



Research article

Leveraging live streaming for effective platform encroachment: Overcoming product quality disadvantages

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Appendix A: The derivation of the outcomes

Proof of Model N

In the benchmark case where the platform does not introduce a private label, we solve the game using backward induction. In the second stage, for a given wholesale price w^N , the platform sets its retail price to maximize its profit: $\pi_P^N = (p_n^N - w^N)(1 - p_n^N)$. The first-order condition yields the retail price response function $p_n^N(w^N) = \frac{1+w^N}{2}$. In the first stage, the manufacturer anticipates this response and sets w^N to maximize its profit: $\pi_M^N = w^N \left(\frac{1-w^N}{2} \right)$. Solving $\frac{\partial \pi_M^N}{\partial w^N} = 0$ yields $w^N = \frac{1}{2}$. Substituting this back gives the equilibrium outcomes: $w^N = \frac{1}{2}$, $p_n^N = \frac{3}{4}$, and $Q_n^{N*} = \frac{1}{4}$. The equilibrium profits are $\pi_P^N = \frac{1}{16}$ and $\pi_M^N = \frac{1}{8}$.

Proof of Model E

When the platform introduces a store brand without live streaming, the store-brand base quality is a_0 . In this main model, a_0 is a given parameter and is not optimized. The inverse demand functions are $p_n^E = 1 - Q_n^E -$

$a_0 Q_s^E$, $p_s^E = a_0(1 - Q_n^E - Q_s^E)$. For a given wholesale price w^E , the platform determines Q_n^E and Q_s^E to maximize $\pi_P^E = (1 - Q_n^E - a_0 Q_s^E - w^E)Q_n^E + [a_0(1 - Q_n^E - Q_s^E) - c_s]Q_s^E$. The first-order conditions in the interior subgame are $\frac{\partial \pi_P^E}{\partial Q_n^E} = 1 - w^E - 2Q_n^E - 2a_0 Q_s^E = 0$, $\frac{\partial \pi_P^E}{\partial Q_s^E} = a_0 - c_s - 2a_0 Q_n^E - 2a_0 Q_s^E = 0$. The

Hessian matrix of the platform's problem with respect to (Q_n^E, Q_s^E) is $H_P^E = \begin{bmatrix} \frac{\partial^2 \pi_P^E}{\partial (Q_n^E)^2} & \frac{\partial^2 \pi_P^E}{\partial Q_n^E \partial Q_s^E} \\ \frac{\partial^2 \pi_P^E}{\partial Q_s^E \partial Q_n^E} & \frac{\partial^2 \pi_P^E}{\partial (Q_s^E)^2} \end{bmatrix} =$

$\begin{bmatrix} -2 & -2a_0 \\ -2a_0 & -2a_0 \end{bmatrix}$. The leading principal minors satisfy $\Delta_1^E = -2 < 0$, $\Delta_2^E = 4a_0(1 - a_0) > 0$. Hence, for $0 <$

$a_0 < 1$, H_P^E is negative definite and the platform's second-stage problem is jointly concave. Solving the first-order conditions $\frac{\partial \pi_P^E}{\partial Q_n^E} = 0$ and $\frac{\partial \pi_P^E}{\partial Q_s^E} = 0$ yields the conditional quantity responses in the false encroachment

subgame: $Q_n^E(w^E) = \frac{1-a_0-w^E+c_s}{2(1-a_0)}$, $Q_s^E(w^E) = \frac{a_0 w^E - c_s}{2a_0(1-a_0)}$. The interior solution is valid when $Q_s^E(w^E) > 0$,

that is, when $w^E > c_s/a_0$. When $w^E \leq c_s/a_0$, the platform chooses the boundary $Q_s^E = 0$ and the national-brand quantity is $Q_n^E(w^E) = \frac{1-w^E}{2}$, $Q_s^E(w^E) = 0$. In the first stage, the manufacturer anticipates the platform

response and chooses w^E . Under false encroachment, the manufacturer's profit is $\pi_M^E(w^E) = w^E Q_n^E(w^E) = w^E \frac{1-a_0-w^E+c_s}{2(1-a_0)}$. Since $\frac{\partial^2 \pi_M^E}{\partial (w^E)^2} = -\frac{1}{1-a_0} < 0$, the manufacturer's problem is concave in w^E . The first-order

condition is $\frac{\partial \pi_M^E}{\partial w^E} = \frac{1-a_0-2w^E+c_s}{2(1-a_0)} = 0$, which gives $w_G^E = \frac{1-a_0+c_s}{2}$. This interior wholesale price must also

satisfy the positive store-brand sales condition $w_G^E > \frac{c_s}{a_0}$. If this condition fails but the platform can still be held

at $Q_s^E = 0$, the manufacturer uses limit pricing and sets $w_F^E = \frac{c_s}{a_0}$. Moreover, if the store-brand production cost

is sufficiently high, the encroachment threat is not binding, and the manufacturer reverts to the benchmark wholesale price $w^E = 1/2$. Consequently, the equilibrium partitions into false encroachment, substantive encroachment, and no encroachment regions according to the cost thresholds reported in the main text. The explicit outcomes in Table 3 follow by substituting Eqs.

Proof of Model ES

Under live streaming, the perceived store-brand quality becomes $g = a_0 + \lambda s$. In the main ES model, s is treated as a given live streaming intensity; the optimization of s is postponed to Extension 1. Replacing a_0 by g in the structure of Model E, the platform's profit is $\pi_P^{ES} = (1 - Q_n^{ES} - gQ_s^{ES} - w^{ES})Q_n^{ES} + [g(1 - Q_n^{ES} - Q_s^{ES}) - c_s]Q_s^{ES} - \frac{s^2}{2}$. Because the investment cost $s^2/2$ does not depend on Q_n^{ES} or Q_s^{ES} , it does not affect the

platform's quantity first-order conditions in this stage. The Hessian matrix is $H_P^{ES} = \begin{bmatrix} -2 & -2g \\ -2g & -2g \end{bmatrix}$. The

principal minors are $\Delta_1^{ES} = -2 < 0$, $\Delta_2^{ES} = 4g(1 - g) > 0$. Thus, for $0 < g < 1$, the platform's quantity problem is jointly concave. Solving the simultaneous first-order conditions yields, in the false encroachment

subgame, $Q_n^{ES}(w^{ES}, s) = \frac{1-g-w^{ES}+c_s}{2(1-g)}$, $(w^{ES}, s) = \frac{gw^{ES}-c_s}{2g(1-g)}$. The positive store-brand sales condition is $w^{ES} > c_s/g$. Under this condition, the manufacturer maximizes $\pi_M^{ES}(w^{ES}, s) = w^{ES} \frac{1-g-w^{ES}+c_s}{2(1-g)}$. The second derivative is $\frac{\partial^2 \pi_M^{ES}}{\partial (w^{ES})^2} = -\frac{1}{1-g} < 0$, and the first-order condition gives $w_G^{ES} = \frac{1-g+c_s}{2} = \frac{1-(a_0+\lambda s)+c_s}{2}$. Moreover, if the false encroachment condition is not satisfied, the manufacturer protects the national-brand market through limit pricing, $w_F^{ES} = \frac{c_s}{g} = \frac{c_s}{a_0+\lambda s}$. When the encroachment threat is not binding, $w^{ES} = 1/2$. The corresponding thresholds are obtained in exactly the same way as in Model E by substituting $g = a_0 + \lambda s$ for a_0 . The equilibrium expressions reported in Table 4 follow from substituting Eqs.

Appendix B: Proofs of propositions

Proof of Proposition 1

The proposition compares the equilibrium outcomes with and without store-brand introduction. Without store brand, Model N gives $w^N = \frac{1}{2}$, $p_n^N = \frac{3}{4}$, $\pi_P^N = \frac{1}{16}$, $\pi_M^N = \frac{1}{8}$. With store-brand introduction, substituting the platform's optimal quantities into the manufacturer's profit yields a piecewise function: $\pi_M^E(w) = \begin{cases} w \frac{1-w}{2}, & Q_s^E = 0, \\ w \frac{1-a_0-w+c_s}{2(1-a_0)}, & Q_s^E > 0. \end{cases}$ In each region, the relevant second derivative is negative, as shown in Appendix A.

Therefore, the regional optimum can be obtained by the corresponding first-order condition, and the global equilibrium is selected by comparing the interior solution with the boundary solutions. This procedure yields two cost thresholds, denoted by c_{s1}^E and c_{s2}^E , as reported in Table 3. The comparative statics are obtained by differentiating the threshold functions: $\frac{\partial c_{s1}^E}{\partial a_0} > 0$, $\frac{\partial c_{s2}^E}{\partial a_0} > 0$. When the thresholds are written as $c_{s1}^E = a_0$ and $c_{s2}^E = (2a_0 - 1)/(1 - a_0)$, $\frac{\partial c_{s1}^E}{\partial a_0} > 0$, $\frac{\partial c_{s2}^E}{\partial a_0} > 0$ becomes $\frac{\partial c_{s1}^E}{\partial a_0} = 1 > 0$, $\frac{\partial c_{s2}^E}{\partial a_0} = \frac{1}{(1-a_0)^2} > 0$. This proves that a higher exogenous base quality expands the parameter region in which store-brand introduction is viable. It should be emphasized again that $\frac{\partial c_{s1}^E}{\partial a_0} = 1 > 0$, $\frac{\partial c_{s2}^E}{\partial a_0} = \frac{1}{(1-a_0)^2} > 0$ is a comparative-static result with respect to a_0 , not an optimality condition for choosing a_0 .

Proof of Proposition 2

Using the equilibrium outcomes of Models E and ES, let $g = a_0 + \lambda s$. The wholesale prices in the false encroachment region are $w_G^E = \frac{1-a_0+c_s}{2}$, $w_G^{ES} = \frac{1-g+c_s}{2} = \frac{1-a_0-\lambda s+c_s}{2}$. Thus, $w_G^{ES} - w_G^E = -\frac{\lambda s}{2} < 0$, which implies that live streaming reduces the wholesale price in the false encroachment region. In the false

encroachment region, $w_F^E = \frac{c_s}{a_0}$, $w_F^{ES} = \frac{c_s}{g} = \frac{c_s}{a_0 + \lambda s}$. Since $g > a_0$, it follows that $w_F^{ES} < w_F^E$. In the no encroachment region, $w^{ES} = w^E = 1/2$. Moreover, when the live streaming effort s is sufficiently low, $w^{ES} < w^E$; and if s is sufficiently large, the comparison is determined by the zero-difference condition $\Delta w = w^{ES} - w^E = 0$, which defines the threshold c_w . Hence, $w^{ES} > w^E$ for $c_s < c_w$ and $w^{ES} < w^E$ for $c_s > c_w$. For national-brand sales, the false encroachment quantities are $Q_n^E = \frac{1-a_0+c_s}{4(1-a_0)}$, $Q_n^{ES} = \frac{1-g+c_s}{4(1-g)}$. In the false encroachment region, $Q_n^E = \frac{a_0-c_s}{2a_0}$, $Q_n^{ES} = \frac{g-c_s}{2g}$. The sign of $Q_n^{ES} - Q_n^E$ is obtained by direct subtraction. When the expression is not signed globally, threshold c_q is defined by $Q_n^{ES} - Q_n^E = 0$. Therefore, $Q_n^{ES} < Q_n^E$ for $c_s < c_q$ and $Q_n^{ES} > Q_n^E$ for $c_s > c_q$. Under no encroachment, $Q_n^{ES} = Q_n^E = \frac{1}{4}$. The proof follows.

Proof of Proposition 3

Under live streaming, the perceived quality is $g = a_0 + \lambda s$. Substituting g into the thresholds in Proposition 1 gives $c_{s1}^{ES} = \frac{g}{2} = \frac{a_0 + \lambda s}{2}$, $c_{s2}^{ES} = \frac{g(1-g)}{2-g}$. Since $g > a_0$ for $\lambda s > 0$, both thresholds increase relative to the no live streaming case. Hence, the false encroachment region expands and the no encroachment region shrinks. For profit comparison in the false encroachment region, platform profits are $\pi_P^{ES} = \frac{(1-g+c_s)^2}{8(1-g)} + \frac{(g-c_s)^2}{8g} - \frac{s^2}{2}$, $\pi_P^E = \frac{(1-a_0+c_s)^2}{8(1-a_0)} + \frac{(a_0-c_s)^2}{8a_0}$. Let $\Delta_P = \pi_P^{ES} - \pi_P^E$. For a given s , Δ_P is not necessarily monotonic in a_0 . Differentiating Δ_P with respect to a_0 gives a critical value a_0^+ , satisfying $\left. \frac{\partial \Delta_P}{\partial a_0} \right|_{a_0=a_0^+} = 0$. Here, a_0^+ is only a threshold parameter value; it is not an optimal decision. Solving $\Delta_P = 0$ similarly defines a cost threshold c_s^+ such that $\pi_P^{ES} \geq \pi_P^E \Leftrightarrow c_s \leq c_s^+$. This proves the stated profit comparison.

Proof of Proposition 4

Let $g(s) = a_0 + \lambda s$. Because $\lambda > 0$ and $s > 0$, $g(s) > a_0$. For the upper boundary separating the store-brand-introduction region from the no encroachment region, differentiating the boundary with respect to g gives $\frac{\partial c_{s1}}{\partial g} > 0$. By the chain rule, $\frac{\partial c_{s1}^{ES}}{\partial s} = \frac{\partial c_{s1}}{\partial g} \cdot \frac{\partial g}{\partial s} = \lambda \frac{\partial c_{s1}}{\partial g} > 0$. For the lower boundary separating genuine and false encroachment, the same calculation yields $\frac{\partial c_{s2}^{ES}}{\partial s} = \lambda \frac{\partial c_{s2}}{\partial g} > 0$. Therefore, live streaming shifts both thresholds upward. This mathematically shows that live streaming expands the false encroachment parameter space while shrinking the single-brand resale region.

Proof of Proposition 5

Let $g = a_0 + \lambda s$. In the false encroachment region, the platform's profit can be written as $\pi_P^{ES}(g, s) = \frac{(1-g+c_s)^2}{8(1-g)} + \frac{(g-c_s)^2}{8g} - \frac{s^2}{2}$. Differentiating with respect to the live streaming intensity gives the marginal effect

$\frac{\partial \pi_P^{ES}}{\partial s} = \lambda \frac{c_s^2(2g-1)}{8g^2(1-g)^2} - s$. For a given s , differentiating $\frac{\partial \pi_P^{ES}}{\partial s}$ with respect to the exogenous base quality a_0 gives $\frac{\partial}{\partial a_0} \left(\frac{\partial \pi_P^{ES}}{\partial s} \right) = \frac{\lambda c_s^2(3g^2-3g+1)}{4g^3(1-g)^3}$. Since $3g^2 - 3g + 1 > 0$ for $g \in (0,1)$, $\frac{\partial}{\partial a_0} \left(\frac{\partial \pi_P^{ES}}{\partial s} \right)$ is positive. Thus, the marginal profit gain from live streaming increases with the store-brand base quality. This conclusion is again a comparative-static result and does not mean that a_0 is chosen optimally. For the manufacturer, under false encroachment, $\pi_M^{ES} = \frac{(1-g+c_s)^2}{8(1-g)}$. Differentiating yields $\frac{\partial \pi_M^{ES}}{\partial s} = \lambda \frac{\partial \pi_M^{ES}}{\partial g} = -\lambda \frac{1-g+c_s}{4(1-g)^2} < 0$. Therefore, live streaming reduces manufacturer profit in this region. Moreover, $\left| \frac{\partial \pi_M^{ES}}{\partial s} \right| = \lambda \frac{1-g+c_s}{4(1-g)^2}$, and the comparative-static relationship with respect to a_0 follows by differentiating $\left| \frac{\partial \pi_M^{ES}}{\partial s} \right|$. This proves that the magnitude of the negative effect varies with the store-brand base quality.

Proof of Proposition 6

In the false encroachment region, let $g = a_0 + \lambda s$. The equilibrium demands can be expressed as $Q_n = 1 - \frac{p_n - p_s}{1-g}$, $Q_s = \frac{p_n - p_s}{1-g} - \frac{p_s}{g}$. Define the sales difference as $\Delta Q = Q_s - Q_n$. Substituting the equilibrium prices and simplifying yields $\Delta Q = \frac{c_s - c_n}{4g(1-g)}$, $c_s = \frac{g(2w+g-1)}{g+1}$. In equilibrium, $w = (1-g+c_s)/2$, so c_s depends only on g . Since $4g(1-g) > 0$, the sign of ΔQ is determined by $c_s - c_n$. Hence, $Q_s < Q_n \Leftrightarrow c_s < c_n$. Total sales are $T = Q_n + Q_s = \frac{1}{2} - \frac{c_s}{4g(1-g)}$. Thus, T decreases in c_s . Total sales are maximized when c_s is sufficiently low, in which case the store brand dominates.

Proof of Proposition 7

Let $g = a_0 + \lambda s$, where $0 < g < 1$. The two relevant cost thresholds in Model ES are $c_{s1}^{ES} = \frac{g}{2} = \frac{a_0 + \lambda s}{2}$, $c_{s2}^{ES} = \frac{g(1-g)}{2-g}$. In the interior dual-brand region, the Hessian matrix of the platform's profit with respect to (p_n, p_s) is $H_P = -\frac{2}{1-g} \begin{pmatrix} 1 & -1 \\ -1 & \frac{1}{g} \end{pmatrix}$. H_P is negative definite for $0 < g < 1$, and the platform's first-order conditions yield a unique interior maximum. In the substantive-encroachment region, the manufacturer's profit is $\pi_M(w) = \frac{w(1+c_s-g-w)}{2(1-g)}$. The first-order condition gives $w^S = \frac{1+c_s-g}{2}$, and $\frac{\partial^2 \pi_M}{\partial w^2} = -\frac{1}{1-g} < 0$. In the no-encroachment region, $w^N = \frac{1}{2}$ and $\frac{\partial^2 \pi_M}{\partial w^2} = -1 < 0$. Differentiating the two thresholds with respect to λ gives $\frac{\partial c_{s2}^{ES}}{\partial \lambda} = \frac{s}{2} > 0$, and $\frac{\partial c_{s1}^{ES}}{\partial \lambda} = s \frac{g^2-4g+2}{(2-g)^2}$. The latter derivative changes sign once at $g = 2 - \sqrt{2}$, and the second derivative is negative at this point. Hence, c_{s2} increases monotonically in λ , whereas c_{s1} exhibits an inverted-U relationship with λ , proving part (i). Next, substituting the equilibrium decisions into the platform-profit function

gives $\pi_P^{ES} = \pi_P(g', c_s, s)$. Since $g = a_0 + \lambda s$, $\frac{\partial \pi_P^{ES}}{\partial \lambda} = s \frac{\partial \pi_P^{ES}}{\partial g}$. When $c_s < c_{s2}^{ES}$, direct differentiation gives $\frac{\partial \pi_P^{ES}}{\partial \lambda} > 0$, $\frac{\partial \pi_P^{ES}}{\partial c_s} > 0$. Therefore, platform profit increases with λ and c_s , proving part (ii). Finally, when c_s is sufficiently high, define the critical efficiency level $\hat{\lambda}$ implicitly by $c_s = c_{s1}(\hat{\lambda}) = \frac{g(\hat{\lambda})[1-g(\hat{\lambda})]}{2-g(\hat{\lambda})}$, where $g(\hat{\lambda}) = a_0 + \hat{\lambda}s$. Thus, the platform does not encroach when $\lambda < \hat{\lambda}$. Regarding the explicit form of λ_1 , it is derived by equating the profits at the boundary between the no-encroachment and false-encroachment regions: $\frac{(1-c_s/g)^2}{4} - \frac{s^2}{2} = \frac{1}{16} - \frac{s^2}{2}$, yielding $g = 2c_s$. Substituting $g = a_0 + \lambda s$ directly gives $\lambda_1 = \frac{2c_s - a_0}{s}$. Direct differentiation yields $\frac{\partial \pi_P^{ES}}{\partial \lambda} < 0$ for $\hat{\lambda} < \lambda < \lambda_1$, and $\frac{\partial \pi_P^{ES}}{\partial \lambda} > 0$ for $\lambda > \lambda_1$. This proves part (iii).

Proof of Proposition 8: Extension 1, endogenous live streaming intensity

This is the part of the analysis in which s is treated as a decision variable. In the main derivations above, s is given; in Extension 1, the platform chooses s to maximize its value function. When $0 \leq c_s < \frac{(a_0 + \lambda s)(1 - a_0 - \lambda s)}{2 - a_0 - \lambda s}$, the platform is in the false encroachment region. Let $g = a_0 + \lambda s$. The platform's objective can be written as $V_G(s) = \frac{-3g^3 + 6g^2c_s + 2g^2 - 3gc_s^2 - 6gc_s + g + 4c_s^2}{16g(1-g)} - \frac{s^2}{2}$. The first-order condition for the optimal live streaming intensity s_G is $\frac{\partial V_G(s)}{\partial s} = 0$. Equivalently, $16(a_0 + \lambda s)^2(1 - a_0 - \lambda s)^2s = \lambda[3(a_0 + \lambda s)^4 - 6(a_0 + \lambda s)^3 + 3(a_0 + \lambda s)^2 - 3(a_0 + \lambda s)^2c_s^2 + 8(a_0 + \lambda s)c_s^2 - 4c_s^2]$.

When $\frac{(a_0 + \lambda s)(1 - a_0 - \lambda s)}{2 - a_0 - \lambda s} \leq c_s < \frac{a_0 + \lambda s}{2}$, the platform is in the false encroachment region. The first-order condition reduces to $2s(a_0 + \lambda s)^3 = \lambda c_s(a_0 + \lambda s - c_s)$. This establishes the implicit form of the optimal live streaming intensity in the false encroachment region.

Proof of Corollary 1

In the false encroachment region, define $F(s, a_0, c_s) = 2s(a_0 + \lambda s)^3 - \lambda c_s(a_0 + \lambda s - c_s)$. The optimal live streaming intensity s_F satisfies $F(s_F, a_0, c_s) = 0$. Let $A_F = a_0 + \lambda s_F$. Under the local second-order condition $D_F = \frac{\partial F}{\partial s} = 2A_F^3 + 6\lambda s_F A_F^2 - \lambda^2 c_s > 0$, the implicit function theorem yields $\frac{\partial s_F}{\partial a_0} = -\frac{\partial F / \partial a_0}{\partial F / \partial s} = \frac{\lambda c_s - 6s_F A_F^2}{D_F} = \frac{2s_F A_F^2(3c_s - 2A_F)}{(A_F - c_s)D_F} < 0$, and $\frac{\partial s_F}{\partial c_s} = -\frac{\partial F / \partial c_s}{\partial F / \partial s} = \frac{\lambda(A_F - 2c_s)}{D_F} \geq 0$. Thus, the optimal live streaming intensity decreases with the store-brand base quality and weakly increases with the store-brand production cost in the false encroachment region.

Proof of Proposition 9

Under agency selling, let $g = a_0 + \lambda s$. Without live streaming, the relevant thresholds are denoted as $c_{s1}^A = a_0 - \frac{\alpha(1-a_0)}{2(1-\alpha)}$, $c_{s2}^A = \frac{a_0(1-\alpha)}{1-\alpha+\alpha a_0}$. With live streaming, substituting g for a_0 gives $c_{s1}^{A,S} = g - \frac{\alpha(1-g)}{2(1-\alpha)}$, $c_{s2}^{A,S} = g$. Since $g > a_0$, both thresholds increase: $c_{s1}^{A,S} > c_{s1}^A$, $c_{s2}^{A,S} > c_{s2}^A$. For a given c_s , if $c_s \in (c_{s1}^A, c_{s1}^{A,S})$, the store brand is not introduced without live streaming but is introduced with live streaming. Moreover, if $c_s \in (c_{s2}^A, c_{s2}^{A,S})$, the store brand has zero sales without live streaming but positive sales with live streaming. Hence, live streaming expands the store-brand introduction region and the false encroachment region.

Proof of Proposition 10

Let $\Delta_c = c_{s1}^{A,S} - c_s^1$, where $c_s^1 = g$ is the false encroachment threshold under reselling. From $c_{s1}^{A,S} = g - \frac{\alpha(1-g)}{2(1-\alpha)}$, $c_{s2}^{A,S} = g$, $\Delta_c = -\frac{\alpha(1-g)}{2(1-\alpha)} < 0$, $\alpha \in (0,1)$. Thus, the threshold under agency selling is lower than that under reselling, which implies that encroachment is less likely under agency selling. Moreover, $\frac{\partial \Delta_c}{\partial s} = -\frac{\alpha\lambda}{2(1-\alpha)} < 0$. Therefore, the gap between the agency-selling and reselling thresholds widens with live streaming intensity.

Proof of Proposition 11

The sequential-pricing structure gives the manufacturer a first-mover advantage because the manufacturer commits to w before the platform determines the store-brand decision. Under simultaneous pricing, the platform and the manufacturer choose prices at the same stage. Solving the corresponding first-order conditions gives the simultaneous-pricing wholesale price w^S and the threshold c_s^S . The comparison is obtained by subtracting the sequential-pricing outcomes from the simultaneous-pricing outcomes. The platform's profit under simultaneous pricing is higher for any given c_s , which relaxes the condition for profitable store-brand introduction: $c_s^S < c_s^1$. Moreover, the equilibrium wholesale price satisfies $w^S < w^1$, because the manufacturer can no longer commit to a high wholesale price before the platform sets its store-brand price. This proves the proposition.



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