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Research article

Material-feasible and dependence-informed distributionally robust lot-sizing for multi-level BOM planning

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Appendix A. Proofs of the master-bound and termination results

Proof of Proposition 4.

For every sample n, the master enforces the semi-infinite epigraph inequality only for the finite subset $\{\widetilde{\xi}^{n\ell} : \ell \in \mathcal{V}_n\} \subseteq \Xi$. The robust material-feasibility constraints are already imposed exactly through Proposition 1; hence the only relaxation is the finite restriction of the epigraph constraints. Every feasible solution of (4.4) is therefore feasible for (4.5), which gives $Z^k \le Z^\star$. Adding an adverse realization appends at least one valid epigraph constraint to the master, so its feasible region can only shrink. Since the master is a minimization problem, its optimal value cannot decrease. □

Proof of Theorem 1.

Let

$$Z^k = C(x^k, z^k) + \lambda^k \varepsilon + \frac{1}{N}\sum_{n=1}^{N} \eta_n^k$$

denote the globally optimal objective value of the finite master problem. By Proposition 4, $Z^k \le Z^\star$, where $Z^\star$ is the global optimum of the exact formulation. The termination condition implies

$$\frac{1}{N}\sum_{n=1}^{N} \theta_n(x^k, \lambda^k) \le \frac{1}{N}\sum_{n=1}^{N} \eta_n^k + \delta.$$

Consequently, the worst-case objective evaluated at (x^k, z^k) and λ^k is at most $Z^k + \delta$. Minimization over $\lambda \ge 0$ cannot increase this value. Hence,

$$Z^\star \le \mathcal{J}(x^k, z^k) \le Z^k + \delta \le Z^\star + \delta,$$

where $\mathcal{J}(x^k, z^k)$ denotes the exact distributionally robust objective of the returned first-stage solution. Thus, the solution is δ-optimal. □



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