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*Research article*

# When should manufacturers introduce live broadcast channels? Impact of asymmetry network externality

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## Appendix A:

### *A.1. Proof of Section 4.1.1*

First, the second-order derivation of online price is carried out respectively:

$$\frac{\partial^2 \pi_a^I}{\partial^2 p_a^I} = -2k_a(1-\phi) < 0.$$

The second derivative is less than 0. The profit function of the agent and the manufacturer are concave functions, and there is an optimal solution. Therefore, the profit of the agent and the manufacturer have the maximum value.

This paper adopts the reverse solution method. Firstly, the agent conducts the first-order derivation of the profit function to determine the optimal online price $p_a$:

$$p_a^{I*} = \frac{\theta v}{2}. \tag{A1}$$

Second, we consider the manufacturer's profit maximization problem, and the manufacturer's profit function is as follows:

$$\pi_{ma}^I = (1-k_a)p_a^I N_a^I + p_{da}^I N_{da}^I. \tag{A2}$$

In the agency dual model, the manufacturer's profit consists of two parts: One part comes from sales in the direct channel; the other part is to earn $1-k_a$ percentage of sales in the agency channel.

After introducing Eq. (A.1) into Eq. (A.2), we get:

$$\pi_{ma}^{I}=(1-k_a)\frac{\theta v}{2}(1-\phi)\frac{\theta v}{2}+p_{da}^{I}\phi\left(v-p_{da}^{I}+\beta(1-\phi)\frac{\theta v}{2}\right). \qquad (A3)$$

We can get $p_a^I$ by solving the objective function (A3):

$$p_{da}^{I*}=\frac{2v+\beta(1-\phi)\theta v}{4}. \qquad (A4)$$

***A.1 Proof of Lemma 1***

(i) $\frac{\partial p_{dl}^{I*}}{\partial\gamma}=\frac{v\beta\theta(1-\phi)^2}{4(1-\gamma+\gamma\phi)^2}$; $\frac{\partial N_{dl}^{I*}}{\partial\gamma}=\frac{v\beta\theta\phi(1-\phi)^2}{4(1-\gamma+\gamma\phi)^2}$; $\frac{\partial N_{l}^{I*}}{\partial\gamma}=\frac{v\theta(1-\phi)^2}{2(1-\gamma+\gamma\phi)^2}$: As can be seen, the results of the derivation are related to the value of $\beta$. When$0<\beta<1$, $\frac{\partial p_{dl}^{I*}}{\partial\gamma}>0$, $\frac{\partial N_{dl}^{I*}}{\partial\gamma}>0$, $\frac{\partial N_{l}^{I*}}{\partial\gamma}>0$; $\frac{\partial p_{l}^{I*}}{\partial\gamma}=0$;

(ii) $\frac{\partial\pi_{ml}^{I*}}{\partial\gamma}=-\frac{v^2\theta(-1+\phi)^2\left(2\beta\phi(-1+\gamma-\gamma\phi)+\theta\left(-2+2\mathrm{kl}(1+\gamma(-1+\phi))-2\gamma(-1+\phi)+\beta^2(-1+\phi)\phi\right)\right)}{8\left(1+\gamma(-1+\phi)\right)^3}$

$>0$; $\frac{\partial\pi_{l}^{I*}}{\partial\gamma}=\frac{4k_l v^2\theta^2(-1+\phi)^2}{(4+4\gamma(-1+\phi))^2}>0$.

***A.2 Proof of Lemma 2***

(i) $p_{dl}^{I*}-p_{da}^{I*}=\frac{v\beta\gamma\theta(-1+\phi)^2}{4+4\gamma(-1+\phi)}$: When $0<\beta<1$, $p_{dl}^{I*}>p_{da}^{I*}$; otherwise, $p_{dl}^{I*}<p_{da}^{I*}$;

(ii) $p_{da}^{I*}-p_{a}^{I*}=\frac{1}{4}v(2+\theta(-2+\beta-\beta\phi))$: When $max\left\{\frac{2\theta-2}{\theta(1-\phi)},-1\right\}<\beta<1$, $p_{da}^{I*}>p_{a}^{I*}$;

$p_{dl}^{I*}-p_{l}^{I*}=\frac{1}{4}v(2-2\theta-\frac{\beta\theta(-1+\phi)}{1+\gamma(-1+\phi)})$: When $max\left\{\frac{(2\theta-2)(1-\gamma+\phi\gamma)}{\theta(1-\phi)},-1\right\}<\beta<1$, $p_{dl}^{I*}>p_{l}^{I*}$.

***A.3. Proof of Proposition 1***

First，let $\pi_{ml}^{I*}-\pi_{ma}^{I*}=\frac{(1-\phi)\theta^2v^2}{4}(\frac{1-k_l}{1-\gamma+\phi\gamma}-1+k_a)=0$. It is easy to find that $\frac{(1-\phi)\theta^2v^2}{4}>0$. Therefore, we only need to determine the size of $\frac{1-k_l}{1-\gamma+\phi\gamma}-1+k_a$. Let $F=\frac{1-k_l}{1-\gamma+\phi\gamma}-1+k_a$. By taking the first-order derivative of $\gamma$, we get $\frac{\partial F}{\partial\gamma}>0$, which means that $F$ increases with $\gamma$. Further, we can find the zero point $\gamma^*=\frac{k_l-k_a}{(1-k_a)(1-\phi)}$ for $F=0$. Next, it is only necessary to determine the magnitude of $\gamma^*$ in relation to 0 and 1.

If $k_l>k_a$, then $\gamma^*>0$. Further, we need to discuss the relationship between $\gamma$ and 1. There are two situations to discuss:

(i) If $0 < \phi \leq \phi^*$, then $0 < \gamma^* \leq 1$. When $\gamma^* < \gamma < 1$, $\pi_{ml}^{I*} > \pi_{ma}^{I*}$; when $0 < \gamma \leq \gamma^*$, $\pi_{ml}^{I*} < \pi_{ma}^{I*}$;

(ii) When $\phi^* < \phi < 1$, then $\gamma^* > 1$, which indicates that $\gamma < 1 < \gamma^*$ is constant, so $\pi_{ml}^{I*} < \pi_{ma}^{I*}$. In this case, $\phi^* = \frac{1-k_l}{1-k_a}$.

***A.4. Proof of Proposition 2***

Consistent with the analytical approach of Proposition 1, let $\pi_{ma}^{I*} - \pi_{ml}^{I*} = \frac{1}{16} v^2 \theta (\frac{4\beta(-1+\phi)\phi}{1+\gamma(-1+\phi)} - \frac{\theta(-1+\phi)(-4+4kl(1+\gamma(-1+\phi))-4\gamma(-1+\phi)+\beta^2(-1+\phi)\phi)}{(1+\gamma(-1+\phi))^2} + (1-\phi)(4\beta\phi + \theta(4-4ka-\beta^2(-1+\phi)\phi)))$, and $\frac{\partial(\pi_{ma}^{I*} - \pi_{ml}^{I*})}{\partial\gamma}$ is monotonically decreasing in $\gamma \in [0,1]$, this is, $\frac{\partial(\pi_{ma}^{I*} - \pi_{ml}^{I*})}{\partial\gamma} < 0$. Let $\pi_{ma}^{I*} - \pi_{ml}^{I*} = 0$, we can get

$$\gamma_1^* =$$

$$-(-(\left(\frac{\theta(\phi-1)^2\begin{pmatrix}4ka-2-2kl+\\ \beta^2(-1+\phi)\phi\end{pmatrix} - 2\beta(\phi-1)^2\phi}{+\sqrt{(\phi-1)^4\begin{pmatrix}4\beta^2\phi^2+4\beta\theta\phi(2-2kl-\beta^2(-1+\phi)\phi)+\\ \theta^2(4+4(-2+kl)kl+\beta^2(\phi-1)\phi(4ka-4+\beta^2(\phi-1)\phi))\end{pmatrix}}}\right)\Big/\frac{((\phi-1)^3}{(\theta(-4+4ka+\beta^2(-1+\phi)\phi))-4\beta\phi)})$$

.

When $0 < \gamma \leq \gamma_1^*$, $\pi_{ml}^{I*} < \pi_{ma}^{I*}$; $\gamma_1^* < \gamma < 1$, $\pi_{ml}^{I*} > \pi_{ma}^{I*}$.

***A.5. Proof of Lemma 3***

$\frac{d\,p_{dl}^{II*}}{d\,\varepsilon} = \frac{v\beta\theta(\varepsilon-\varepsilon\phi+(1+\varepsilon(-1+\phi))Log[1+\varepsilon(-1+\phi)])}{4\varepsilon^2(1+\varepsilon(-1+\phi))}$: When $0 < \beta < 1$, $\frac{d\,p_{dl}^{II*}}{d\,\varepsilon} > 0$; when $-1 < \beta < 0$, $\frac{d\,p_{dl}^{II*}}{d\,\varepsilon} < 0$.

***A.6. Proof of Lemma 4***

$p_{dl}^{II*} - p_{da}^{II*} = \frac{v\beta\theta(\varepsilon(-1+\phi)-Log[1+\varepsilon(-1+\phi)])}{4\varepsilon}$: When $\beta[z_2 + (1-\phi)\varepsilon] < 0$, $p_{dl}^{II*} > p_{da}^{II*}$; when $\beta[z_2 + (1-\phi)\varepsilon] > 0$, $p_{dl}^{II*} < p_{da}^{II*}$.

***A.7. Proof of Corollary 1***

$p_{dl}^{II*} - p_{dl}^{I*} = -\frac{v\beta\theta(\varepsilon-\varepsilon\phi+(1+\gamma(-1+\phi))Log[1+\varepsilon(-1+\phi)])}{4(\varepsilon+\gamma\varepsilon(-1+\phi))}$: When $\beta[z_1 z_2 + (1-\phi)\varepsilon] > 0$, $p_{dl}^{II*} < p_{dl}^{I*}$.

## Appendix B: Proof of extension

### *B.1. Proof of Proposition 6*

The solution process is consistent with Proposition 4. By comparing the profit, we get:

$$\overline{\beta_3} = \frac{-2\phi(\theta_a+\frac{\theta_l}{z_1})+2\sqrt{\left(\phi\theta_a+\frac{\phi\theta_a}{z_1}\right)^2-\phi(1-\phi)\left(\theta_a^2-\theta_l^2\right)\left[(1-k_a)\theta_a^2-(1-k_l)\frac{\theta_l^2}{z_1}\right]}}{\phi(1-\phi)\left(\theta_l^2+\theta_a^2\right)},$$

$$\underline{\beta_3} = \frac{-2\phi(\theta_a+\frac{\theta_l}{z_1})-2\sqrt{\left(\phi\theta_a+\frac{\phi\theta_a}{z_1}\right)^2-\phi(1-\phi)\left(\theta_a^2-\theta_l^2\right)\left[(1-k_a)\theta_a^2-(1-k_l)\frac{\theta_l^2}{z_1}\right]}}{\phi(1-\phi)\left(\theta_l^2+\theta_a^2\right)}.$$

### *B.2. Proof of Proposition 7*

The solution process is consistent with Proposition 6. By comparing the profit, we get：

$$\overline{\beta_4} = \frac{-2\varepsilon(\varepsilon\phi(1-\phi)\theta_a-\theta_l \ln z_1)+2\sqrt{[\phi\theta_a\varepsilon^2(1-\phi)-\varepsilon\theta_l \ln z_1]^2-\phi H\left[\varepsilon^2\theta_a^2(1-k_a)(1-\phi)+\varepsilon\theta_l^2(1-k_l)\ln z_1\right]}}{\phi H},$$

$$\underline{\beta_4} = \frac{-2\varepsilon(\varepsilon\phi(1-\phi)\theta_a-\theta_l \ln z_1)-2\sqrt{[\phi\theta_a\varepsilon^2(1-\phi)-\varepsilon\theta_l \ln z_1]^2-\phi H\left[\varepsilon^2\theta_a^2(1-k_a)(1-\phi)+\varepsilon\theta_l^2(1-k_l)\ln z_1\right]}}{\phi H}.$$

Here, $H_3 = \varepsilon^2(1-\phi)^2\theta_a^2 - \theta_l^2 \ln^2 z_1$.