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Research article

Game-theoretic analysis of green investment decisions under service competition in shipping supply chains

Huawei Sun[1,2], Jing Xu[1,2*], Yang Lu[3,4] and Zhuanzhuan Ren[5]

[1] School of Management, Xi'an Jiaotong University, Xi'an Shaanxi 710049, China

[2] The Key Lab of the Ministry of Education for Process Management & Efficiency Engineering, Xi'an 710049, China

[3] School of Business, Sichuan University, Chengdu Sichuan 610065, China

[4] Department of Logistics and Maritime Studies, The Hong Kong Polytechnic University, Hong Kong SAR 999077, China

[5] School of Management, Xi'an Polytechnic University, Xi'an Shaanxi 710048, China

* **Correspondence:** Email: xujing@stu.xjtu.edu.cn.

Appendix

The calculation of the equilibrium solutions

Scenario NN

Following the order of backward induction, we first consider the freight price decisions of both freight forwarders. Given the service levels and shipping fees determined by ocean shipping companies, the profit function of each freight forwarder is concave with respect to its freight price, p_i^{NN}, due to $\frac{\partial^2 \pi_{Fi}^{NN}}{\partial p_i^{NN2}} = -2 < 0$. By solving the first-order condition,

$$\frac{\partial \pi_{Fi}^{NN}}{\partial p_i^{NN}} = a - 2p_i^{NN} + s_i^{NN} - bs_j^{NN} + w_i^{NN} = 0,\ i = \{1, 2\},\ j = 3 - i,$$

we obtain $p_i^{NN} = \frac{a+s_i^{NN}-bs_j^{NN}+w_i^{NN}}{2}$. We then consider each ocean shipping company's transport service

level and shipping service fee decisions. The Hessian matrix of π_{OSi}^{NN} with respect to s_i^{NN} and w_i^{NN} is

$$H^{NN} = \begin{vmatrix} -1 & \frac{1}{2} \\ \frac{1}{2} & -m \end{vmatrix}.$$

The first-order leading principal minor is $\Delta_1 = -1 < 0$, and the second-order leading principal minor is $\Delta_2 = m - \frac{1}{4}$. To ensure that π_{OSi}^{NN} is a differentiable concave function of s_i^{NN} and w_i^{NN}, $m > \frac{1}{4}$ is required. Solving the following first-order conditions $\frac{\partial \pi_{OSi}^{NN}}{\partial s_i^{NN}} = 0$ and $\frac{\partial \pi_{OSi}^{NN}}{\partial w_i^{NN}} = 0$, we can obtain

$$w_i^{NN^*} = \frac{2(a+c)m - (1-b)c}{4m-1+b},\ s_i^{NN^*} = \frac{a-c}{4m-1+b}.$$

It can be concluded that the equilibrium transport demand of each freight forwarder is $q_i^{NN*} = \frac{(a-c)m}{4m-1+b}$. The equilibrium profits of the stakeholders are $\pi_{Fi}^{NN*} = \frac{(a-c)^2m^2}{(4m-1+b)^2}$ and $\pi_{OSi}^{NN*} = \frac{(a-c)^2m(4m-1)}{2(4m-1+b)^2}$.

To ensure all equilibrium solutions take positive values, the following condition must be satisfied as derived from calculations: $m > \frac{1}{4}$ along with $a > c$.

Scenario EN

We first consider the freight price decisions of both freight forwarders. Each freight forwarder's profit function is still concave with respect to p_i^{EN} due to $\frac{\partial^2 \pi_{Fi}^{EN}}{\partial p_i^{EN^2}} = -2 < 0$. By solving the first-order conditions,

$$\begin{cases} \frac{\partial \pi_{F1}^{EN}}{\partial p_1^{EN}} = a + kg_1^{EN} - 2p_1^{EN} + s_1^{EN} - bs_2^{EN} + w_1^{EN} = 0, \\ \frac{\partial \pi_{F2}^{EN}}{\partial p_2^{EN}} = a - 2p_2^{EN} - bs_1^{EN} + s_2^{EN} + w_2^{EN} = 0, \end{cases}$$

we obtain $p_1^{EN} = \frac{a+s_1^{EN}-bs_2^{EN}+w_1^{EN}+kg_1^{EN}}{2}$ and $p_2^{EN} = \frac{a+s_2^{EN}-bs_1^{EN}+w_2^{EN}}{2}$. We then consider each ocean shipping company's transport service level and shipping service fee decisions. The Hessian matrix of π_{OSi}^{EN} with respect to s_i^{EN} and w_i^{EN} is

$$H^{EN} = \begin{vmatrix} -1 & \frac{1}{2} \\ \frac{1}{2} & -m \end{vmatrix}.$$

The first-order leading principal minor is $\Delta_1 = -1 < 0$, and the second-order leading principal minor is $\Delta_2 = m - \frac{1}{4}$. To ensure that π_{OSi}^{EN} is a differentiable concave function of s_i^{EN} and w_i^{EN}, $m > \frac{1}{4}$ is still required. Solving the following first-order conditions $\frac{\partial \pi_{OSi}^{EN}}{\partial s_i^{EN}} = 0$ and $\frac{\partial \pi_{OSi}^{EN}}{\partial w_i^{EN}} = 0$, we can obtain

$$w_1^{EN} = \frac{(1+b-4m)[2am + (2m-1+b)c] - \{b^2 + (4m-1)[1+2(k-1)m]\}g_1}{b^2 - (4m-1)^2}.$$

$$s_1^{EN} = \frac{(a-c)(1+b-4m) - (1+k)(4m-1)g_1}{b^2 - (4m-1)^2}.$$

$$w_2^{EN} = \frac{(1+b-4m)\,(2am + c\,(-1+b+2m)) + 2b(1+k)mg_1}{b^2 - (4m-1)^2},\ s_2^{EN} = \frac{(a-c)(1+b-4m) + b(1+k)g_1}{b^2 - (4m-1)^2}.$$

Finally, we consider the green investment decision of OS1. As $\frac{\partial^2 \pi_{OS1}^{EN}}{\partial g_1^{EN}} = \frac{(1+k)^2 m(4m-1)^3}{(16m^2-8m-b^2+1)^2} - n$, to ensure OS1's profit function π_{OS1}^{EN} is concave with respect to g_1^{EN}, we require $n > \frac{(1+k)^2 m(4m-1)^3}{(16m^2-8m-b^2+1)^2}$. Solving the first-order condition $\frac{\partial \pi_{OS1}^{EN}}{\partial g_1^{EN}} = 0$, we obtain $g_1^* = \frac{(a-c)(4m-1)^2(1+k)(4m-1-b)m}{(4m-1-b)^2(4m-1+b)^2 n - m(4m-1)^3(1+k)^2}$.

It can be concluded that the equilibrium service level of each ocean shipping company is $s_1^{EN*} = \frac{(a-c)(4m-1-b)^2(4m-1+b)n}{(4m-1-b)^2(4m-1+b)^2 n - m(4m-1)^3(1+k)^2}$ and $s_2^{EN*} = \frac{(a-c)[(4m-1-b)^2(4m-1+b)^2 n + b(1+k)^2(4m-1)^2 m - (1+k)^2 m(4m-1)^3]}{(4m-1+b)[(4m-1-b)^2(4m-1+b)^2 n - m(4m-1)^3(1+k)^2]}$. The equilibrium transport demand of each freight forwarder is $q_1^{EN*} = \frac{(a-c)(4m-1-b)^2 m(4m-1+b)n}{(4m-1-b)^2(4m-1+b)^2 n - m(4m-1)^3(1+k)^2}$ and $q_2^{EN*} = \frac{(a-c)m[(4m-1-b)^2(4m-1+b)n - m(4m-1)^2(1+k)^2]}{(4m-1-b)^2(4m-1+b)^2 n - m(4m-1)^3(1+k)^2}$. The equilibrium profits of all stakeholders are $\pi_{OS1}^{EN*} = \frac{(a-c)^2(4m-1-b)^2(4m-1)mn}{2[(4m-1-b)^2(4m-1+b)^2 n - m(4m-1)^3(1+k)^2]}$, $\pi_{OS2}^{EN*} = \frac{(a-c)^2 m(4m-1)[(4m-1-b)^2(4m-1+b)n - m(4m-1)^2(1+k)^2]^2}{2[(4m-1-b)^2(4m-1+b)^2 n - m(4m-1)^3(1+k)^2]^2}$, $\pi_{F1}^{EN*} = \frac{[(a-c)(4m-1-b)^2 m(4m-1+b)n]^2}{[(4m-1-b)^2(4m-1+b)^2 n - m(4m-1)^3(1+k)^2]^2}$, and $\pi_{F2}^{EN*} = \frac{\{(a-c)m[(1+k)^2(4m-1)^2 m - (4m-1-b)^2(4m-1+b)n]\}^2}{[(4m-1-b)^2(4m-1+b)^2 n - m(4m-1)^3(1+k)^2]^2}$.

To ensure all equilibrium solutions take positive values, the following conditions must be satisfied as derived from calculations: $m > \frac{1+b}{4}$ and $n > \frac{m(4m-1)^2(1+k)^2}{(4m-1-b)^2(4m-1+b)}$, along with $a > c$.

Scenario EE

We first consider the freight price decisions of both freight forwarders. Each freight forwarder's profit function is concave with respect to p_i^{EE} due to $\frac{\partial^2 \pi_{Fi}^{EE}}{\partial p_i^{EE2}} = -2 < 0$. By solving the first-order condition, $\frac{\partial \pi_{Fi}^{EE}}{\partial p_i^{EE}} = 0$, we obtain $p_i^{EE} = \frac{a + s_i^{EE} - b s_j^{EE} + w_i^{EE} + k g_i^{EE}}{2}$. We then consider each ocean shipping company's transport service level and shipping service fee decisions. The Hessian matrix of π_{OSi}^{EE} with respect to s_i^{EE} and w_i^{EE} is

$$H^{EE} = \begin{vmatrix} -1 & \frac{1}{2} \\ \frac{1}{2} & -m \end{vmatrix}.$$

The first-order leading principal minor is $\Delta_1 = -1 < 0$, and the second-order leading principal minor is $\Delta_2 = m - \frac{1}{4}$. To ensure that π_{OSi}^{EE} is a differentiable concave function of s_i^{EE} and w_i^{EE}, $m > \frac{1}{4}$ is still required. Solving the following first-order conditions $\frac{\partial \pi_{OSi}^{EE}}{\partial s_i^{EE}} = 0$ and $\frac{\partial \pi_{OSi}^{EE}}{\partial w_i^{EE}} = 0$, we can obtain

$$w_i^{EE*} = \frac{(1+b-4m)[2am + c(2m-1+b)] - \{b^2 + (4m-1)[1 + 2(k-1)m]\} g_i + 2b(1+k) m g_j}{b^2 - (4m-1)^2}.$$

$$s_i^{EE*} = \frac{(a-c)(1+b-4m) - (1+k)(4m-1) g_i + b(1+k) g_j}{b^2 - (4m-1)^2}.$$

Finally, we consider each ocean shipping company's green investment decision. As $\frac{\partial^2 \pi_{OSi}^{EE}}{\partial g_i^{EE2}} = \frac{(1+k)^2 m(4m-1)^3}{(16m^2-8m-b^2+1)^2} - n$, to ensure each ocean shipping company's profit function π_{OSi}^{EE} is concave with respect to g_i^{EE}, we require $n > \frac{(1+k)^2 m(4m-1)^3}{(16m^2-8m-b^2+1)^2}$. Solving the first-order condition $\frac{\partial \pi_{OSi}^{EE}}{\partial g_1^{EE}} = 0$, we obtain $g_i^{EE*} = \frac{(a-c)A^2 Bm}{(A-b)(A+b)^2 n - m A^2 B^2}$.

It can be concluded that the equilibrium service level of each ocean shipping company is $s_i^{EE*} = \frac{(a-c)(4m-1-b)(4m-1+b)n}{(4m-1-b)(4m-1+b)^2n-m(4m-1)^2(1+k)^2}$. The optimal transport demand of each freight forwarder is $q_i^{EE*} = \frac{(a-c)(4m-1-b)m(4m-1+b)n}{(4m-1-b)(4m-1+b)^2n-m(4m-1)^2(1+k)^2}$. The equilibrium profits of the stakeholders are $\pi_{OSi}^{EE*} = \frac{(a-c)^2mn(4m-1)[(4m-1-b)^2(4m-1+b)^2n-m(4m-1)^3(1+k)^2]}{2[(4m-1-b)(4m-1+b)^2n-m(4m-1)^2(1+k)^2]^2}$ and $\pi_{Fi}^{EE*} = \frac{[(a-c)(4m-1-b)m(4m-1+b)n]^2}{[(4m-1-b)(4m-1+b)^2n-m(4m-1)^2(1+k)^2]^2}$.

To ensure all equilibrium solutions take positive values, the following conditions must be satisfied as derived from calculations: $m > \frac{1+b}{4}$ and $n > \frac{(1+k)^2m(4m-1)^3}{((4m-1)^2-b^2)^2}$, along with $a > c$.

Proof of Lemma 1

As $a > c$ and $m > \frac{1}{4}$ are required,

(1) $\frac{\partial s_i^{NN*}}{\partial b} = -\frac{a-c}{(4m-1+b)^2} < 0$;

(2) $\frac{\partial q_i^{NN*}}{\partial b} = -\frac{(a-c)m}{(4m-1+b)^2} < 0$;

(3) $\frac{\partial \pi_{OSi}^{NN*}}{\partial b} = -\frac{(a-c)^2m(4m-1)}{(4m-1+b)^3} < 0$, $\frac{\partial \pi_{Fi}^{NN*}}{\partial b} = -\frac{2(a-c)^2m^2}{(4m-1+b)^3} < 0$.

Proof of Lemma 2

In scenario EN, the required conditions are $m > \frac{1+b}{4}$ and $n > \frac{m(4m-1)^2(1+k)^2}{(4m-1-b)^2(4m-1+b)}$, alongside $a > c$. Thus, $4m-1-b > 0$ and $(4m-1-b)^2(4m-1+b)n - m(4m-1)^2(1+k)^2 > 0$.

(1) $\frac{\partial g_1^{EN*}}{\partial b} = \frac{(a-c)(1+k)(4m-1)^2m[(1+k)^2m(4m-1)^3+(4m-1-b)^2(1+3b-4m)(4m-1+b)n]}{[(4m-1-b)^2(4m-1+b)^2n-(1+k)^2m(4m-1)^3]^2}$. We only need to analyze the sign of $(1+k)^2m(4m-1)^3 + (4m-1-b)^2(1+3b-4m)(4m-1+b)n$, and other terms are positive. Let $f(n) = (1+k)^2m(4m-1)^3 + (4m-1-b)^2(1+3b-4m)(4m-1+b)n$. If $1+3b-4m \geq 0$, that is, $m \leq \frac{1+3b}{4}$, $f(n) > f(n=0) = (1+k)^2m(4m-1)^3 > 0$. If $1+3b-4m < 0$, that is, $m > \frac{1+3b}{4}$, by solving $f(n) > 0$, we obtain $n < \frac{m(1+k)^2(4m-1)^3}{(4m-1-b)^2(4m-1-3b)(4m-1+b)}$; by solving $f(n) < 0$, we obtain $n > \frac{m(1+k)^2(4m-1)^3}{(4m-1-b)^2(4m-1-3b)(4m-1+b)}$. Moreover, it is easy to verify that $\frac{m(4m-1)^2(1+k)^2}{(4m-1-b)^2(4m-1+b)} < \frac{m(1+k)^2(4m-1)^3}{(4m-1-b)^2(4m-1-3b)(4m-1+b)}$. Therefore, if ① $\frac{1+b}{4} < m \leq \frac{1+3b}{4}$, or ② $m > \frac{1+3b}{4}$ and $\frac{m(4m-1)^2(1+k)^2}{(4m-1-b)^2(4m-1+b)} < n < \frac{m(1+k)^2(4m-1)^3}{(4m-1-b)^2(4m-1-3b)(4m-1+b)}$, $\frac{\partial g_1^{EN*}}{\partial b} > 0$; if $m > \frac{1+3b}{4}$ and $n > \frac{m(1+k)^2(4m-1)^3}{(4m-1-b)^2(4m-1-3b)(4m-1+b)}$, $\frac{\partial g_1^{EN*}}{\partial b} < 0$.

(2) For OS1, $\frac{\partial s_1^{EN*}}{\partial b} = \frac{(a-c)(4m-1-b)n[(1+k)^2m(4m-1)^3(4m-1+3b)-(4m-1-b)^3(4m-1+b)^2n]}{[(4m-1-b)^2(4m-1+b)^2n-(1+k)^2m(4m-1)^3]^2}$ and $\frac{\partial q_1^{EN*}}{\partial b} = \frac{(a-c)(4m-1-b)mn[(1+k)^2m(4m-1)^3(4m-1+3b)-(4m-1-b)^3(4m-1+b)^2n]}{[(4m-1-b)^2(4m-1+b)^2n-(1+k)^2m(4m-1)^3]^2}$. We only need to analyze the sign of $(1+k)^2m(4m-1)^3(4m-1+3b) - (4m-1-b)^3(4m-1+b)^2n$, and other terms are positive. By solving $(1+k)^2m(4m-1)^3(4m-1+3b) - (4m-1-b)^3(4m-1+b)^2n > 0$, we obtain $n < \frac{(1+k)^2m(4m-1)^3(4m-1+3b)}{(4m-1-b)^3(4m-1+b)^2}$; by solving $(1+k)^2m(4m-1)^3(4m-1+3b) - (4m-1-b)^3(4m-1+b)^2n < 0$, we can obtain $n > \frac{(1+k)^2m(4m-1)^3(4m-1+3b)}{(4m-1-b)^3(4m-1+b)^2}$. It is easy to verify that $\frac{m(4m-1)^2(1+k)^2}{(4m-1-b)^2(4m-1+b)} < \frac{(1+k)^2m(4m-1)^3(4m-1+3b)}{(4m-1-b)^3(4m-1+b)^2}$. Thus, if $\frac{m(4m-1)^2(1+k)^2}{(4m-1-b)^2(4m-1+b)} < n < \frac{(1+k)^2m(4m-1)^3(4m-1+3b)}{(4m-1-b)^3(4m-1+b)^2}$, $\frac{\partial s_1^{EN*}}{\partial b} > 0$ and $\frac{\partial q_1^{EN*}}{\partial b} > 0$; if $n > \frac{(1+k)^2m(4m-1)^3(4m-1+3b)}{(4m-1-b)^3(4m-1+b)^2}$, $\frac{\partial s_1^{EN*}}{\partial b} < 0$ and $\frac{\partial q_1^{EN*}}{\partial b} < 0$.

For OS2, $\frac{\partial s_2^{EN*}}{\partial b} = \frac{(a-c)(4m-1-b)n\{(1+k)^2(4m-1)^2m[b-4b^2+(4m-1)^2-4bm]-(4m-1-b)^3(4m-1+b)^2n\}}{[(4m-1-b)^2(4m-1+b)^2n-(1+k)^2m(4m-1)^3]^2}$, $\frac{\partial q_2^{EN*}}{\partial b} =$

$\frac{(a-c)(4m-1-b)mn\{(1+k)^2(4m-1)^2m[b-4b^2+(4m-1)^2-4bm]-(4m-1-b)^3(4m-1+b)^2n\}}{[(4m-1-b)^2(4m-1+b)^2n-(1+k)^2m(4m-1)^3]^2}$. We only need to analyze the sign of $(1+k)^2(4m-1)^2m[b-4b^2+(4m-1)^2-4bm]-(4m-1-b)^3(4m-1+b)^2n$, and other terms are positive. Let $h(n)=(1+k)^2(4m-1)^2m[b-4b^2+(4m-1)^2-4bm]-(4m-1-b)^3(4m-1+b)^2n$. $h(n)<h\left(n=\frac{m(4m-1)^2(1+k)^2}{(4m-1-b)^2(4m-1+b)}\right)=-b(1+k)^2(4m-1)^2m(4m-1+3b)<0$. Thus, $\frac{\partial s_2^{EN*}}{\partial b}<0$ and $\frac{\partial q_2^{EN*}}{\partial b}<0$;

(3) $\frac{\partial \pi_{OS1}^{EN*}}{\partial b}=\frac{(a-c)^2(4m-1-b)mn(4m-1)[4(1+k)^2m(4m-1)^3-4(4m-1-b)^3(4m-1+b)n]}{[2(1+k)^2m(4m-1)^3-2(4m-1-b)^2(4m-1+b)^2n]^2}$. By solving $\frac{\partial \pi_{OS1}^{EN*}}{\partial b}>0$, we can obtain $\frac{m(4m-1)^2(1+k)^2}{(4m-1-b)^2(4m-1+b)}<n<\frac{(1+k)^2m(4m-1)^3}{(4m-1-b)^3(4m-1+b)}$; by solving $\frac{\partial \pi_{OS1}^{EN*}}{\partial b}<0$, we can obtain $n>\frac{(1+k)^2m(4m-1)^3}{(4m-1-b)^3(4m-1+b)}$.
$\frac{\partial \pi_{F1}^{EN*}}{\partial b}=\frac{2(a-c)^2(4m-1-b)^3m^2(4m-1+b)n^2[(4m-1-b)^3(4m-1+b)^2n-(1+k)^2m(4m-1)^3(4m-1+3b)]}{[(1+k)^2m(4m-1)^3-(4m-1-b)^2(4m-1+b)^2n]^3}$. By solving $\frac{\partial \pi_{F1}^{EN*}}{\partial b}>0$, we can obtain $\frac{m(4m-1)^2(1+k)^2}{(4m-1-b)^2(4m-1+b)}<n<\frac{(1+k)^2m(4m-1)^3(4m-1+3b)}{(4m-1-b)^3(4m-1+b)^2}$; by solving $\frac{\partial \pi_{F1}^{EN*}}{\partial b}<0$, we can obtain $n>\frac{(1+k)^2m(4m-1)^3(4m-1+3b)}{(4m-1-b)^3(4m-1+b)^2}$.
$\frac{\partial \pi_{OS2}^{EN*}}{\partial b}=\frac{(a-c)^2(4m-1-b)m(4m-1)n[(1+k)^2(4m-1)^2m-(4m-1-b)^2(4m-1+b)n][(1+k)^2(4m-1)^2m[b-4b^2+(4m-1)^2-4bm]-(4m-1-b)^3(4m-1+b)^2n]}{[(1+k)^2m(4m-1)^3-(4m-1-b)^2(4m-1+b)^2n]^3}<0$.
$\frac{\partial \pi_{F2}^{EN*}}{\partial b}=\frac{2(a-c)^2(4m-1-b)m^2n[(1+k)^2(4m-1)^2m-(4m-1-b)^2(4m-1+b)n][(1+k)^2(4m-1)^2m[b-4b^2+(4m-1)^2-4bm]-(4m-1-b)^3(4m-1+b)^2n]}{[(1+k)^2m(4m-1)^3-(4m-1-b)^2(4m-1+b)^2n]^3}<0$.

Proof of Lemma 3

In scenario EE, the required conditions are $m>\frac{1+b}{4}$ and $n>\frac{(1+k)^2m(4m-1)^3}{[(4m-1)^2-b^2]^2}$, alongside $a>c$. Thus, $4m-1-b>0$ and $(4m-1+b)^2(4m-1-b)^2n-(1+k)^2m(4m-1)^3>0$.

(1) $\frac{\partial g_i^{EE*}}{\partial b}=\frac{(a-c)(1+k)(4m-1)^2(4m-1+b)mn(1+3b-4m)}{[(4m-1-b)(4m-1+b)^2n-m(1+k)^2(4m-1)^2]^2}$. By solving $\frac{\partial g_i^{EE*}}{\partial b}>0$, we can obtain $\frac{1+b}{4}<m<\frac{1+3b}{4}$; by solving $\frac{\partial g_i^{EE*}}{\partial b}<0$, we can obtain $m>\frac{1+3b}{4}$.

(2) $\frac{\partial s_i^{EE*}}{\partial b}=\frac{(a-c)n[2b(1+k)^2(4m-1)^2m-(4m-1-b)^2(4m-1+b)^2n]}{[(4m-1-b)(4m-1+b)^2n-m(1+k)^2(4m-1)^2]^2}$, $\frac{\partial q_i^{EE*}}{\partial b}=\frac{(a-c)mn[2b(1+k)^2(4m-1)^2m-(4m-1-b)^2(4m-1+b)^2n]}{[(4m-1-b)(4m-1+b)^2n-m(1+k)^2(4m-1)^2]^2}$. We only need to analyze the sign of $2b(1+k)^2(4m-1)^2m-(4m-1-b)^2(4m-1+b)^2n$, and other terms are positive. Let $l(n)=2b(1+k)^2(4m-1)^2m-(4m-1-b)^2(4m-1+b)^2n$. Within the range of $n>\frac{m(4m-1)^2(1+k)^2}{(4m-1-b)^2(4m-1+b)}$, $l(n)<l\left(n=\frac{m(4m-1)^2(1+k)^2}{(4m-1-b)^2(4m-1+b)}\right)=(1+k)^2(4m-1)^2(1+2b-4m)m$. If $1+2b-4m<0$, that is, $m>\frac{1+2b}{4}$, we have $l(n)<l\left(n=\frac{m(4m-1)^2(1+k)^2}{(4m-1-b)^2(4m-1+b)}\right)=(1+k)^2(4m-1)^2(1+2b-4m)m<0$. If $1+2b-4m>0$, that is, $m<\frac{1+2b}{4}$, by solving $l(n=0)$, we can obtain $n=\frac{2b(1+k)^2(4m-1)^2m}{(4m-1-b)^2(4m-1+b)^2}$. It is easy to see that when $\frac{(1+k)^2m(4m-1)^3}{[(4m-1)^2-b^2]^2}<n<\frac{2b(1+k)^2(4m-1)^2m}{(4m-1-b)^2(4m-1+b)^2}$, $l(n)>0$; and when $n>\frac{2b(1+k)^2(4m-1)^2m}{(4m-1-b)^2(4m-1+b)^2}$, $l(n)<0$. Therefore, if $\frac{1+b}{4}<m<\frac{1+2b}{4}$ and $\frac{(1+k)^2m(4m-1)^3}{[(4m-1)^2-b^2]^2}<n<\frac{2b(1+k)^2(4m-1)^2m}{(4m-1-b)^2(4m-1+b)^2}$, $\frac{\partial s_i^{EE*}}{\partial b}>0$ and $\frac{\partial q_i^{EE*}}{\partial b}>0$; if ① $m>\frac{1+2b}{4}$, or ② $\frac{1+b}{4}<m<\frac{1+2b}{4}$ and $n>\frac{2b(1+k)^2(4m-1)^2m}{(4m-1-b)^2(4m-1+b)^2}$, $\frac{\partial s_i^{EE*}}{\partial b}<0$ and $\frac{\partial q_i^{EE*}}{\partial b}<0$.

(3) $\frac{\partial \pi_{OSi}^{EE*}}{\partial b}=\frac{(a-c)^2m(4m-1)(4m-1+b)^2n^2[(4m-1-b)^3(4m-1+b)n-(1+k)^2(4m-1)^2(4m-1-2b)m]}{[(4m-1-b)(4m-1+b)^2n-(1+k)^2(4m-1)^2m]^3}<0$,
$\frac{\partial \pi_{Fi}^{EE*}}{\partial b}=\frac{2(a-c)^2(4m-1+b)m^2(4m-1+b)n^2[2b(1+k)^2(4m-1)^2m-(4m-1-b)^2(4m-1+b)^2n]}{[(4m-1-b)(4m-1+b)^2n-(1+k)^2(4m-1)^2m]^3}$.

We have analyzed the sign of $2b(1+k)^2(4m-1)^2m-(4m-1-b)^2(4m-1+b)^2n$, and other terms are positive. Therefore, if $\frac{1+b}{4} < m < \frac{1+2b}{4}$ and $\frac{(1+k)^2m(4m-1)^3}{[(4m-1)^2-b^2]^2} < n < \frac{2b(1+k)^2(4m-1)^2m}{(4m-1-b)^2(4m-1+b)^2}$, $\frac{\partial\pi_{Fi}^{EE*}}{\partial b} > 0$; if ① $m > \frac{1+2b}{4}$, or ② $\frac{1+b}{4} < m < \frac{1+2b}{4}$ and $n > \frac{2b(1+k)^2(4m-1)^2m}{(4m-1-b)^2(4m-1+b)^2}$, $\frac{\partial\pi_{Fi}^{EE*}}{\partial b} < 0$.

Proof of Proposition 1

(1) $s_1^{EN*}-s_1^{EE*}=\frac{b(a-c)(1+k)^2(4m-1)^2(4m-1-b)m(4m-1+b)n}{[(4m-1-b)(4m-1+b)^2n-(1+k)^2(4m-1)^2m][(4m-1-b)^2(4m-1+b)^2n-(1+k)^2m(4m-1)^3]} > 0$, and $s_1^{EE*}-s_1^{NN*}=\frac{(a-c)(1+k)^2(4m-1)^2m}{-(1+k)^2(4m-1)^2m(4m-1+b)+(4m-1-b)(4m-1+b)^3n} > 0$. Therefore, $s_1^{EN*} > s_1^{EE*} > s_1^{NN*}$.

(2) $s_2^{EE*}-s_2^{NN*}=\frac{(a-c)(1+k)^2(4m-1)^2m}{-(1+k)^2(4m-1)^2m(4m-1+b)+(4m-1-b)(4m-1+b)^3n} > 0$, and $s_2^{NN*}-s_2^{EN*}=\frac{b(a-c)(1+k)^2(4m-1)^2m}{(4m-1-b)^2(4m-1+b)^3n-(1+k)^2m(4m-1)^3(4m-1+b)} > 0$. Therefore, $s_2^{EE*} > s_2^{NN*} > s_2^{EN*}$.

Proof of Proposition 2

(1) $q_1^{EN*}-q_1^{EE*}=\frac{(a-c)b(1+k)^2(4m-1)^2(4m-1-b)m^2(4m-1+b)n}{[(4m-1-b)(4m-1+b)^2n-(1+k)^2(4m-1)^2m][(4m-1-b)^2(4m-1+b)^2n-(1+k)^2m(4m-1)^3]} > 0$, and $q_1^{EE*}-q_1^{NN*}=\frac{(a-c)m^2(a-c)(1+k)^2(4m-1)^2m^2}{(4m-1-b)(4m-1+b)^3n-(1+k)^2m(4m-1)^2(4m-1+b)} > 0$. Therefore, $q_1^{EN*} > q_1^{EE*} > q_1^{NN*}$.

(2) $q_2^{EE*}-q_2^{NN*}=\frac{(a-c)m^2(a-c)(1+k)^2(4m-1)^2m^2}{(4m-1-b)(4m-1+b)^3n-(1+k)^2m(4m-1)^2(4m-1+b)} > 0$, and $q_2^{NN*}-q_2^{EN*}=\frac{b(a-c)(1+k)^2(4m-1)^2m^2}{(4m-1-b)^2(4m-1+b)^3n-(1+k)^2m(4m-1)^3(4m-1+b)} > 0$. Therefore, $q_2^{EE*} > q_2^{NN*} > q_2^{EN*}$.

Proof of Proposition 3

(1) $\pi_{F1}^{EN*}-\pi_{F1}^{EE*}=\frac{b(a-c)^2(1+k)^2(4m-1)^2(4m-1-b)^2m^3(4m-1+b)^2n^2[2(4m-1-b)^2(4m-1+b)^2n-(1+k)^2(8m-2-b)(4m-1)^2m]}{[(4m-1-b)(4m-1+b)^2n-(1+k)^2(4m-1)^2m]^2[(4m-1-b)^2(4m-1+b)^2n-(1+k)^2m(4m-1)^3]^2} > 0$, and $\pi_{F1}^{EE*}-\pi_{F1}^{NN*}=\frac{(a-c)^2(1+k)^2(4m-1)^2m^3[2(4m-1-b)(4m-1+b)^2n-(1+k)^2(4m-1)^2m]}{[(4m-1-b)(4m-1+b)^3n-(1+k)^2(4m-1)^2m(4m-1+b)]^2} > 0$. Thus, $\pi_{F1}^{EN*} > \pi_{F1}^{EE*} > \pi_{F1}^{NN*}$.

(2) $\pi_{F2}^{EE*}-\pi_{F2}^{NN*}=\frac{(a-c)^2(1+k)^2(4m-1)^2m^3[2(4m-1-b)(4m-1+b)^2n-(1+k)^2(4m-1)^2m]}{[(4m-1-b)(4m-1+b)^3n-(1+k)^2(4m-1)^2m(4m-1+b)]^2} > 0$ and $\pi_{F2}^{NN*}-\pi_{F2}^{EN*}=\frac{b(a-c)^2(1+k)^2(4m-1)^2m^3[2(4m-1-b)^2(4m-1+b)^2n-(1+k)^2(4m-1)^2m(8m-2+b)]}{[(4m-1-b)^2(4m-1+b)^3n-(1+k)^2m(4m-1)^3(4m-1+b)]^2} > 0$. Thus, $\pi_{F2}^{EE*} > \pi_{F2}^{NN*} > \pi_{F2}^{EN*}$.

Proof of Proposition 4

(1) $\pi_{OS1}^{EN*}-\pi_{OS1}^{NN*}=\frac{(a-c)^2(1+k)^2(4m-1)^4m^2}{2(4m-1-b)^2(4m-1+b)^4n-2(1+k)^2m(4m-1)^3(4m-1+b)^2} > 0$ and $\pi_{OS1}^{EN*}-\pi_{OS1}^{EE*}=\frac{b(a-c)^2(1+k)^2m^2(4m-1)^3n[2(1+b-4m)^2(4m-1+b)^2n-(1+k)^2(8m-2-b)(4m-1)^2m]}{2[(1+k)^2(4m-1)^2m-(4m-1-b)(4m-1+b)^2n]^2[(4m-1-b)^2(4m-1+b)^2n-(1+k)^2m(4m-1)^3]} > 0$.

Moreover, $\pi_{OS1}^{NN*}-\pi_{OS1}^{EE*}=\frac{(a-c)^2(1+k)^2m^2(4m-1)^3[(4m-1-2b)(4m-1+b)^2n-(1+k)^2(4m-1)^2m]}{2[(4m-1-b)(4m-1+b)^3n-(1+k)^2(4m-1)^2m(4m-1+b)]^2}$. By solving $\pi_{OS1}^{NN*}-\pi_{OS1}^{EE*} > 0$, we can obtain ① $\frac{1+b}{4} < m \le \frac{1+2b}{4}$, or ② $m > \frac{1+2b}{4}$ and $\frac{(1+k)^2m(4m-1)^3}{[(4m-1)^2-b^2]^2} < n < \frac{(1+k)^2(4m-1)^2m}{(4m-1-2b)(4m-1+b)^2}$; solving $\pi_{OS1}^{NN*}-\pi_{OS1}^{EE*} < 0$, we can obtain $m > \frac{1+2b}{4}$ and $n > \frac{(1+k)^2(4m-1)^2m}{(4m-1-2b)(4m-1+b)^2}$.

(2) $\pi_{OS2}^{NN*}-\pi_{OS2}^{EN*}=-\frac{b(a-c)^2(1+k)^2m^2(4m-1)^3[(1+k)^2(4m-1)^2m(8m-2+b)-2(4m-1-b)^2(4m-1+b)^2n]}{2[(1+k)^2m(4m-1)^3(4m-1+b)-(4m-1-b)^2(4m-1+b)^3n]^2} > 0$,

$$\pi_{OS2}^{EE*}-\pi_{OS2}^{EN*}=\frac{(a-c)^2(1+k)^2(4m-1)^4m^2\left(\begin{array}{l}(4m-1-b)^4(4m-1+b)^4n^3-(1+k)^6m^3(4m-1)^5+\\(1+k)^4[3(4m-1)^2-4b^2](4m-1)^4m^2n-\\(1+k)^2[3(4m-1)^2-2b^2](4m-1-b)^2m(4m-1)(4m-1+b)^2n^2\end{array}\right)}{2[(4m-1-b)(4m-1+b)^2n-(1+k)^2(4m-1)^2m]^2[(4m-1-b)^2(4m-1+b)^2n-(1+k)^2m(4m-1)^3]^2}>$$

0. Moreover, $\pi_{OS2}^{NN*}-\pi_{OS2}^{EE*}=\frac{(a-c)^2(1+k)^2m^2(4m-1)^3[(4m-1-2b)(4m-1+b)^2n-(1+k)^2(4m-1)^2m]}{2[(4m-1-b)(4m-1+b)^3n-(1+k)^2(4m-1)^2m(4m-1+b)]^2}$. By solving $\pi_{OS2}^{NN*}-\pi_{OS2}^{EE*}>0$, we can obtain ① $\frac{1+b}{4}<m\leq\frac{1+2b}{4}$, or ② $m>\frac{1+2b}{4}$ and $\frac{(1+k)^2m(4m-1)^3}{[(4m-1)^2-b^2]^2}<n<\frac{(1+k)^2(4m-1)^2m}{(4m-1-2b)(4m-1+b)^2}$; solving $\pi_{OS2}^{NN*}-\pi_{OS2}^{EE*}<0$, we can obtain $m>\frac{1+2b}{4}$ and $n>\frac{(1+k)^2(4m-1)^2m}{(4m-1-2b)(4m-1+b)^2}$.

Proof of Proposition 5

We verify the Nash equilibrium by examining the unilateral deviations from each possible strategy profile. First, consider NN. If OS1 unilaterally deviates from N to E, the resulting strategy profile becomes EN. By Proposition 4(1), $\pi_{OS1}^{EN*}>\pi_{OS1}^{NN*}$. Hence, OS1 has a profitable unilateral deviation from NN. By symmetry, OS2 also has a profitable unilateral deviation. Therefore, NN is not a Nash equilibrium.

Next, consider EN. If OS2 unilaterally deviates from N to E, the resulting strategy profile becomes EE. By Proposition 4(2), $\pi_{OS2}^{EE*}>\pi_{OS2}^{EN*}$. Thus, OS2 has a profitable unilateral deviation from EN, and EN is not a Nash equilibrium. By symmetry, NE is also not a Nash equilibrium.

Finally, consider EE. If OS1 unilaterally deviates from E to N, the resulting strategy profile becomes NE. By Proposition 4(2) and the symmetry between the two shipping supply chains, $\pi_{OS1}^{EE*}>\pi_{OS1}^{NE*}$. Similarly, if OS2 unilaterally deviates from E to N, the resulting strategy profile becomes EN, and $\pi_{OS2}^{EE*}>\pi_{OS2}^{EN*}$. Therefore, neither OS1 nor OS2 has an incentive to deviate unilaterally from EE. Hence, EE is a Nash equilibrium. Moreover, the above comparisons show that each ocean shipping company prefers E to *N* regardless of its rival's decision. Therefore, E is a strictly dominant strategy for both ocean shipping companies, and EE is the unique Nash equilibrium.

Proof of Proposition 6

$$\pi_{OS1}^{EE*}-\pi_{OS1}^{NN*}=\pi_{OS2}^{EE*}-\pi_{OS2}^{NN*}=\frac{(a-c)^2(1+k)^2m^2}{2[(1-b)^2-4m]\{m[(1+k)^2-4n]+(1-b)^2n\}}>0.$$

Proof of Proposition 7

Let the common freight forwarder F^C purchase shipping services from both carriers and set the two freight prices p_1 and p_2. Its profit is $\pi_F^C=(p_1-w_1)q_1+(p_2-w_2)q_2$. Under the benchmark demand functions, $q_i=a-p_i+s_i-bs_j+kg_i$, the demand for service from OSi does not depend directly on the price of service from OSj. Therefore, the common freight forwarder's pricing problem is separable across the two shipping services. The first-order condition yields $p_i^{C*}=(a+s_i-bs_j+kg_i+w_i)/2$, and consequently $q_i^{C*}=(a+s_i-bs_j+kg_i-w_i)/2$. These expressions are identical to those obtained when the two carriers are matched with separate freight forwarders. Therefore, under the benchmark demand structure, replacing the one-to-one matching structure with a common freight forwarder does not alter the carriers' subsequent optimization problems or the equilibrium results.

Proof of Corollary 1

$n_7 = \frac{(1+k)^2(4m-1)^2 m}{(4m-1-2b)(4m-1+b)^2}$. $\frac{\partial n_7}{\partial b} = \frac{6b(1+k)^2(4m-1)^2 m}{(4m-1-2b)^2(4m-1+b)^3} > 0$. $\frac{\partial n_7}{\partial k} = \frac{2(1+k)(4m-1)^2 m}{(4m-1-2b)(4m-1+b)^2}$. By solving $\frac{\partial n_7}{\partial k} > 0$, we can obtain $m > \frac{1}{4}(1+2b)$; and by solving $\frac{\partial n_7}{\partial k} < 0$, we can obtain $\frac{1+b}{4} < m < \frac{1+2b}{4}$.

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