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Research article

Research on dual outsourcing strategies for multi-component products in a competitive equipment manufacturing platform

Jinzhou He[1,2,*] and Xiaojuan Li[3]

[1] School of Management Science and Real Estate, Chongqing University, Chongqing, China
[2] Chongqing Chang'an Industry (Group) Co., Ltd., Chongqing, China
[3] Department of Mechanical and Vehicle Engineering, Chongqing Jiaotong University, Chongqing, China

* **Correspondence:** Email: 202403042374@stu.cqu.edu.cn; Tel: +86 15378397839.

Appendix

Proof of Theorem 1.

To establish the following inequality,

$$\frac{(1-2d)\left[2(1-2d)\alpha_a-\alpha_b\right]^2}{\left(16d^2-16d+3\right)^2}-\frac{2\left[(1-2d)\alpha_a-(1-d)\alpha_b\right]^2}{(1-2d)(10d-3)^2}>0$$

note that the admissible ranges of d and $\frac{\alpha_a}{\alpha_b}$ imply $16d^2-16d+3>0$, $2(1-2d)\alpha_a-\alpha_b>0$, $(1-2d)\alpha_a-(1-d)\alpha_b>0$, $1-2d>0$, and $10d-3<0$. Taking square roots and rearranging yields

$$\frac{(1-2d)(3-10d)(2-4d)-\sqrt{2}(1-2d)\left(16d^2-16d+3\right)}{\sqrt{1-2d}\,(3-10d)\left(16d^2-16d+3\right)}\alpha_a$$
$$>\frac{(1-2d)(3-10d)-\sqrt{2}(1-d)\left(16d^2-16d+3\right)}{\sqrt{1-2d}\,(3-10d)\left(16d^2-16d+3\right)}\alpha_b \tag{28}$$

For Equation (28), the admissible range of d gives

$(1-2d)(3-10d)(2-4d)>(1-2d)(3-10d)$ and

$\sqrt{2}(1-2d)(16d^2-16d+3)<\sqrt{2}(1-d)(16d^2-16d+3)$. Therefore, Equation (28) holds, which establishes

$$\frac{(1-2d)\left[2(1-2d)\alpha_a-\alpha_b\right]^2}{(16d^2-16d+3)^2}-\frac{2\left[(1-2d)\alpha_a-(1-d)\alpha_b\right]^2}{(1-2d)(10d-3)^2}>0$$

Proof of Theorem 2.

If the OEM is willing to outsource procurement, then $\pi_a^{I*}>\pi_a^{D*}$; equivalently,

$$\frac{2\left[(1-2d)\alpha_a-(1-d)\alpha_b\right]^2}{(1-2d)(10d-3)^2}-\frac{2(8d^2-8d+1)\left[(1-2d)\alpha_a-\alpha_b\right]^2}{(1-2d)(32d^2-32d+3)^2}>0 \tag{29}$$

For Equation (29), the admissible ranges of d and $\frac{\alpha_a}{\alpha_b}$ imply $1-2d>0$, $10d-3<0$, $8d^2-8d+1>0$, $32d^2-32d+3>0$, $(1-2d)\alpha_a-(1-d)\alpha_b>0$ and $(1-2d)\alpha_a-\alpha_b>0$. Taking square roots and rearranging yields

$$\frac{(1-2d)\left[32d^2-32d+3-(3-10d)\sqrt{8d^2-8d+1}\right]}{(3-10d)(32d^2-32d+3)}\alpha_a$$

$$>\frac{(1-d)(32d^2-32d+3)-(3-10d)\sqrt{8d^2-8d+1}}{(3-10d)(32d^2-32d+3)}\alpha_b \tag{30}$$

For Equation (30), the admissible range of d gives $32d^2-32d+3-(3-10d)\sqrt{8d^2-8d+1}<0$ and $(1-d)(32d^2-32d+3)-(3-10d)\sqrt{8d^2-8d+1}<0$. Therefore,

$$\frac{\alpha_a}{\alpha_b}<\frac{(1-d)(32d^2-32d+3)-(3-10d)\sqrt{8d^2-8d+1}}{(1-2d)\left[32d^2-32d+3-(3-10d)\sqrt{8d^2-8d+1}\right]} \tag{31}$$

Let $f(d)=\frac{(1-d)(32d^2-32d+3)-(3-10d)\sqrt{8d^2-8d+1}}{(1-2d)\left[32d^2-32d+3-(3-10d)\sqrt{8d^2-8d+1}\right]}$. The OEM is therefore willing to outsource procurement when $\frac{\alpha_a}{\alpha_b}<f(d)$, but is unwilling to do so when $\frac{\alpha_a}{\alpha_b}>f(d)$.

Figure 2 establishes the ordering of $f_L(d)$, $f_H(d)$, and $f(d)$. Hence, when $f_L(d) < \frac{\alpha_a}{\alpha_b} < f(d)$, the OEM prefers procurement outsourcing; when $f(d) \leqslant \frac{\alpha_a}{\alpha_b} < f_H(d)$, it prefers in-house procurement.

When $f_L(d) < \frac{\alpha_a}{\alpha_b} < f(d)$, if the OEM prefers production outsourcing, then $\pi_a^{I*} > \pi_a^{N*}$; equivalently,

$$\frac{2\left[(1-2d)\alpha_a-(1-d)\alpha_b\right]^2}{(1-2d)(10d-3)^2}-\frac{(1-2d)\left[2(1-2d)\alpha_a-\alpha_b\right]^2}{\left(16d^2-16d+3\right)^2}+k>0 \tag{32}$$

$$k>\frac{(1-2d)\left[2(1-2d)\alpha_a-\alpha_b\right]^2}{\left(16d^2-16d+3\right)^2}-\frac{2\left[(1-2d)\alpha_a-(1-d)\alpha_b\right]^2}{(1-2d)(10d-3)^2}>0 \tag{33}$$

Define $Q_{IN}=\frac{(1-2d)\left[2(1-2d)\alpha_a-\alpha_b\right]^2}{\left(16d^2-16d+3\right)^2}-\frac{2\left[(1-2d)\alpha_a-(1-d)\alpha_b\right]^2}{(1-2d)(10d-3)^2}$. Therefore, when $f_L(d) < \frac{\alpha_a}{\alpha_b} < f(d)$, the OEM chooses Strategy N if $0 < k \leqslant Q_{IN}$, and Strategy I if $k > Q_{IN}$.

When $f(d) \leqslant \frac{\alpha_a}{\alpha_b} < f_H(d)$, if the OEM prefers production outsourcing, then $\pi_a^{D*} > \pi_a^{N*}$; equivalently,

$$\frac{2\left(8d^2-8d+1\right)\left[(1-2d)\alpha_a-\alpha_b\right]^2}{(1-2d)\left(32d^2-32d+3\right)^2}-\frac{(1-2d)\left[2(1-2d)\alpha_a-\alpha_b\right]^2}{\left(16d^2-16d+3\right)^2}+k>0 \tag{34}$$

$$k>\frac{(1-2d)\left[2(1-2d)\alpha_a-\alpha_b\right]^2}{\left(16d^2-16d+3\right)^2}-\frac{2\left(8d^2-8d+1\right)\left[(1-2d)\alpha_a-\alpha_b\right]^2}{(1-2d)\left(32d^2-32d+3\right)^2} \tag{35}$$

The comparison reduces to a threshold condition in k. To show that this threshold is positive, it suffices to establish

$$\frac{(1-2d)\left[2(1-2d)\alpha_a-\alpha_b\right]^2}{\left(16d^2-16d+3\right)^2}-\frac{2\left(8d^2-8d+1\right)\left[(1-2d)\alpha_a-\alpha_b\right]^2}{(1-2d)\left(32d^2-32d+3\right)^2}>0$$

The admissible ranges of d and $\frac{\alpha_a}{\alpha_b}$ imply $16d^2-16d+3>0$, $1-2d>0$, $2(1-2d)\alpha_a-\alpha_b>0$, $(1-2d)\alpha_a-\alpha_b>0$, $8d^2-8d+1>0$ and $32d^2-32d+3>0$. Taking square roots and rearranging yields

$$\frac{(2-4d)(1-2d)(32d^2-32d+3)-\sqrt{2}\sqrt{8d^2-8d+1}(1-2d)(16d^2-16d+3)}{\sqrt{1-2d}(16d^2-16d+3)(32d^2-32d+3)}\alpha_a$$
$$>\frac{(1-2d)(32d^2-32d+3)-\sqrt{2}\sqrt{8d^2-8d+1}(16d^2-16d+3)}{\sqrt{1-2d}(16d^2-16d+3)(32d^2-32d+3)}\alpha_b \quad (36)$$

Because $2-4d>1$ and $1-2d<1$, the preceding inequality holds.

Define $Q_{DN}=\frac{(1-2d)\left[2(1-2d)\alpha_a-\alpha_b\right]^2}{(16d^2-16d+3)^2}-\frac{2(8d^2-8d+1)\left[(1-2d)\alpha_a-\alpha_b\right]^2}{(1-2d)(32d^2-32d+3)^2}$. Therefore, when $f(d)\leqslant\frac{\alpha_a}{\alpha_b}<f_H(d)$, the OEM chooses Strategy N if $0<k\leqslant Q_{DN}$, and Strategy D if $k>Q_{DN}$.

This completes the proof of Theorem 2.

Proof of Corollary 3

Let $\rho=\frac{\alpha_a}{\alpha_b}$. The admissible range of ρ is determined by the nonnegative-output conditions: $f_L(d)<\rho<f(d)$, where $f_L(d)=\frac{1}{1-2d}$. For Q_{IN}, differentiating twice with respect to ρ gives $\frac{\partial^2 Q_{IN}}{\partial\rho^2}=-\frac{4\alpha_b^2(2d-1)(544d^4-768d^3+400d^2-96d+9)}{(4d-3)^2(4d-1)^2(10d-3)^2}$. Because $0<d<\frac{1}{2}-\frac{\sqrt{10}}{8}<\frac{1}{2}$, we have $2d-1<0$. Moreover, $544d^4-768d^3+400d^2-96d+9>0$ over the admissible range of d. Therefore, $\frac{\partial^2 Q_{IN}}{\partial\rho^2}>0$. Hence, $\frac{\partial Q_{IN}}{\partial\rho}$ is increasing in ρ. Its minimum is therefore attained at the lower boundary $\rho=f_L(d)$. Substituting $\rho=f_L(d)$ yields

$\left.\frac{\partial Q_{IN}}{\partial\rho}\right|_{\rho=f_L(d)}=-\frac{4\alpha_b^2(256d^5-912d^4+992d^3-472d^2+105d-9)}{(4d-3)^2(4d-1)^2(10d-3)^2}$. The polynomial in the numerator is negative over the admissible range of d. Therefore, $\left.\frac{\partial Q_{IN}}{\partial\rho}\right|_{\rho=f_L(d)}>0$. Since this is the minimum value

of $\frac{\partial Q_{IN}}{\partial \rho}$, it follows that $\frac{\partial Q_{IN}}{\partial \rho} > 0$ throughout the admissible domain of Q_{IN}.

Proof of Corollary 6

Let $\rho = \frac{\alpha_a}{\alpha_b}$ and define $G_{IN}(\rho, d) = \frac{1}{\alpha_b^2}\frac{\partial Q_{IN}}{\partial d}$. According to Theorem 2, the applicable domain of Q_{IN} is $0 < d < \frac{1}{2} - \frac{\sqrt{10}}{8}$, $f_L(d) < \rho < f(d)$, where $f_L(d) = \frac{1}{1-2d}$. For a given d, $G_{IN}(\rho, d)$ is a quadratic function of ρ. Differentiating twice with respect to ρ gives

$$\frac{\partial^2 G_{IN}}{\partial \rho^2} = \frac{8\left(87040d^7 - 219648d^6 + 249024d^5 - 165792d^4 + 70144d^3 - 18720d^2 + 2862d - 189\right)}{(4d-3)^3(4d-1)^3(10d-3)^3}.$$ Over the admissible range of d, both the polynomial in the numerator and the denominator are negative. Therefore, $\frac{\partial^2 G_{IN}}{\partial \rho^2} > 0$. Hence, $\frac{\partial G_{IN}}{\partial \rho}$ is increasing in ρ. At the lower boundary $\rho = f_L(d) = \frac{1}{1-2d}$, we obtain

$$\left.\frac{\partial G_{IN}}{\partial \rho}\right|_{\rho = f_L(d)} = \frac{4\left(81920d^8 - 344064d^7 + 599040d^6 - 593152d^5 + 368192d^4 - 145856d^3 + 35820d^2 - 4968d + 297\right)}{(2d-1)(4d-3)^3(4d-1)^3(10d-3)^3}.$$

Over the admissible range of d, the polynomial in the numerator and the denominator are positive. Therefore, $\left.\frac{\partial G_{IN}}{\partial \rho}\right|_{\rho = f_L(d)} > 0$. It follows that $\frac{\partial G_{IN}}{\partial \rho} > 0$ throughout the applicable domain. Thus, $G_{IN}(\rho, d)$ is increasing in ρ, and its minimum is attained at $\rho = f_L(d)$. Substituting $\rho = f_L(d)$ into $G_{IN}(\rho, d)$ gives

$$G_{IN}(f_L(d), d) = -\frac{2\left(81920d^9 - 385024d^8 + 798208d^7 - 929536d^6 + 646688d^5 - 270576d^4 + 65420d^3 - 7902d^2 + 216d + 27\right)}{(2d-1)^2(4d-3)^3(4d-1)^3(10d-3)^3}.$$ The polynomial in the numerator is positive over the admissible range of d, whereas $(2d-1)^2(4d-3)^3(4d-1)^3(10d-3)^3 < 0$. Consequently, $G_{IN}(f_L(d), d) > 0$. Because this is the minimum value of $G_{IN}(\rho, d)$, we have $G_{IN}(\rho, d) > 0$ throughout the applicable domain. Finally, since $\alpha_b^2 > 0$, $\frac{\partial Q_{IN}}{\partial d} > 0$ throughout the applicable domain. This completes the proof.

Proof of Theorem 3.

If the CM prefers Strategy I, then $\pi_b^{I*} > \pi_b^{D*}$; equivalently,

$$\frac{\left[(1-2d)\alpha_a-(1-d)\alpha_b\right]^2}{(2d-1)^2(3-10d)}-\frac{\left[(1-2d)\alpha_a-\alpha_b\right]^2}{(1-2d)\left(32d^2-32d+3\right)}>0 \tag{37}$$

The admissible ranges of d and $\frac{\alpha_a}{\alpha_b}$ imply $1-2d>0$, $3-10d>0$, $32d^2-32d+3>0$, $(1-2d)\alpha_a-\alpha_b>0$ and $(1-2d)\alpha_a-(1-d)\alpha_b>0$. Taking square roots and rearranging yields

$$\frac{(1-2d)\left(\sqrt{32d^2-32d+3}-\sqrt{1-2d}\sqrt{3-10d}\right)}{\sqrt{1-2d}\sqrt{3-10d}\sqrt{32d^2-32d+3}}\alpha_a>\frac{(1-d)\sqrt{32d^2-32d+3}-\sqrt{1-2d}\sqrt{3-10d}}{\sqrt{1-2d}\sqrt{3-10d}\sqrt{32d^2-32d+3}}\alpha_b \tag{38}$$

The admissible range of d further gives $1-2d>0$, $\sqrt{32d^2-32d+3}-\sqrt{1-2d}\sqrt{3-10d}<0$ and $(1-d)\sqrt{32d^2-32d+3}-\sqrt{1-2d}\sqrt{3-10d}<0$. Therefore,

$$\frac{\alpha_a}{\alpha_b}<\frac{(1-d)\sqrt{32d^2-32d+3}-\sqrt{1-2d}\sqrt{3-10d}}{(1-2d)\left(\sqrt{32d^2-32d+3}-\sqrt{1-2d}\sqrt{3-10d}\right)} \tag{39}$$

Define $g(d)=\frac{(1-d)\sqrt{32d^2-32d+3}-\sqrt{1-2d}\sqrt{3-10d}}{(1-2d)\left(\sqrt{32d^2-32d+3}-\sqrt{1-2d}\sqrt{3-10d}\right)}$. Figure 4 establishes the ordering of $f_L(d)$, $f_H(d)$, and $g(d)$. Consequently, when $f_L(d)<\frac{\alpha_a}{\alpha_b}<g(d)$, the CM prefers the OEM to choose Strategy I; when $g(d)\leqslant\frac{\alpha_a}{\alpha_b}<f_H(d)$, it prefers the OEM to choose Strategy D.

This completes the proof of Theorem 3.

Proof of the monotonicity of $q_a^{I*}-q_a^{D*}$.

Let $\rho=\frac{\alpha_a}{\alpha_b}$, and define $\Delta q_a(\rho,d)=q_a^{I*}-q_a^{D*}$.

Substituting $\alpha_a=\rho\alpha_b$ into the equilibrium order quantities gives

$$\Delta q_a(\rho,d)=\frac{\alpha_b\left[(2-4d)(12d^2-16d)\rho-2\left((1-d)(32d^2-32d+3)-(20d^2-16d+3)\right)\right]}{(20d^2-16d+3)(32d^2-32d+3)};$$

$$\frac{\partial \Delta q_a(\rho,d)}{\partial \rho}=\frac{\alpha_b(2-4d)(12d^2-16d)}{(20d^2-16d+3)(32d^2-32d+3)}.$$

Under the admissible condition $0<d<\frac{1}{2}-\frac{\sqrt{10}}{8}$, the denominator of $\frac{\partial \Delta q_a(\rho,d)}{\partial \rho}$ is positive. Moreover, $(2-4d)(12d^2-16d)<0$ and $\alpha_b>0$. The numerator of $\frac{\partial \Delta q_a(\rho,d)}{\partial \rho}$ is negative. Therefore, $\frac{\partial \Delta q_a(\rho,d)}{\partial \rho}<0$. Hence, $\Delta q_a(\rho,d)$ is strictly decreasing in ρ throughout the admissible domain. This completes the proof.

Equilibrium solutions under the extended model.

Strategy I

$$\bar{\pi}_a^I=\left[A_a-\left(\bar{q}_a^I+\gamma\bar{q}_b^I\right)-\left(a_1-d\bar{q}_a^I\right)-\bar{w}_b^I-c_{a1}\right]\bar{q}_a^I \tag{40}$$

$$\bar{\pi}_b^I=\left[A_b-\left(\gamma\bar{q}_a^I+\bar{q}_b^I\right)-\left(a_1-d\bar{q}_b^I\right)-\left(a_2-d\bar{q}_a^I-d\bar{q}_b^I\right)-c_{b1}-c_{b2}\right]\bar{q}_b^I+\left[\bar{w}_b^I-\left(a_2-d\bar{q}_a^I-d\bar{q}_b^I\right)-c_{b2}\right]\bar{q}_a^I \tag{41}$$

$$\bar{w}_b^{I^*}=\frac{\left[12d^2+(8\gamma-16)d+4-3\gamma^2\right]\left[(1-2d)(\alpha_a+c_{a2}-c_{b2})+(d-\gamma)\alpha_b\right]}{(1-2d)\left[20d^2+(12\gamma-28)d+8-5\gamma^2\right]}-\frac{(2d-\gamma)\alpha_b}{2(1-2d)}+a_2+c_{b2} \tag{42}$$

$$\bar{q}_a^{I^*}=\frac{2\left[(1-2d)(\alpha_a+c_{a2}-c_{b2})+(d-\gamma)\alpha_b\right]}{20d^2+(12\gamma-28)d+8-5\gamma^2} \tag{43}$$

$$\bar{q}_b^{I^*}=\frac{\alpha_b\left[20d^2+(12\gamma-28)d+8-5\gamma^2\right]+2(2d-\gamma)\left[(1-2d)(\alpha_a+c_{a2}-c_{b2})+(d-\gamma)\alpha_b\right]}{2(1-2d)\left[20d^2+(12\gamma-28)d+8-5\gamma^2\right]} \tag{44}$$

$$\bar{\pi}_a^{I^*}=\frac{2\left[4d^2+(2\gamma-6)d+2-\gamma^2\right]\left[(1-2d)(\alpha_a+c_{a2}-c_{b2})+(d-\gamma)\alpha_b\right]^2}{(1-2d)\left[20d^2+(12\gamma-28)d+8-5\gamma^2\right]^2} \tag{45}$$

$$\bar{\pi}_b^{I^*}=\frac{\alpha_b^2}{4(1-2d)}+\frac{\left[(1-2d)(\alpha_a+c_{a2}-c_{b2})+(d-\gamma)\alpha_b\right]^2}{(1-2d)\left[20d^2+(12\gamma-28)d+8-5\gamma^2\right]} \tag{46}$$

Strategy D

$$\bar{\pi}_a^D=\left[A_a-\left(\bar{q}_a^D+\gamma\bar{q}_b^D\right)-\left(a_1-d\bar{q}_a^D\right)-\left(a_2-d\bar{q}_a^D\right)-\bar{w}_b^D-c_{a1}\right]\bar{q}_a^D \tag{47}$$

$$\bar{\pi}_b^D = \left[A_b - \left(\gamma \bar{q}_a^D + \bar{q}_b^D\right) - \left(a_1 - d\bar{q}_b^D\right) - \left(a_2 - d\bar{q}_b^D\right) - c_{b1} - c_{b2} \right] \bar{q}_b^D + \left(\bar{w}_b^D - c_{b2}\right) \bar{q}_a^D \tag{48}$$

$$\bar{w}_b^{D*} = \frac{\left[4(1-2d)^2 - 3\gamma^2\right]\left[(1-2d)(\alpha_a + c_{a2} - c_{b2}) - \gamma\alpha_b\right]}{(1-2d)\left[8(1-2d)^2 - 5\gamma^2\right]} + \frac{\gamma\alpha_b}{2(1-2d)} + c_{b2} \tag{49}$$

$$\bar{q}_a^{D*} = \frac{2\left[(1-2d)(\alpha_a + c_{a2} - c_{b2}) - \gamma\alpha_b\right]}{8(1-2d)^2 - 5\gamma^2} \tag{50}$$

$$\bar{q}_b^{D*} = \frac{\alpha_b\left[8(1-2d)^2 - 5\gamma^2\right] - 2\gamma\left[(1-2d)(\alpha_a + c_{a2} - c_{b2}) - \gamma\alpha_b\right]}{2(1-2d)\left[8(1-2d)^2 - 5\gamma^2\right]} \tag{51}$$

$$\bar{\pi}_a^{D*} = \frac{2\left[2(1-2d)^2 - \gamma^2\right]\left[(1-2d)(\alpha_a + c_{a2} - c_{b2}) - \gamma\alpha_b\right]^2}{(1-2d)\left[8(1-2d)^2 - 5\gamma^2\right]^2} \tag{52}$$

$$\bar{\pi}_b^{D*} = \frac{\alpha_b^2}{4(1-2d)} + \frac{\left[(1-2d)(\alpha_a + c_{a2} - c_{b2}) - \gamma\alpha_b\right]^2}{(1-2d)\left[8(1-2d)^2 - 5\gamma^2\right]} \tag{53}$$

Strategy O

$$\bar{\pi}_a^O = \left[A_a - \left(\bar{q}_a^O + \gamma \bar{q}_b^O\right) - \left(a_1 - d\bar{q}_a^O\right) - \bar{w}_b^O - c_{a1} - c_{a2} \right] \bar{q}_a^O - k \tag{54}$$

$$\bar{\pi}_b^O = \left[A_b - \left(\gamma \bar{q}_a^O + \bar{q}_b^O\right) - \left(a_1 - d\bar{q}_b^O\right) - \left(a_2 - d\bar{q}_a^O - d\bar{q}_b^O\right) - c_{b1} - c_{b2} \right] \bar{q}_b^O + \left[\bar{w}_b^O - \left(a_2 - d\bar{q}_a^O - d\bar{q}_b^O\right) \right] \bar{q}_a^O \tag{55}$$

$$\bar{w}_b^{O*} = \frac{\left[12d^2 + (8\gamma - 16)d + 4 - 3\gamma^2\right]\left[(1-2d)\alpha_a + (d-\gamma)\alpha_b\right]}{(1-2d)\left[20d^2 + (12\gamma - 28)d + 8 - 5\gamma^2\right]} + a_2 - \frac{(2d-\gamma)\alpha_b}{2(1-2d)} \tag{56}$$

$$\bar{q}_a^{O*} = \frac{2\left[(1-2d)\alpha_a + (d-\gamma)\alpha_b\right]}{20d^2 + (12\gamma - 28)d + 8 - 5\gamma^2} \tag{57}$$

$$\bar{q}_b^{O*} = \frac{\alpha_b\left[20d^2 + (12\gamma - 28)d + 8 - 5\gamma^2\right] + 2(2d-\gamma)\left[(1-2d)\alpha_a + (d-\gamma)\alpha_b\right]}{2(1-2d)\left[20d^2 + (12\gamma - 28)d + 8 - 5\gamma^2\right]} \tag{58}$$

$$\bar{\pi}_a^{O*} = \frac{2\left[4d^2 + (2\gamma - 6)d + 2 - \gamma^2\right]\left[(1-2d)\alpha_a + (d-\gamma)\alpha_b\right]^2}{(1-2d)\left[20d^2 + (12\gamma - 28)d + 8 - 5\gamma^2\right]^2} - k \tag{59}$$

$$\bar{\pi}_b^{O*} = \frac{\alpha_b^2}{4(1-2d)} + \frac{\left[(1-2d)\alpha_a + (d-\gamma)\alpha_b\right]^2}{(1-2d)\left[20d^2 + (12\gamma - 28)d + 8 - 5\gamma^2\right]} \tag{60}$$

Strategy N

$$\bar{\pi}_a^N = \left[A_a - \left(\bar{q}_a^N + \gamma \bar{q}_b^N\right) - \left(a_1 - d\bar{q}_a^N\right) - \left(a_2 - d\bar{q}_a^N\right) - c_{a1} - c_{a2} \right] \bar{q}_a^N - k \tag{61}$$

$$\bar{\pi}_b^N = \left[A_b - \left(\gamma \bar{q}_a^N + \bar{q}_b^N \right) - \left(a_1 - d\bar{q}_b^N \right) - \left(a_2 - d\bar{q}_b^N \right) - c_{b1} - c_{b2} \right] \bar{q}_b^N \tag{62}$$

$$\bar{q}_a^{N*} = \frac{2(1-2d)\alpha_a - \gamma\alpha_b}{4(1-2d)^2 - \gamma^2} \tag{63}$$

$$\bar{q}_b^{N*} = \frac{2(1-2d)\alpha_b - \gamma\alpha_a}{4(1-2d)^2 - \gamma^2} \tag{64}$$

$$\bar{\pi}_a^{N*} = \frac{(1-2d)\left[2(1-2d)\alpha_a - \gamma\alpha_b\right]^2}{\left[4(1-2d)^2 - \gamma^2\right]^2} - k \tag{65}$$

$$\bar{\pi}_b^{N*} = \frac{(1-2d)\left[2(1-2d)\alpha_b - \gamma\alpha_a\right]^2}{\left[4(1-2d)^2 - \gamma^2\right]^2} \tag{66}$$

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