_c = q_{n1}$, $q_{r2} = (c_r - c_n\alpha)/(2\alpha(\alpha-1))$, and $\eta_1 = -k_c q_{n1}$. We find that $\eta_1 = -k_c q_{n1} < 0$, which does not satisfy the former condition $\eta_1 > 0$. Therefore, the scenario *C=R+* will not exist.

Scenario *C=R=* ($q_c = q_{n1}$, $q_{r2} = q_c$)

In this scenario, the manufacturer collects all end-of-life products in the second period with refurbishment of all. Therefore, we use the first-order conditions $\partial L_2 / \partial q_{n2} = \overline{v} - c_n - 2q_{n2} - 2q_{r2}\alpha$, $\partial L_2 / \partial q_c = -k_c q_c - \eta_1 + \eta_2 + \eta_3 + \eta_4$, $\partial L_2 / \partial q_{r2} = -c_r - q_{n2}\alpha - q_{r2}\alpha - \eta_3 + \eta_5 - \alpha(q_{n2} + q_{r2} - \overline{v})$, and $\partial L_2 / \partial \eta_1 = q_{n1} - q_c$, $\partial L_2 / \partial \eta_3 = q_c - q_{r2}$.

Solving the optimal solutions in the second period, we can get $q_{n2} = (\overline{v} - c_n - 2q_{n1}\alpha)/2$, $q_c = q_{n1}$, $q_{r2} = q_{n1}$, $\eta_1 = \alpha(c_n - 2q_{n1}(1-\alpha)) - c_r - k_c q_{n1}$, and $\eta_3 = \alpha(c_n - 2q_{n1}(1-\alpha)) - c_r$. The optimal pricing decisions in the second period can be written as $p_{n2}^* = (c_n + \overline{v})/2$ and $p_{r2}^* = \alpha(c_n + \overline{v} - 2q_{n1}(1-\alpha))/2$. We put p_{n2}^* back into $\overline{v} = (p_{n1} - \delta p_{n2}^*)/(1-\delta)$ and we have $\overline{v} = (2p_{n1} - c_n\delta)/(2-\delta)$.

When the demand in the first period is $q_{n1} = 1 - \overline{v}$, the optimal price of the new products in the first period should be $p_{n1} = 1 - q_{n1} + \delta(c_n + q_{n1} - 1)/2$. We put p_{n1} back into the manufacturer's problem (P1) and set $\partial M_1 / \partial q_{n1} = 0$. The optimal decision for the new product in the first period is

$q_{n1}^{*}=(1-2c_r-\delta+c_n(2\alpha+\delta-1))/(3+2k_c+4\alpha(1-\alpha)-2\delta)$. We present the equilibrium for *C*₌*R*₌ scenario in Table A5.

Table A5. The equilibrium for the *C*₌*R*₌ scenario.

Parameters	Value	Parameters	Value
q_{n1}	$\frac{1-2c_r-\delta+c_n(2\alpha+\delta-1)}{3+2k_c+4\alpha(1-\alpha)-2\delta}$	M_1	$\frac{(\delta-2)\,(-2\,F_{16}+c_n\,F_{21}+\delta)\,F_{22}}{2\,(3+2k_c\,+4\,(1-\alpha)\,\alpha-2\,\delta)^2}$
q_{n2}	$\frac{2(1+c_r+k_c+\alpha+F_{14})-F_{15}}{6+4k_c+8\alpha(1-\alpha)-4\delta}$	M_2	$\frac{F_{23}+F_{25}c_n^2+2c_n(2F_{26}+F_{29})}{4(3+2k_c+4\alpha(1-\alpha)-2\delta)^2}$
	$\frac{1-2c_r-\delta+c_n(2\alpha+\delta-1)}{3+2k_c+4\alpha(1-\alpha)-2\delta}$	η_1	$\frac{c_r(2\delta-3)+F_{30}}{3+2k_c+4(1-\alpha)-2\delta}$
q_{r2}	$\frac{1-2c_r-\delta+c_n(2\alpha+\delta-1)}{3+2k_c+4\alpha(1-\alpha)-2\delta}$	η_2	0
p_{n1}	$\frac{(2-\delta)(\,2\,F_{16}-\delta)+c_nF_{17}}{6+4k_c+8\alpha(1-\alpha)-4\delta}$	η_3	$\frac{F_{31}-c_r(3+2k_c-2\delta)}{3+2k_c+4\alpha(1-\alpha)-2\delta}$
p_{n2}	$\frac{2(F_{16}+c_nF_{18})-(1+3c_n)\delta}{6+4k_c+8\alpha(1-\alpha)-4\delta}$	η_4	0
p_{r2}	$\frac{\alpha(6c_r+2k_c+6\alpha+c_nF_{19}+F_{20})}{6+4k_c+8\alpha(1-\alpha)-4\delta}$	η_5	0

where $F_{14}=2\alpha(c_r-\alpha)-c_n(1+k_c+2\alpha)$, $F_{15}=\delta(1-c_n)(1-2\alpha)$, $F_{16}=1+c_r+k_c+2\,\alpha(1-\alpha)$, $F_{17}=2-\alpha(4-6\delta)+2k_c\delta-4\delta\alpha^2-\delta^2$, $F_{18}=2+k_c+\alpha-2\alpha^2$, $F_{19}=6+2k_c-2\alpha(2-\delta)-5\delta$, $F_{20}=\delta-2\alpha(2(c_r+\alpha)+\delta)$, $F_{21}=2+2\,k_c+6\,\alpha-4\,\alpha^2-\delta$, $F_{22}=1-2\,c_r-\delta-c_n\,(1-2\,\alpha-\delta)$, $F_{23}=2(2+3kc+6\alpha+2F_{24})-4\delta(1-2c_r)^2+\delta^2((1-2\alpha)^2-8c_r-2k_c)$, $F_{24}=k_c^2+c_r(2k_c-(1-2\alpha)^2)+4k_c\alpha(1-\alpha)+\alpha^2(1-4\alpha(2-\alpha))+c_r^2(7+2k_c+4\alpha(1-\alpha))$, $F_{25}=4+4k_c^2+k_c(6+8\alpha(3-\alpha))+8\alpha(1+6\alpha-4\alpha^2)-4\delta(1-2\alpha)^2+\delta^2(1-2k_c-4\alpha(3-\alpha))$, $F_{26}=F_{27}-2\alpha^2(3+2c_r-4k_c)+4\alpha^3(5+2c_r)-8\alpha^4$, $F_{27}=c_r-2-3k_c-2c_rk_c-2k_c^2-F_{28}$, $F_{28}=\alpha(5+10k_c+2c_r(9+2k_c))$, $F_{29}=4\delta(1-2c_r)(1-2\alpha)+\delta^2(4c_r+2k_c-1+4\alpha(2-\alpha))$, $F_{30}=k_c(c_n+\delta-1-c_n\delta)+\alpha(c_n(5-2\alpha-2\delta(2-\alpha))-2(1-\alpha-\delta+\alpha\delta))$, and $F_{31}=\alpha(c_n(5+2k_c-2\alpha-2\delta(2-\alpha))-2(1-\alpha-\delta+\alpha\delta))$.

However, $q_{n1}>0$, $q_{n2}>0$, $q_c>0$, $q_{r2}>0$, $\eta_2>0$, $M_1>0$, and $M_2>0$ should hold. To sum up these conditions, we have (i) $\frac{2-c_n(5+2k_c)}{2(1-c_n)}<\alpha<1$ and $0<c_r\le\frac{\alpha(c_n(5+2k_c-2\alpha)-2(1-\alpha))}{3+2k_c}$; (ii) $\frac{2-c_n(5+2k_c)}{2(1-c_n)}<\alpha<1$, $\frac{\alpha(c_n(5+2k_c-2\alpha)-2(1-\alpha))}{3+2k_c}<c_r<c_n\alpha$, and $\delta>1-\frac{(1+2k_c)(c_n\alpha-c_r)}{2(c_r+\alpha(1-c_n(2-\alpha)-\alpha))}$; and (iii) $c_r<c_n\alpha$, $c_n\le\frac{2(1-\alpha)}{5+2k_c-2\alpha}$, and $\delta>1-\frac{(1+2k_c)(c_n\alpha-c_r)}{2(c_r+\alpha(1-c_n(2-\alpha)-\alpha))}$.

Scenario *C*₊*R₀* ($\beta_c q_{n1}<q_c<q_{n1}$, $q_{r2}=0$)

In this scenario, the manufacturer collects as many products as possible in the second period without refurbishment. Therefore, we use the first-order conditions $\partial L_2/\partial q_{n2}=\overline{v}-c_n-2q_{n2}-2q_{r2}\alpha$,

$\partial L_2/\partial q_c = -k_c q_c - \eta_1 + \eta_2 + \eta_3 + \eta_4$, $\partial L_2/\partial q_{r2} = -c_r - q_{n2}\alpha - q_{r2}\alpha - \eta_3 + \eta_5 - \alpha(q_{n2} + q_{r2} - \bar{v})$, and $\partial L_2/\partial \eta_5 = q_{r2}$.

Solving the optimal solutions in the second period, we get $q_{n2} = (\bar{v} - c_n)/2$, $q_c = 0$, $q_{r2} = 0$, and $\eta_5 = c_r - c_n\alpha$. However, we find that $q_c = 0$ does not satisfy the condition $\beta_c q_{n1} < q_c < q_{n1}$, which indicates the C_+R_0 scenario will not exist.

Scenario C_+R_+ ($\beta_c q_{n1} < q_c < q_{n1}$, $0 < q_{r2} < q_c$)

In this scenario, the manufacturer collects as many products as possible in the second period and refurbishes some. Therefore, we use the first-order conditions $\partial L_2/\partial q_{n2} = \bar{v} - c_n - 2q_{n2} - 2q_{r2}\alpha$, $\partial L_2/\partial q_c = -k_c q_c - \eta_1 + \eta_2 + \eta_3 + \eta_4$, and $\partial L_2/\partial q_{r2} = -c_r - q_{n2}\alpha - q_{r2}\alpha - \eta_3 + \eta_5 - \alpha(q_{n2} + q_{r2} - \bar{v})$.

Solving the optimal solutions in the second period, we get $q_{n2} = (c_r - c_n + (1-\alpha)\bar{v})/(2(1-\alpha))$, $q_c = 0$, and $q_{r2} = (c_n\alpha - c_r)/(2\alpha(1-\alpha))$. However, we find that $q_c = 0$ does not satisfy the condition $\beta_c q_{n1} < q_c < q_{n1}$, which indicates the C_+R_+ scenario will not exist.

Scenario $C_+R_=$ ($\beta_c q_{n1} < q_c < q_{n1}$, $q_{r2} = q_c$)

In this scenario, the manufacturer collects as many products as possible in the second period and refurbishes all of them. Therefore, we use the first-order conditions $\partial L_2/\partial q_{n2} = \bar{v} - c_n - 2q_{n2} - 2q_{r2}\alpha$, $\partial L_2/\partial q_c = -k_c q_c - \eta_1 + \eta_2 + \eta_3 + \eta_4$, $\partial L_2/\partial q_{r2} = -c_r - q_{n2}\alpha - q_{r2}\alpha - \eta_3 + \eta_5 - \alpha(q_{n2} + q_{r2} - \bar{v})$, and $\partial L_2/\partial \eta_3 = q_c - q_{r2}$.

Solving the optimal solutions in the second period, we get $q_{n2} = (2c_r\alpha - c_n(k_c + 2\alpha) + \bar{v}(k_c + 2\alpha(1-\alpha)))/(2(k_c + 2\alpha(1-\alpha)))$, $q_c = (c_n\alpha - c_r)/(k_c + 2\alpha(1-\alpha))$, $q_{r2} = (c_n\alpha - c_r)/(k_c + 2\alpha(1-\alpha))$, and $\eta_3 = k_c(c_n\alpha - c_r)/(k_c + 2\alpha(1-\alpha))$. The optimal pricing decisions in the second period can be written as $p_{n2}^* = (c_n + \bar{v})/2$ and $p_{r2}^* = \alpha(c_n k_c + 2c_r(1-\alpha) + \bar{v}(k_c + 2\alpha(1-\alpha)))/(2(k_c + 2\alpha(1-\alpha)))$. We put p_{n2}^* back into $\bar{v} = (p_{n1} - \delta p_{n2}^*)/(1-\delta)$ and we have $\bar{v} = (2p_{n1} - c_n\delta)/(2-\delta)$.

Table A6. The equilibrium for the $C_+R_=$ scenario.

Parameters	Value	Parameters	Value
q_{n1}	$(1-c_n)/2$	M_1	$\frac{8c_r^2 - 16c_n c_r\alpha + (k_c + 2\alpha(1-\alpha))F_{11}}{16(k_c + 2\alpha(1-\alpha))}$
q_{n2}	$\frac{k_c(1-c_n) - 2\alpha(c_n\alpha + \alpha - 2c_r - 1)}{4(k_c + 2\alpha(1-\alpha))}$	M_2	$\frac{(k_c + 2\alpha)(1-c_n)^2 + 2F_{12}F_{13}}{16(k_c + 2\alpha(1-\alpha))}$
q_c	$\frac{c_n\alpha - c_r}{k_c + 2\alpha(1-\alpha)}$	η_1	0
q_{r2}	$\frac{c_n\alpha - c_r}{k_c + 2\alpha(1-\alpha)}$	η_2	0
p_{n1}	$(2 - \delta + c_n(2+\delta))/4$	η_3	$\frac{k_c(c_n\alpha - c_r)}{k_c + 2\alpha(1-\alpha)}$
p_{n2}	$(1 + 3c_n)/4$	η_4	0
p_{r2}	$\frac{\alpha(F_{10} + 2\alpha(1-\alpha)(1+c_n))}{4(k_c + 2\alpha(1-\alpha))}$	η_5	0

When the demand in the first period is $q_{n1} = 1 - \bar{v}$, the optimal price of the new products in the first period should be $p_{n1} = 1 - q_{n1} + \delta(c_n + q_{n1} - 1)/2$. We put p_{n1} back into the manufacturer's problem (P1) and set $\partial M_1 / \partial q_{n1} = 0$. The optimal decision for the new product in the first period is $q_{n1}^* = (1-c_n)/2$. We show the equilibrium for $C_+R_=$ scenario in Table A6.

Here, $F_{10} = k_c + 3c_n k_c + 4c_r(1-\alpha)$, $F_{11} = (5-2\delta)(1-c_n)^2$, $F_{12} = 2c_r + \alpha - 3c_n\alpha$, and $F_{13} = 2c_r - \alpha(1+c_n)$.

However, $q_{n1} > 0$, $q_{n2} > 0$, $q_c > 0$, $q_{r2} > 0$, $\eta_2 > 0$, $M_1 > 0$, and $M_2 > 0$ should hold. To sum up these conditions, we have $0 < c_r < \alpha < 1$, $0 < \beta_c < 1$, $0 < \delta < 1$, and (i) $\frac{c_r}{c_n} < \alpha \le \frac{2(1+c_r)}{1+c_n} - 1$; or (ii) $\frac{2(1+c_r)}{1+c_n} - 1 < \alpha$ and $2\alpha(c_n - 1 - 2c_r + \alpha + c_n\alpha) < k(1-c_n)$.

Appendix B

Proof of Proposition 3

According to the equilibria from Tables A2, A3, and A4, we can obtain the first-order partial derivatives of the relevant parameters with respect to the consumers' willingness and the collection target, as shown in Table B1.

Table B1. The first-order partial derivatives of the relevant parameters.

Parameters	Parameters	C_0R_0	C_0R_+	$C_0R_=$
p_{n1}	δ	<0	<0	<0
	β_c	0	>0	0
p_{n2}	δ	0	>0	0
	β_c	0	>0	0
p_{r2}	δ	0	>0	0
	β_c	0	>0	<0
q_{n1}	δ	0	<0	0
	β_c	0	<0	0
q_{n2}	δ	0	>0	0
	β_c	0	>0	<0
q_c	δ	0	<0	0
	β_c	>0	Depends	>0
q_{r2}	δ	0	0	0
	β_c	0	0	>0

We get the following first-order condition for q_c in the C_0R_+ scenario:

$$\frac{\partial q_c}{\partial \beta_c} = \frac{(1-c_n)(1-\delta)(3-2k_c\beta_c^2-2\delta)}{(3+2k_c\beta_c^2-2\delta)^2}$$

We then know that $\frac{\partial q_c}{\partial \beta_c} < 0$ while $\delta < \frac{3-2k_c}{2}$ and $\sqrt{\frac{3-2\delta}{2k_c}} < \beta_c < 1$; in other cases, $\frac{\partial q_c}{\partial \beta_c} > 0$.

Proof of Proposition 4

We now analyze the profits gained by the manufacturer within the different periods and explore how the consumer's willingness to wait affects profits in each period. According to Corollary 2, under the C_+ and $C_=$ strategies, we analyze the $R_=$ case only. Our results are shown in Table B2.

Table B2. The effects of the consumer's willingness to wait on the manufacturer's profits.

Scenario	Profits	Effects of δ
C_0R_0	M_1	<0
	M_2	—
C_0R_+	M_1	<0
	M_2	>0
$C_0R_=$	M_1	<0
	M_2	—
$C_+R_=$	M_1	<0
	M_2	—
$C_=R_=$	M_1	<0
	M_2	>0

Therefore, we find that in the C_0R_+ and $C_=R_=$ scenarios only, the manufacturer's profit will increase in δ; however, in the first period, the manufacturer's profit always decreases in δ.

Proof of Proposition 5

Using the results in Table A4 and A6, we have

$$M_2^{<C_+R_=>} - M_2^{<C_0R_=>} = \frac{\left(2c_n\alpha - 2c_r - \beta_c(1-c_n)(k_c + 2\alpha(1-\alpha))\right)^2}{8(k_c + 2\alpha(1-\alpha))} > 0 .$$

From the formulation above, we have the conclusion that the collection strategy could affect the manufacturer's profits in the second period, and the more items collected, the better.

$$M_2^{<C_0R_=>} - M_2^{<C_0R_0>} = -\frac{1}{4}\beta_c(1-c_n)(2c_r - 2c_n\alpha + (1-c_n)(1-\alpha)\alpha\beta_c)$$

However, whether the manufacturer should adopt refurbishing when completing the goal set by collection target needs to be carefully considered. As long as $\beta_c < \frac{2(c_n\alpha - c_r)}{(1-c_n)(1-\alpha)}$ and $\alpha \in \left(\frac{3-\sqrt{1+8k_c}}{4}, 1\right)$, refurbishment could make $M_2^{<C_0R_=>} - M_2^{<C_0R_0>} > 0$, which also means that it is worth refurbishing. In other cases, the manufacturer should choose the no-refurbishment strategy (R_0).

Proof of Proposition 6

First, we define all the parameters' restraints as

$$U = \left\{\alpha, \beta_c, k_c, \delta, c_n, c_r \middle| 0 < \alpha < 1, 0 < \beta_c < 1, 0 < k_c < 1, 0 < \delta < 1, 0 < c_n < 1, 0 < c_r < 1\right\}$$

Using the results from Appendix A, we give the differences between two profits from C_0R_0 and C_0R_+ as shown below:

$$M_1^{<C_0R_+>} - M_1^{<C_0R_0>} = \frac{1}{16}(1-c_n)^2\left(2k_c\beta_c^2 - 5 + 2\delta + \frac{8(2+2k_c\beta_c^2-\delta)(1-\delta)(2-\delta)}{(3+2k_c\beta_c^2-2\delta)^2}\right).$$

We get $M_1^{<C_0R_+>} < M_1^{<C_0R_0>}$ if $0<\beta_c<1$, $0<k_c<1$, $0<c_n<1$, and $0<\delta<1$. From this, we know that refurbishing just some of the collected items is not beneficial, and no refurbishment is a better choice.

Next, we compare the profits from C_0R_0 and $C_0R_=$ as follows:

$$M_1^{<C_0R_=>} - M_1^{<C_0R_0>} = \frac{1}{4}(c_n-1)\beta_c\left(2c_r - 2c_n\alpha + (1-c_n)(1-\alpha)\alpha\beta_c\right).$$

Letting $M_1^{<C_0R_=>} - M_1^{<C_0R_0>} > 0$, we get the conditions which lead the manufacturer to refurbish all the collected items: (i) $\frac{c_r}{c_n} < \alpha < \alpha_0$ and $0 < \beta_c < \frac{2(c_n\alpha - c_r)}{\alpha(1-\alpha)(1-c_n)}$; ($ii$) $\alpha_0 < \alpha < 1$, where $\alpha_0 = \frac{1-3c_n+\sqrt{1-6c_n+9c_n^2+8c_r-8c_nc_r}}{2(1-c_n)}$. Otherwise, $M_1^{<C_0R_0>}$ will be higher than $M_1^{<C_0R_=>}$.

Last, we compare the profits from C_0R_+ and $C_0R_=$ as follows:

$$M_1^{<C_0R_=>} - M_1^{<C_0R_+>} = \frac{c_n-1}{16}\left(\begin{array}{l} 5-8c_r\beta_c - c_n\left(5-2\beta_c\left(2\alpha\left(2+(1-\alpha)\beta_c\right)+k_c\beta_c\right)-2\delta\right) \\ +\frac{8(c_n-1)(2+2k_c\beta_c^2-\delta)(2-\delta)(1-\delta)}{(3+2k_c\beta_c^2-2\delta)^2} - 2\delta - 2\beta_c^2\left(k_c+2\alpha(1-\alpha)\right) \end{array}\right).$$

It is obvious that $c_n - 1 < 0$, so the positivity and negativity of the formulation above is equivalent to

$$\Delta = 5-8c_r\beta_c - c_n\left(5-2\beta_c\left(2\alpha\left(2+(1-\alpha)\beta_c\right)+k_c\beta_c\right)-2\delta\right)+\frac{8(c_n-1)\left(2+2k_c\beta_c^2-\delta\right)(2-\delta)(1-\delta)}{\left(3+2k_c\beta_c^2-2\delta\right)^2}-2\delta-2\beta_c^2\left(k_c+2\alpha(1-\alpha)\right) \quad (4)$$

However, it is difficult to judge whether Equation (13) is greater or less than 0. Therefore, let $\Delta = 0$, and we have

$$\delta_1 = \frac{7u_1+48u_2\beta_c-4u_1\beta_c^2(k_c-6u_3)+32k_cu_2\beta_c^3-4u_1k_c\beta_c^4(k_c-4u_3)+\sqrt{u_1u_4\left(1+2k_c\beta_c^2\right)^2}}{4\left(u_1+8u_2\beta_c-2u_1\beta_c^2(k_c-2u_3)\right)}$$

$$\delta_1 = \frac{7u_1+48u_2\beta_c-4u_1\beta_c^2(k_c-6u_3)+32k_cu_2\beta_c^3-4u_1k_c\beta_c^4(k_c-4u_3)-\sqrt{u_1u_4\left(1+2k_c\beta_c^2\right)^2}}{4\left(u_1+8u_2\beta_c-2u_1\beta_c^2(k_c-2u_3)\right)},$$

where $u_1 = c_n - 1$, $u_2 = c_r - c_n\alpha$, $u_3 = \alpha(\alpha-1)$, and $u_4 = -3u_1 - 32u_2\beta_c + 4u_1(5k_c - 4u_3)\beta_c^2 + 64k_cu_2\beta_c^3 - 4u_1k_c(3k_c - 8u_3)\beta_c^4$.

Through collating the equation $\Delta = 0$, we can see that the coefficient of δ^2 is

$$\phi = -4\left(-1 + c_n + 8\beta_c(c_r - c_n\alpha) + 2(1-c_n)(k_c + 2\alpha(1-\alpha))\beta_c^2\right)$$

By calculating $\phi < 0$, we have the necessary and sufficient condition (A) as follows:

(1) (i) $\frac{\sqrt{2}}{2} < \beta_c < 1$, or ($ii$) $\frac{\sqrt{3}}{3} < \beta_c < \frac{\sqrt{2}}{2}$ and $\frac{1}{2} - \frac{\sqrt{3-\frac{1}{\beta_c^2}}}{2} < \alpha < \frac{1}{2} + \frac{\sqrt{3-\frac{1}{\beta_c^2}}}{2}$;

(2) $k_c > \frac{1}{2\beta_c^2} - 2\alpha + 2\alpha^2$;

(3) (*i*) $g(\alpha) < c_n < 1$ and $f(c_n) < c_r < c_n\alpha$; or (*ii*) $c_n \le g(\alpha)$, and $c_r < c_n\alpha$.

where $f(c_n) = \frac{1-c_n}{8\beta_c} + c_n\alpha - \frac{\beta_c(1-c_n)(k_c + 2\alpha(1-\alpha)\beta_c)}{4}$ and $g(\alpha) = 1 - \frac{8\alpha\beta_c}{2\beta_c(k_c\beta_c + 2\alpha(2+\beta_c - \alpha\beta_c)) - 1}$.

As Condition (A) is satisfied, we know that $\Delta > 0$ can be true if and only if $Max\{0, \ \delta_1\} < \delta < Min\{1, \ \delta_2\}$, which means $M_1^{<C_0R_=>} - M_1^{<C_0R_+>} < 0$. On the other side, if $0 < \delta < Max\{0, \ \delta_1\}$ or $Min\{1, \ \delta_2\} < \delta < 1$, $\Delta < 0$ can be true and $M_1^{<C_0R_=>} - M_1^{<C_0R_+>} > 0$.

However, by Condition (B) where $B = C_U A$, we have $\phi > 0$. At this time, if and only if $Max\{0, \ \delta_1\} < \delta < Min\{1, \ \delta_2\}$, $\Delta < 0$ can be true and $M_1^{<C_0R_=>} - M_1^{<C_0R_+>} > 0$.

We summarize the results as follows:

(1) $M_1^{<C_0R_=>} > M_1^{<C_0R_0>} > M_1^{<C_0R_+>}$, if (*i*) $\frac{c_r}{c_n} < \alpha < \alpha_0$, and $0 < \beta_c < \frac{2(c_n\alpha - c_r)}{\alpha(1-\alpha)(1-c_n)}$; (*ii*) $\alpha_0 < \alpha < 1$;

(2) $M_1^{<C_0R_0>} > M_1^{<C_0R_+>} > M_1^{<C_0R_=>}$, if (*i*) $Max\{0, \ \delta_1\} < \delta < Min\{1, \ \delta_2\}$ with Condition (A); (*ii*) $0 < \delta < Max\{0, \ \delta_1\}$ or $Min\{1, \ \delta_2\} < \delta < 1$ with Condition (B), where $B = C_U A$;

(3) $M_1^{<C_0R_0>} > M_1^{<C_0R_=>} > M_1^{<C_0R_+>}$, in other cases.

Proof of Corollary 1

Using the results from Appendix A, we can find the difference between the two profits from *$C_0R_=$* and *$C_+R_=$* as follows:

$$M_1^{<C_+R_=>} - M_1^{<C_0R_=>} = \frac{(2c_n\alpha - 2c_r + (c_n - 1)\beta_c(k_c + 2\alpha(1-\alpha)))^2}{8(k_c + 2\alpha(1-\alpha))} > 0,$$

so $M_1^{<C_+R_=>} > M_1^{<C_0R_=>}$.

***Proof of Proposition** 7*

According to the consumer surplus formulation mentioned in our manuscript, we use the equilibrium from Appendix A and give the consumer surplus under the *C_0R_0* and *$C_0R_=$* strategies as follows:

$$CS^{<C_0R_0>} = \frac{1}{32}(1-c_n)^2(4+5\delta);$$

$$CS^{<C_0R_=>} = \frac{1}{32}(1-c_n)^2\left(4 + \left(5 + 4(1-\alpha)\alpha\beta_c^2\right)\delta\right).$$

We calculate the first-order condition of each consumer surplus as follows:

$$\frac{\partial CS^{<C_0R_0>}}{\partial\delta} = \frac{5}{32}(1-c_n)^2 > 0$$

$$\frac{\partial CS^{<C_0R_=>}}{\partial\delta} = \frac{1}{32}(1-c_n)^2\left(5 + 4(1-\alpha)\alpha\beta_c^2\right) > 0$$

Thus, as the consumer's willingness to wait δ increases, the consumer surplus always increases when the manufacturer adopts the *C_0R_0* or *$C_0R_=$* strategy.

Proof of Corollary 2

Using the results from Appendix A, have the differences between two consumer surplus from $C_0R_=$ and $C_+R_=$ as follows:

$$CS^{<C_+R_=>}-CS^{<C_0R_=>}=\frac{\alpha\delta(\alpha-1)(2c_r-2c_n\alpha+(1-c_n)(k_c+2\alpha(1-\alpha))\beta_c)(2c_n\alpha-2c_r+(1-c_n)(k_c+2\alpha(1-\alpha))\beta_c)}{8(k_c+2\alpha(1-\alpha))^2}.$$

However, there will be three cases with $CS^{<C_+R_=>}>CS^{<C_0R_=>}$ as follows:

(1) $0<c_n\le\frac{k_c+2\alpha-2\alpha^2}{k_c+4\alpha-2\alpha^2}$ and $0<\beta_c<\frac{2c_n\alpha-2c_r}{k_c-c_nk_c+2\alpha-2c_n\alpha-2\alpha^2+2c_n\alpha^2}$;

(2) $\frac{k_c+2\alpha-2\alpha^2}{k_c+4\alpha-2\alpha^2}<c_n<1$ and $0<c_r\le\frac{k_cc_n-k_c-2\alpha+4c_n\alpha+2\alpha^2-2c_n\alpha^2}{2}$;

(3) $\frac{k_c+2\alpha-2\alpha^2}{k_c+4\alpha-2\alpha^2}<c_n<1$, $\frac{k_cc_n-k_c-2\alpha+4c_n\alpha+2\alpha^2-2c_n\alpha^2}{2}<c_r<c_n\alpha$, and $0<\beta_c<\frac{2c_n\alpha-2c_r}{k_c-c_nk_c+2\alpha-2c_n\alpha-2\alpha^2+2c_n\alpha^2}$.

From the conditions above, we find that the collection target may affect the consumer's benefits. Especially when the collection target is low, the consumers will benefit from the manufacturer's $C_+R_=$ strategy.

Proof of Proposition 8

As Proposition 6 has proved that C_0R_+ can never be an optimal choice, we only consider the total social welfare in the C_0R_0 and $C_0R_=$ scenarios. According to the results before, we get the following total social welfare in the C_0R_0 and $C_0R_=$ scenarios:

$$SW^{<C_0R_0>}=\frac{(1-c_n)\left(14-24e_m-24e_s-8e_d(3-2\beta_c)-4\beta_c(k_c\beta_c-4e_c)+\delta-c_n\left(14+\delta-4k_c\beta_c^2\right)\right)}{32}$$

$$SW^{<C_0R_=>}=\frac{(c_n-1)(2(7c_n-7+12(e_d+e_m+e_s)+8\beta_c(c_r-e_c+e_r+e_s-\alpha(c_n+e_d+e_m+e_s))+S_1))}{32},$$

where $S_1=2\beta_c(1-c_n)(k_c+2\alpha(1-\alpha))-\delta(1-c_n)\left(1+4\alpha\beta_c^2(1-\alpha)\right)$.

We then have $\frac{\partial SW^{<C_0R_0>}}{\partial\delta}=\frac{(1-c_n)^2}{32}>0$ and $\frac{\partial SW^{<C_0R_=>}}{\partial\delta}=\frac{(1-c_n)^2}{32}\left(1+4\alpha\beta_c^2(1-\alpha)\right)>0$, which means that in compliance mode (C_0), the total social welfare always rises as the consumer's willingness to wait increases. Similarly, we have $\frac{\partial SW^{<C_+R_=>}}{\partial\delta}=-\frac{(1-c_n)^2}{8}<0$, which indicates that in voluntary mode (C_+), the total social welfare may decline as the consumer's willingness to wait increases.

Proof of Proposition 9

In order to analyze the problem in different scenarios, we consider the Lagrangian for the refurbisher's problem as follows:

$$L_{R2}=(p_{r2}-c_r)q_r-k_cq_r^2/2+\lambda_1((1-\tau)q_{n1}-q_r)+\lambda_2q_r$$

For the refurbisher, she/he has two collection strategies: One is collecting some but not all, and the other one is collecting all available products. However, her/his unique refurbishment strategy is to make full use of the collected items ($R_=$).

(1) The first situation (collecting some but not all) means $\lambda_1 = 0$ and $\lambda_2 = 0$. We first solve the refurbisher's problem and let $\partial L_{R2} / \partial q_r = 0$. We then have

$$q_r = -\frac{c_r + (q_{n2} + q_{r2})\alpha - \alpha \bar{v}}{k_c + 2\alpha}.$$

Subsequently, the equilibrium in all cases can be resolved by following the steps in the proof of Proposition 1. We list the just equilibrium for each scenario below.

***C₀R₀* scenario**

Through calculating, we find $\eta_5 = \frac{(k_c + \alpha)(c_r - c_n \alpha)}{k_c + \alpha(2 - \alpha)} < 0$, which indicates that the C_0R_0 scenario will not exist.

(2) The second situation (collecting all available products) means $\lambda_1 > 0$ and $\lambda_2 = 0$. We first solve the refurbisher's problem and let $\partial L_{R2} / \partial q_r = 0$. We then have $q_r = (1-\tau) q_{n1}$ and $\lambda_1 = \alpha \bar{v} + 2 q_{n1} \alpha \tau - c_r - \alpha(2 q_{n1} + q_{n2} + q_{r2}) - k_c q_{n1}(1-\tau)$. Next, the equilibrium in all cases can be resolved by following the steps in the proof of Proposition 1.

***C₀R₀* scenario**

Through calculating, we reach the equilibrium shown in Table B3.

Table B3. The equilibrium in the C_0R_0 scenario.

Parameters	Optimal value
q_{n1}	$\frac{(1-c_n)(1-\alpha-\delta+\alpha\tau)}{3+2k_c\beta_c^2-2\delta+\alpha(2(1-\delta)+\alpha(1-\tau))(\tau-1)}$
q_{n2}	$\frac{(1-c_n)(2+2k_c\beta_c^2-\delta+\alpha(2-3\delta)(1-\tau))}{2(3+2k_c\beta_c^2-2\delta-\alpha(1-\tau)(2(1-\delta)+\alpha(1-\tau)))}$
q_c	$\frac{\beta_c(1-c_n)(1-\alpha-\delta+\alpha\tau)}{3+2k_c\beta_c^2-2\delta+\alpha(2(1-\delta)+\alpha(1-\tau))(\tau-1)}$
q_{r2}	0
q_r	$\frac{(1-c_n)(1-\alpha-\delta+\alpha\tau)(1-\tau)}{3+2k_c\beta_c^2-2\delta+\alpha(2(1-\delta)+\alpha(1-\tau))(\tau-1)}$

However, $q_{n1} > 0$, $q_{n2} > 0$, and $\lambda_1 > 0$ should hold. We then see that if and only if (1) $0 < c_n < c_n^*$, or $c_n^* < c_n$ and $c_r^* < c_r$ and (2) $\delta \le \tau$, or $\delta > \tau$ and $\alpha < \frac{1-\delta}{1-\tau}$, the C_0R_0 scenario could exist, where $c_r^* = \frac{\alpha((1-\tau)(1-\alpha)(1-\delta-\alpha(1-\tau)) + c_n(3\delta+\tau-4-2k_c\beta_c^2-\delta\tau+\alpha(1-\tau)(4-3\delta-\tau)))}{2\delta-3-2k_c\beta_c^2+\alpha(2(1-\delta)+\alpha(1-\tau))(1-\tau)}$ and $c_n^* = \frac{(1-\alpha)(1-\tau)(1-\delta-\alpha(1-\tau))}{4+2k_c\beta_c^2-3\delta-\tau(1-\delta)-\alpha(1-\tau)(4-3\delta-\tau)}$.