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Research article

Strategic pricing and coordination in a closed-loop supply chain under cap-and-trade regulation: a quality improvement perspective

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Supplementary

Appendix A. Proof of Proposition 1

The first derivative of optimal solutions concerning k is obtained when $k > \frac{l^2-2\theta l+1}{8(1-\theta^2)}$ and $l > \theta$, as follows.

$$\frac{\partial \omega_n{}^*}{\partial k} = \frac{4(1-\theta^2)((-Dl-B)\theta+Bl+D)}{(8k\theta^2+l^2-2l\theta-8k+1)^2} < 0$$

$$\frac{\partial \omega_r{}^*}{\partial k} = \frac{4l(1-\theta^2)((-Dl-B)\theta+Bl+D)}{(8k\theta^2+l^2-2l\theta-8k+1)^2} < 0$$

$$\frac{\partial p_n{}^*}{\partial k} = \frac{6(1-\theta^2)((-Dl-B)\theta+Bl+D)}{(8k\theta^2+l^2-2l\theta-8k+1)^2} < 0$$

$$\frac{\partial p_r{}^*}{\partial k} = \frac{6l(1-\theta^2)((-Dl-B)\theta+Bl+D)}{(8k\theta^2+l^2-2l\theta-8k+1)^2} < 0$$

$$\frac{\partial {D_n}^*}{\partial k}=\frac{2(1-l\theta)((-Dl-B)\theta+Bl+D)}{(8k\theta^2+l^2-2l\theta-8k+1)^2}<0$$

$$\frac{\partial {D_r}^*}{\partial k}=\frac{2(l-\theta)((-Dl-B)\theta+Bl+D)}{(8k\theta^2+l^2-2l\theta-8k+1)^2}<0$$

$$\frac{\partial q^*}{\partial k}=\frac{8(1-\theta^2)((-Dl-B)\theta+Bl+D)}{(8k\theta^2+l^2-2l\theta-8k+1)^2}<0$$

Appendix B. Proof of Proposition 2

The first derivative of the optimal solutions concerning pc is obtained as follows.

$$\frac{\partial {\omega_n}^*}{\partial p_c}=\frac{8k\theta^2e_n+l^2e_n-3l\theta e_n-8ke_n+le_r-\theta e_r+2e_n}{16k\theta^2+2l^2-4l\theta-16k+2} \tag{1}$$

$$\frac{\partial {\omega_r}^*}{\partial p_c}=\frac{8k\theta^2e_r-l^2\theta e_n-3l\theta e_r-8ke_r+2l^2e_r+le_n+e_r}{16k\theta^2+2l^2-4l\theta-16k+2} \tag{2}$$

$$\frac{\partial {p_n}^*}{\partial p_c}=\frac{8k\theta^2e_n+l^2e_n-5l\theta e_n-8ke_n+4e_n+3le_r-3\theta e_r}{32k\theta^2+4l^2-8l\theta-32k+4} \tag{3}$$

$$\frac{\partial {p_r}^*}{\partial p_c}=\frac{-3l^2\theta e_n+4l^2e_r-5l\theta e_r+3le_n+8k\theta^2e_r-8ke_r+e_r}{32k\theta^2+4l^2-8l\theta-32k+4} \tag{4}$$

$$\frac{\partial q^*}{\partial p_c}=\frac{-l\theta e_n+le_r-\theta e_r+e_n}{8k\theta^2+l^2-2l\theta-8k+1} \tag{5}$$

For equation (5), when $8k\theta^2+l^2-2l\theta-8k+1<0$, which implies $k>\frac{l^2-2\theta l+1}{8(1-\theta^2)}=k_0$, the condition $\frac{\partial q^*}{\partial p_c}<0$ holds. For equation (1), it is established that when $8k\theta^2e_n+l^2e_n-3l\theta e_n-8ke_n+le_r-\theta e_r+2e_n>0$, or equivalently $k<\frac{l^2e_n-3e_n\theta l+le_r-e_r\theta+2e_n}{8(1-\theta^2)e_n}=k_1$, then $\frac{\partial {\omega_n}^*}{\partial p_c}<0$. Conversely, when $k>k_1$, $\frac{\partial {\omega_n}^*}{\partial p_c}>0$. Regarding equation (2), it is observed that when $8k\theta^2e_r-l^2\theta e_n-3l\theta e_r-8ke_r+2l^2e_r+le_n+e_r>0$, or equivalently $k<\frac{l^2\theta e_n+3e_r\theta l-le_n-e_r-2l^2e_r}{8(\theta^2-1)e_r}=k_2$, then $\frac{\partial {\omega_r}^*}{\partial p_c}<0$. Conversely, when $k>k_2$, $\frac{\partial {\omega_r}^*}{\partial p_c}>0$. For equation (3), it is evident that when $8k\theta^2e_n+l^2e_n-5l\theta e_n-8ke_n+4e_n+3le_r-3\theta e_r>0$, or equivalently $k<\frac{l^2e_n-5e_n\theta l+3le_r-3\theta e_r+4e_n}{8(1-\theta^2)e_n}=k_3$, then $\frac{\partial {p_n}^*}{\partial p_c}<0$. Conversely, when $k>\frac{l^2e_n-5e_n\theta l+3le_r-3\theta e_r+4e_n}{8(1-\theta^2)e_n}=k_3$, $\frac{\partial {p_n}^*}{\partial p_c}>0$. For equation (4), it is evident that when $-3l^2\theta e_n+4l^2e_r-5l\theta e_r+3le_n+8k\theta^2e_r-8ke_r+e_r>0$, or equivalently $k<\frac{3l^2\theta e_n-4l^2e_r+5e_r\theta l-3le_n-e_r}{8(\theta^2-1)e_r}=k_4$, then $\frac{\partial {p_r}^*}{\partial p_c}<0$. Conversely, when$k>k_4$, $\frac{\partial {p_r}^*}{\partial p_c}>0$. When $le_n>e_r$, it follows that $k_0>k_1>k_2>k_3>k_4$. Therefore, the conclusion of Proposition 2 is readily established.

Appendix C. Proof of Proposition 3

The partial derivatives of the optimal solutions are taken concerning the degree of quality upgrading when $k>\frac{l^2-2\theta l+1}{4(1-\theta^2)}$ and $l>\theta$.

$$\frac{\partial {p_n}^{**}}{\partial k}=\frac{2(1-\theta^2)((-Dl-B)\theta+Bl+D)}{(4k\theta^2+l^2-2l\theta-4k+1)^2}<0$$

$$\frac{\partial {p_r}^{**}}{\partial k} = \frac{2l(1-\theta^2)((-Dl-B)\theta+Bl+D)}{(4k\theta^2+l^2-2l\theta-4k+1)^2} < 0$$

$$\frac{\partial {D_n}^{**}}{\partial k} = \frac{2(1-l\theta)((-Dl-B)\theta+Bl+D)}{(4k\theta^2+l^2-2l\theta-4k+1)^2} < 0$$

$$\frac{\partial {D_r}^{**}}{\partial k} = \frac{2(l-\theta)((-Dl-B)\theta+Bl+D)}{(4k\theta^2+l^2-2l\theta-4k+1)^2} < 0$$

$$\frac{\partial q^{**}}{\partial k} = \frac{4(1-\theta^2)((-Dl-B)\theta+Bl+D)}{(4k\theta^2+l^2-2l\theta-4k+1)^2} < 0$$

Appendix D. Proof of Proposition 4

The partial derivatives of the optimal solutions are calculated concerning pc.

$$\frac{\partial {p_n}^{**}}{\partial p_c} = \frac{4k\theta^2 e_n+l^2 e_n-3l\theta e_n-4ke_n+2e_n+le_r-\theta e_r}{8k\theta^2+2l^2-4l\theta-8k+2} \tag{6}$$

$$\frac{\partial {p_r}^{**}}{\partial p_c} = \frac{-l^2\theta e_n+2l^2 e_r-3l\theta e_r+le_n+4k\theta^2 e_r-4ke_r+e_r}{8k\theta^2+2l^2-4l\theta-8k+2} \tag{7}$$

$$\frac{\partial q^{**}}{\partial p_c} = \frac{-l\theta e_n+le_r-\theta e_r+e_n}{4k\theta^2+l^2-2l\theta-4k+1} \tag{8}$$

For equation (8), when $4k\theta^2 + l^2 - 2l\theta - 4k + 1 < 0$, which implies $k > \frac{l^2-2\theta l+1}{4(1-\theta^2)} = k_5$, the condition $\frac{\partial q^{**}}{\partial p_c} < 0$ holds. For equation (6), it is established that when $4k\theta^2 e_n + l^2 e_n - 3l\theta e_n - 4ke_n + 2e_n + le_r - \theta e_r > 0$, or equivalently $k < \frac{l^2 e_n-3e_n\theta l+le_r-e_r\theta+2e_n}{4(1-\theta^2)e_n} = k_6$, then $\frac{\partial {p_n}^{**}}{\partial p_c} < 0$. Conversely, when $k > k_6$, $\frac{\partial {p_n}^{**}}{\partial p_c} > 0$. Regarding equation (7), it is observed that when $-l^2\theta e_n + 2l^2 e_r - 3l\theta e_r + le_n + 4k\theta^2 e_r - 4ke_r + e_r > 0$, or equivalently $k < \frac{l^2\theta e_n+3e_r\theta l-le_n-e_r-2l^2 e_r}{4(\theta^2-1)e_r} = k_7$, then $\frac{\partial {p_r}^{**}}{\partial p_c} < 0$. Conversely, when $k > k_7$, $\frac{\partial {p_r}^{**}}{\partial p_c} > 0$. When $le_n > e_r$, it follows that $k_5 > k_6 > k_7$. Therefore, the conclusion of Proposition 4 is readily established.

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