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Research article

Joint operational decisions for a supply chain considering demand-dependent green investment

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Appendix

Proof of Lemma 1

From Equation (3.7), it is calculated that

$$\frac{\partial \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q_1} = o + e - w - \frac{1}{\eta}(e+\alpha-v)F(q_1 \mid \theta), \tag{0.1}$$

$$\frac{\partial \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q} = p - o - e - \alpha - \frac{1}{\eta}(p-\alpha-e)F(q \mid \theta), \tag{0.2}$$

$$\frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q_1^2} = -\frac{1}{\eta}(e+\alpha-v)f(q_1 \mid \theta) < 0,$$

$$\frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q^2} = -\frac{1}{\eta}(p-\alpha-e)f(q \mid \theta) < 0,$$

$$\frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q_1 \partial q} = \frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q \partial q_1} = 0.$$

The Hessian matrix of $\mathrm{CVaR}_\eta(\pi_r(X;q_1,q))$ is

$$\begin{bmatrix} \frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q_1^2} & \frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q_1 \partial q} \\ \frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q \partial q_1} & \frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q^2} \end{bmatrix}.$$

Since $\frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q_1^2} < 0$ and

$$\begin{vmatrix} \frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q_1^2} & \frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q_1 \partial q} \\ \frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q \partial q_1} & \frac{\partial^2 \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q^2} \end{vmatrix} = \frac{1}{\eta^2}(e+\alpha-v)(p-\alpha-e)f(q \mid \theta)f(q_1 \mid \theta) > 0,$$

it follows that the Hessian matrix of $\mathrm{CVaR}_\eta(\pi_r(X;q_1,q))$ is a negative definite matrix. This shows that $\mathrm{CVaR}_\eta(\pi_r(X;q_1,q))$ is a jointly concave function with respect to the decision variables q_1 and q. □

Proof of Proposition 1

From Lemma 1, $\mathrm{CVaR}_\eta(\pi_r(X;q_1,q))$ is a jointly concave function with respect to the decision variables q_1 and q. From Equation (0.1), let $\frac{\partial \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q_1} = 0$ and $A = \frac{(e+o-w)}{e+\alpha-v}\eta$, it is derived that $q_1^* = F^{-1}(A \mid \theta)$. From Equation (0.2), let $\frac{\partial \mathrm{CVaR}_\eta(\pi_r(X;q_1,q))}{\partial q} = 0$ and $B = \frac{(p-\alpha-o-e)}{p-\alpha-e}\eta$, and it is shown that $q^* = F^{-1}(B \mid \theta)$. Since $q_2^* = q^* - q_1^*$, and it is calculated that $q_2^* = F^{-1}(B \mid \theta) - F^{-1}(A \mid \theta)$. □

Proof of Proposition 2

From Proposition 1, $F(q_1^*|\theta) = A = \frac{(e+o-w)}{e+\alpha-v}\eta$, where $q_1^* = q_1^*(\theta)$ is an implicit function of θ and $\frac{(e+o-w)}{e+\alpha-v}\eta$ is a constant function with respect to θ. According to implicit differentiation and properties of cumulative distribution functions, it is derived that

$$\frac{\partial F(q_1^*|\theta)}{\partial q_1^*} \cdot \frac{dq_1^*}{d\theta} + \frac{\partial F(q_1^*|\theta)}{\partial \theta} = 0.$$

Further, it can be calculated that $\frac{dq_1^*}{d\theta} = -\frac{\frac{\partial F(q_1^*|\theta)}{\partial \theta}}{\frac{\partial F(q_1^*|\theta)}{\partial q_1^*}}$. Since $\frac{\partial F(q_1^*|\theta)}{\partial q_1^*} > 0$ and $\frac{\partial F(q_1^*|\theta)}{\partial \theta} < 0$, it is demonstrated that $\frac{dq_1^*}{d\theta} > 0$. The same method can be employed to prove that $\frac{dq^*}{d\theta} > 0$. □

Proof of Proposition 3

Since $X = f(\theta)\varepsilon$, $f(\theta) = \theta^{1/\beta} + \gamma$, and $F(x \mid \theta) = \begin{cases} 0, & x < 0, \\ \dfrac{x}{\bar{M} f(\theta)}, & 0 \le x \le \bar{M} f(\theta), \\ 1, & x > \bar{M} f(\theta), \end{cases}$ it is derived that $f'(\theta) = \frac{d}{d\theta}(\theta^{1/\beta} + \gamma) = \frac{1}{\beta}\theta^{\frac{1}{\beta}-1} > 0$, $\frac{dq_1^*}{d\theta} = \bar{M}Af'(\theta) = \bar{M}\frac{A}{\beta}\theta^{\frac{1}{\beta}-1} > 0$, $\frac{dq^*}{d\theta} = \bar{M}Bf'(\theta) = \bar{M}\frac{B}{\beta}\theta^{\frac{1}{\beta}-1} > 0$, $f''(\theta) = \frac{1-\beta}{\beta^2}\theta^{\frac{1}{\beta}-2} < 0$, $\frac{d^2q_1^*}{d\theta^2} = \bar{M}Af''(\theta) < 0$, and $\frac{d^2q^*}{d\theta^2} = \bar{M}Bf''(\theta) < 0$. Since $M_1 = w - o - e + (e-v)A$ and $M_2 = o + e - c - (e-v)B$, from Equation (3.9), it is derived that the first and second derivatives of $\mathrm{E}[\pi_s(X; q_1^*(\theta), q^*(\theta), \theta)]$ with respect to θ are respectively expressed as follows:

$$\frac{d\mathrm{E}[\pi_s(X; q_1^*(\theta), q^*(\theta), \theta)]}{d\theta} = \bar{M}f'(\theta)\left[AM_1 + BM_2 + \frac{e-v}{2}(B^2 - A^2)\right] - (1-\delta)g'(\theta), \tag{0.3}$$

$$\frac{d^2\mathrm{E}[\pi_s(X; q_1^*(\theta), q^*(\theta), \theta)]}{d\theta^2} = \bar{M}f''(\theta)\left[AM_1 + BM_2 + \frac{e-v}{2}(B^2 - A^2)\right] - (1-\delta)g''(\theta). \tag{0.4}$$

Since $K = AM_1 + BM_2 + \frac{e-v}{2}(B^2 - A^2)$, $K = A(w - o - e) + B(o + e - c) + \frac{e-v}{2}(A^2 - B^2)$. Since $B > A$, $K > A(w - o - e) + B(o + e - c) + (e - v)(A - B) = A(w - o - v) + B(o + v - c) > [\frac{w-o-v}{e+\alpha-v} - \frac{c-o-v}{e-v}]\eta = \frac{(w-o-v)(e-v)-(e+\alpha-v)(c-o-v)}{(e+\alpha-v)(e-v)}\eta = \frac{(w-c)(e-v)-\alpha(c-o-v)}{(e+\alpha-v)(e-v)}\eta$. Since $w - c > \alpha$ and $e + o > c$, it can be deduced that $K > 0$. Since $f''(\theta) = \frac{1-\beta}{\beta^2}\theta^{\frac{1}{\beta}-2} < 0$ and $\frac{d^2g(\theta)}{d^2\theta} > 0$, from Equation (0.4), it follows that $\frac{d^2\mathrm{E}[\pi_s(X;q_1^*(\theta),q^*(\theta),\theta)]}{d^2\theta} < 0$. Then $\mathrm{E}[\pi_s(X; q_1^*(\theta), q^*(\theta), \theta)]$ is a concave function with respect to θ. Since $\frac{dg(\theta)}{d\theta} > 0$ and $f'(\theta) = \frac{1}{\beta}\theta^{\frac{1}{\beta}-1} > 0$, it can be deduced that $\overline{\theta}$ satisfies $\frac{d\mathrm{E}[\pi_s(X;q_1^*(\theta),q^*(\theta),\theta)]}{d\theta} = \bar{M}f'(\overline{\theta})\left[AM_1 + BM_2 + \frac{e-v}{2}(B^2 - A^2)\right] - (1-\delta)g'(\overline{\theta}) = 0$, it is demonstrated that $\bar{M}f'(\overline{\theta})K - (1-\delta)g'(\overline{\theta}) = 0$. Due to the constraints $0 \le \theta \le 1$, it follows that with Karush–Kuhn–Tucker (KKT) optimality conditions, the optimal green investment strategy of the manufacturer is

$$\theta^* = \begin{cases} 0, & \overline{\theta} \le 0, \\ \overline{\theta}, & 0 < \overline{\theta} < 1, \\ 1, & 1 \le \overline{\theta}. \end{cases}$$

□

Proof of Proposition 4

Equation (3.13) can be reformulated as

$$\mathrm{E}[\pi_c(X; Q, \Theta)] = (p - \alpha)Q - cQ - g(\Theta) - (p - \alpha - v)\int_0^Q F(x \mid \Theta)\mathrm{d}x.$$

Under the centralized supply chain, the first-order and second-order partial derivatives of $\mathrm{E}[\pi_c(Q, \Theta)]$ with respect to Θ are

$$\begin{aligned} \frac{\partial \mathrm{E}[\pi_c(X; Q, \Theta)]}{\partial \Theta} &= -\frac{dg(\Theta)}{d\Theta} - (p - \alpha - v)\int_0^Q \frac{\partial F(x \mid \Theta)}{\partial \Theta}\mathrm{d}x, \\ \frac{\partial \mathbb{E}[\pi_c(X; Q, \Theta)]}{\partial \Theta} &= -g'(\Theta) - (p - \alpha - v)\int_0^Q \frac{\partial F(x \mid \Theta)}{\partial \Theta}\mathrm{d}x \\ &= -g'(\Theta) + \frac{(p - \alpha - v)\, Q^2\, f'(\Theta)}{2\,\bar{M}\, f^2(\Theta)}, \end{aligned} \tag{0.5}$$

and

$$\frac{\partial^2 \mathrm{E}[\pi_c(X; Q, \Theta)]}{\partial^2 \Theta} = -\frac{d^2g(\Theta)}{d^2\Theta} - (p - \alpha - v)\int_0^Q \frac{\partial^2 F(x \mid \Theta)}{\partial^2 \Theta}\mathrm{d}x < 0, \tag{0.6}$$

$$\begin{aligned} \frac{\partial^2 \mathbb{E}[\pi_c(X; Q, \Theta)]}{\partial \Theta^2} &= -g''(\Theta) - (p - \alpha - v)\int_0^Q \frac{\partial^2 F(x \mid \Theta)}{\partial \Theta^2}\mathrm{d}x \\ &= -g''(\Theta) - \frac{(p - \alpha - v)\, Q^2}{2} \cdot \frac{2(f'(\Theta))^2 - f(\Theta)f''(\Theta)}{\bar{M}\, f^3(\Theta)}. \end{aligned}$$

The first-order and second-order partial derivatives of $\mathrm{E}[\pi_c(Q, \Theta)]$ with respect to Q are

$$\frac{\partial \mathrm{E}[\pi_c(X; Q, \Theta)]}{\partial Q} = p - \alpha - c - (p - \alpha - v)F(Q \mid \Theta) \tag{0.7}$$

and

$$\frac{\partial^2 \mathrm{E}[\pi_c(X;Q,\Theta)]}{\partial^2 Q} = -(p-\alpha-v)\frac{\partial F(Q \mid \Theta)}{\partial Q} = -\frac{p-\alpha-v}{\bar{M}\, f(\Theta)} < 0. \tag{0.8}$$

The first-order partial derivative of $\frac{\partial \mathrm{E}[\pi_c(Q,\Theta)]}{\partial \Theta}$ with respect to Q is

$$\frac{\partial^2 \mathrm{E}[\pi_c(X;Q,\Theta)]}{\partial \Theta \partial Q} = -(p-\alpha-v)\frac{\partial F(Q \mid \Theta)}{\partial \Theta} = \frac{(p-\alpha-v)\, Q\, f'(\Theta)}{\bar{M}\, f^2(\Theta)} > 0, \tag{0.9}$$

and the first-order partial derivative of $\frac{\partial \mathrm{E}[\pi_c(Q,\Theta)]}{\partial Q}$ with respect to Θ is

$$\frac{\partial^2 \mathrm{E}[\pi_c(X;Q,\Theta)]}{\partial Q \partial \Theta} = -(p-\alpha-v)\frac{\partial F(Q \mid \Theta)}{\partial \Theta} = \frac{(p-\alpha-v)\, Q\, f'(\Theta)}{\bar{M}\, f^2(\Theta)} > 0. \tag{0.10}$$

The Hessian matrix of $\mathrm{E}[\pi_c(X;Q,\Theta)]$ is $H = \begin{pmatrix} \dfrac{\partial^2 \mathrm{E}[\pi_c(X;Q,\Theta)]}{\partial^2 \Theta} & \dfrac{\partial^2 \mathrm{E}[\pi_c(X;Q,\Theta)]}{\partial \Theta \partial Q} \\ \dfrac{\partial^2 \mathrm{E}[\pi_c(X;Q,\Theta)]}{\partial \Theta \partial Q} & \dfrac{\partial^2 \mathrm{E}[\pi_c(X;Q,\Theta)]}{\partial^2 Q} \end{pmatrix}$.

It is proved that

$$\begin{aligned} \det(H) &= \left(g''(\Theta) + \frac{(p-\alpha-v)Q^2}{2}\frac{2(f')^2 - ff''}{\bar{M}f^3}\right)\left(\frac{(p-\alpha-v)}{\bar{M}f}\right) - \left(\frac{(p-\alpha-v)Qf'}{\bar{M}f^2}\right)^2 \\ &= g''(\Theta)\left(\frac{(p-\alpha-v)}{\bar{M}f}\right) - \frac{f''}{2f^3}\left(\frac{(p-\alpha-v)Q}{\bar{M}}\right)^2. \end{aligned}$$

Since $g''(\Theta) > 0$ and $f''(\Theta) = \frac{1-\beta}{\beta^2}\Theta^{\frac{1}{\beta}-2} < 0$, it follows that $\det(H) > 0$, and the Hessian matrix of $\mathrm{E}[\pi_c(X;Q,\Theta)]$ is strictly negative definite. Then $\mathrm{E}[\pi_c(X;Q,\Theta)]$ is a jointly concave function with respect to Q and Θ. Let

$$\frac{\partial \mathrm{E}[\pi_c(X;Q,\Theta)]}{\partial \Theta} = -g'(\Theta) + \frac{(p-\alpha-v)\, Q^2\, f'(\Theta)}{2\,\bar{M}\, f^2(\Theta)} = 0$$

and

$$\frac{\partial \mathrm{E}[\pi_c(X;Q,\Theta)]}{\partial Q} = p-\alpha-c-(p-\alpha-v)F(Q \mid \Theta) = 0.$$

It can be deduced that the optimal production quantity is $Q^* = F^{-1}(C \mid \Theta^*) > 0$, where $C = \frac{p-\alpha-c}{p-\alpha-v}$, and the optimal green investment is $\frac{\partial \mathbb{E}[\pi_c(X;Q,\Theta)]}{\partial \Theta} = -g'(\overline{\Theta}) + \frac{(p-\alpha-v)\, Q^{*2}\, f'(\overline{\Theta})}{2\,\bar{M}\, f^2(\overline{\Theta})} = 0$. Since $\frac{Q^*}{\bar{M}f(\overline{\Theta})} = C$ and $Q^* = C\,\bar{u}\, f(\overline{\Theta})$, it is demonstrated that $g'(\overline{\Theta}) - \frac{(p-\alpha-v)}{2}C^2\bar{M}f'(\overline{\Theta}) = 0$. Due to the constraints $0 \le \Theta \le 1$, it follows that with Karush–Kuhn–Tucker (KKT) optimality conditions, the optimal green investment strategy of the centralized system is

$$\Theta^* = \begin{cases} 0, & \overline{\Theta} \le 0, \\ \overline{\Theta}, & 0 < \overline{\Theta} < 1, \\ 1, & 1 \le \overline{\Theta}. \end{cases}$$

□

Proof of Proposition 5

From the equation $p-\alpha-c-(p-\alpha-v)F(Q^* \mid \Theta^*)=0$ in Proposition 4, it is derived that $F(Q^* \mid \Theta^*)=\dfrac{p-\alpha-c}{p-\alpha-v}=C$. From Proposition 1, $q_1^*=F^{-1}(A \mid \theta)$ and $q_2^*=F^{-1}(B \mid \theta)-F^{-1}(A \mid \theta)$, and it is deduced that $q^*=F^{-1}(B \mid \theta)$, where $A=\dfrac{(e+o-w)}{e+\alpha-v}\eta$ and $B=\dfrac{(p-\alpha-o-e)}{p-\alpha-e}\eta$. Based on $Q^*=q^*$ and $\Theta^*=\theta^*$, it is demonstrated that $B=C$ and $(p-\alpha-c)(p-\alpha-e)=(p-\alpha-v)(p-\alpha-o-e)\eta$, i.e., Equation (3.17). From Propositions 3 and 4, $\bar{M}f'(\bar{\theta})K-(1-\delta)g'(\bar{\theta})=0$ and $g'(\overline{\Theta})-\frac{(p-\alpha-v)}{2}C^2\bar{M}f'(\overline{\Theta})=0$. It is derived that $\bar{\theta}=\overline{\Theta}$ and $2K=(p-\alpha-v)(1-\delta)B^2$, i.e., Equation (3.18). □

Proof of Proposition 7

Proposition 6 indicates that $\frac{(p-w-\alpha)}{p-v}\eta=F(q_0^* \mid \theta)$. According to Equation (3.15), it is derived that $\frac{p-\alpha-c}{p-\alpha-v}=F(Q^* \mid \Theta^*)$. Since $w>c$ and $w>v$, it is shown that $p-\alpha-c>p-w-\alpha$. Further, it is derived that $\frac{p-\alpha-c}{p-\alpha-v}>\frac{(p-w-\alpha)}{p-v}\eta$. When $\Theta^*=\theta_0^*$, $F(Q^* \mid \Theta^*)>F(q_0^* \mid \theta_0^*)$. According to the properties of the function $F(\cdot \mid \theta)$, it is deduced that $q_0^*<Q^*$, which means that the carbon abatement cost-sharing and wholesale price contracts cannot achieve joint carbon abatement and coordination of the supply chain. □

Proof of Proposition 8

From Equation (4.5), it is demonstrated that $g'(\overline{\theta_1})=\bar{M}f'(\overline{\theta_1})K$. From Equation (3.15), it is demonstrated that $g'(\overline{\Theta})=\frac{(p-\alpha-v)}{2}C^2\bar{M}f'(\overline{\Theta})$. We will proceed by contradiction. Suppose the statement of Proposition 8 does not hold, i.e., $\theta_1^*>\Theta^*$. It is derived that $\overline{\theta_1}>\overline{\Theta}$, $g'(\overline{\theta_1})>g'(\overline{\Theta})$, and $f'(\overline{\theta_1})<f'(\overline{\Theta})$. When $q^*=Q^*$, it is deduced that $B<C$. Since $K=A(w-o-e)+B(o+e-c)+\frac{e-v}{2}(A^2-B^2)$ and $A(o+e-w)>\frac{e-v}{2}A^2$, it is derived that $K<B(o+e-c)-\frac{e-v}{2}B^2=\frac{e-v}{2}B^2<\frac{p-\alpha-v}{2}C^2$. Further, it is demonstrated that $\bar{M}f'(\overline{\theta_1})K<\frac{(p-\alpha-v)}{2}C^2\bar{M}f'(\overline{\Theta})$, which violates $g'(\overline{\theta_1})>g'(\overline{\Theta})$. Therefore, the initial assumption that $\theta_1^*>\Theta^*$ is false, and it follows that $\theta_1^*\leq\Theta^*$.

□

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