



Research article

Dynamic competition and stability analysis of dual-track battery recycling systems under low-carbon transition

Haohao Song¹, Wenping Liu¹, Xiaobo Wang¹, Jing Zhang¹ and Ying Wang^{2,*}

¹ School of Business, NanJing Xiaozhuang University, Nanjing, Jiangsu, China

² College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu, China

* **Correspondence:** Email:yingwang@nuaa.edu.cn

Supplementary Material

Proof of Proposition 1: Calculating the partial derivatives of results with respect to parameter μ , it is obtained that $\frac{\partial P_1}{\partial \mu} = \frac{\gamma(1+\beta\xi)(\eta^2-\beta^2\sigma^2)U_1}{M^2}$, $\frac{\partial P_2}{\partial \mu} = \frac{\beta(2+\beta\xi)(\eta^2-\beta^2\sigma^2)U_1}{M^2}$, $\frac{\partial e}{\partial \mu} = -\frac{(\eta+\beta\sigma)U_1U_3}{M^2}$, $\frac{\partial Q_1}{\partial \mu} = -\frac{\beta\gamma(\eta^2-\beta^2\sigma^2)U_1}{M^2}$, $\frac{\partial Q_2}{\partial \mu} = -\frac{\beta U_1(\eta+\beta\sigma)[\beta^3\xi\sigma-\beta\gamma^2\xi\sigma+2\beta^2\sigma-\gamma^2\sigma+\beta^2\eta\xi+2\beta\eta]}{M^2}$, $\frac{\partial \pi_1}{\partial \mu} = -\frac{\beta\gamma(2+\beta\xi)(\eta^2-\beta^2\sigma^2)U_1U_2}{M^3}$, and $\frac{\partial \pi_2}{\partial \mu} = \frac{U_1^2(\eta+\beta\sigma)\{\mu(\eta+\beta\sigma)^2[\beta\eta(2+\beta\xi)+\sigma A_6]-\beta\sigma(U_3-2\gamma^2-2\beta\gamma^2\xi)-\eta U_4\}}{2M^3}$. To simplify the analysis, we will set $U_1 = A_2A_6 + \alpha A_5 - \beta\gamma A_1$, $U_2 = A_1A_7 + \alpha A_3 - \gamma(\beta\mu - \eta^2 - \beta\eta\sigma)A_2$, $U_3 = 4\beta^2 - \gamma^2 + 2\beta^3\xi - \beta\gamma^2\xi$, and $U_4 = 4\beta^2 + \gamma^2 + 2\beta^3\xi + \beta\gamma^2\xi$. Combining the equilibrium results of Q_1 and Q_2 , it is obtained that $U_1, U_2, U_3, U_4 > 0$ and then $\frac{\partial e}{\partial \mu}, \frac{\partial Q_2}{\partial \mu} < 0$. When $\eta > \beta\sigma$, it is obtained that $\frac{\partial P_1}{\partial \mu}, \frac{\partial P_2}{\partial \mu} > 0$ and $\frac{\partial Q_1}{\partial \mu}, \frac{\partial \pi_1}{\partial \mu} < 0$. When $\eta \leq \beta\sigma$, it is obtained that $\frac{\partial P_1}{\partial \mu}, \frac{\partial P_2}{\partial \mu} \leq 0$ and $\frac{\partial Q_1}{\partial \mu}, \frac{\partial \pi_1}{\partial \mu} \geq 0$. Letting $\mu_5 = \frac{(\eta+\beta\sigma)^2[\beta^2\eta\xi+2\beta\eta+\sigma A_6]}{\beta\sigma U_3+\eta U_4}$, we can obtain that $\mu_5 - \mu_1 = -\frac{\gamma^2(1+\beta\xi)(\eta-\beta\sigma)(\eta+\beta\sigma)^2}{2\beta(\beta\sigma U_3+\eta U_4)}$ and $\mu_5 - \mu_2 = -\frac{\gamma^2(1+\beta\xi)(\eta^2-\beta^2\sigma^2)[2\beta\eta(2+\beta\xi)+\sigma U_3]}{U_3(\beta\sigma U_3+\eta U_4)}$. Thus, when $\eta \geq \beta\sigma$, we can obtain that $\mu_5 \leq \mu_1, \mu_2$ and then $\frac{\partial \pi_2}{\partial \mu} < 0$. When $\eta < \beta\sigma$, we can obtain that $\frac{\partial \pi_2}{\partial \mu} > 0$ if $\mu \in (\mu_1, \mu_5)$ and $\frac{\partial \pi_2}{\partial \mu} \leq 0$ if $\mu \in [\mu_5, +\infty)$. Therefore, **Proposition 1** is proved.

Proof of Proposition 2: Calculating the partial derivatives of results with respect to parameter ξ , it is obtained that $\frac{\partial P_1}{\partial \xi} = -\frac{\beta^2 A_4 U_2}{M^2}$, $\frac{\partial P_2}{\partial \xi} = -\frac{\beta^2 \gamma[\mu-\sigma(\eta+\beta\sigma)]U_2}{M^2}$, $\frac{\partial e}{\partial \xi} = \frac{\beta^2 \gamma(\eta+\beta\sigma)U_2}{M^2}$, $\frac{\partial Q_1}{\partial \xi} = -\frac{\beta^2 A_7 U_2}{M^2}$, $\frac{\partial Q_2}{\partial \xi} = \frac{\beta^3 \gamma \mu U_2}{M^2}$, $\frac{\partial \pi_1}{\partial \xi} = -\frac{\beta^2 U_2^2\{\mu U_3-(\eta+\beta\sigma)[-3\gamma^2\sigma+\beta^3\xi\sigma+\beta^2(\eta\xi+2\sigma)+\beta(2\eta-\gamma^2\xi\sigma)]\}}{2M^3}$, and $\frac{\partial \pi_2}{\partial \xi} = \frac{\beta^2 \gamma \mu A_4 U_1 U_2}{M^3}$. It is obvious that $\frac{\partial e}{\partial \xi}, \frac{\partial Q_2}{\partial \xi}, \frac{\partial \pi_2}{\partial \xi} > 0$ and $\frac{\partial P_1}{\partial \xi} < 0$. The relationship between Q_1 and ξ is highly correlated with the value of A_7 . Hence, when

$\eta \leq \beta\sigma$, we can obtain that $\frac{\partial Q_1}{\partial \xi} < 0$; when $\eta > \beta\sigma$, $\frac{\partial Q_1}{\partial \xi} > 0$ holds if $\mu \in (\mu_2, \mu_4)$, and $\frac{\partial Q_1}{\partial \xi} < 0$ holds if $\mu \in [\mu_4, +\infty)$. Letting $\mu_6 = \sigma(\eta + \beta\sigma)$, it is obtained that $\mu_6 - \mu_1 = -\frac{\eta^2 - \beta^2\sigma^2}{2\beta}$ and $\mu_6 - \mu_2 = -\frac{\beta(2+\beta\xi)(\beta^2\sigma^2 - \eta^2)}{U_3}$. Thus, when $\eta \geq \beta\sigma$, we can obtain that $\mu_6 \leq \mu_1, \mu_2$ and then $\frac{\partial P_2}{\partial \xi} < 0$. When $\eta < \beta\sigma$, $\frac{\partial P_2}{\partial \xi} > 0$ holds if $\mu \in (\mu_1, \mu_6)$, and $\frac{\partial P_2}{\partial \xi} \leq 0$ holds if $\mu \in [\mu_6, +\infty)$. Letting $\mu_7 = \frac{(\eta + \beta\sigma)[-3\gamma^2\sigma + \beta^3\xi\sigma + \beta^2(\eta\xi + 2\sigma) + \beta(2\eta - \gamma^2\xi\sigma)]}{U_3}$, it is obtained that $\mu_7 - \mu_1 = \frac{\gamma^2(3+\beta\xi)(\eta^2 - \beta^2\sigma^2)}{2\beta U_3}$ and $\mu_7 - \mu_2 = \frac{2\beta\gamma^2(2+\beta\xi)(\eta^2 - \beta^2\sigma^2)}{(4\beta^2 - 3\gamma^2 + 2\beta^3\xi - \beta\gamma^2\xi)U_3}$. Hence, when $\eta \leq \beta\sigma$, we can obtain that $\frac{\partial \pi_1}{\partial \xi} < 0$; when $\eta > \beta\sigma$, $\frac{\partial \pi_1}{\partial \xi} > 0$ holds if $\mu \in (\mu_2, \mu_7)$, and $\frac{\partial \pi_1}{\partial \xi} \leq 0$ holds if $\mu \in [\mu_7, +\infty)$. Therefore, **Proposition 2** is proved.

Proof of Proposition 3: Calculating the partial derivatives of results with respect to parameter η , it is obtained that $\frac{\partial P_1}{\partial \eta} = -\frac{\gamma(1+\beta\xi)[2\eta\mu - \sigma(\eta + \beta\sigma)^2]U_1}{M^2}$, $\frac{\partial P_2}{\partial \eta} = -\frac{\beta(2+\beta\xi)[2\eta\mu - \sigma(\eta + \beta\sigma)^2]U_1}{M^2}$, $\frac{\partial e}{\partial \eta} = \frac{[\mu U_3 + \beta(2+\beta\xi)(\eta + \beta\sigma)^2]U_1}{M^2}$, $\frac{\partial Q_1}{\partial \eta} = -\frac{\beta\gamma U_1[2\eta\mu - \sigma(\eta + \beta\sigma)^2]}{M^2}$, $\frac{\partial Q_2}{\partial \eta} = \frac{\beta\mu U_1(2\beta^3\xi\sigma - \beta\gamma^2\xi\sigma + 4\beta^2\gamma^2\sigma + 2\beta^2\eta\xi + 4\beta\eta)}{M^2}$, $\frac{\partial \pi_1}{\partial \eta} = \frac{\beta\gamma(2+\beta\xi)U_1 U_2[2\eta\mu - \sigma(\eta + \beta\sigma)^2]}{M^3}$, and $\frac{\partial \pi_2}{\partial \eta} = \frac{\mu U_1^2[(\beta\sigma U_3 + \eta U_4)\mu - \beta(2+\beta\xi)(\eta + \beta\sigma)^3]}{M^3}$. Obviously, $\frac{\partial e}{\partial \eta}, \frac{\partial Q_2}{\partial \eta} > 0$. Letting $\mu_8 = \frac{\sigma(\eta + \beta\sigma)^2}{2\eta}$, it is obtained that $\mu_8 - \mu_1 = -\frac{(\eta - \beta\sigma)(\eta + \beta\sigma)^2}{2\beta\eta}$ and $\mu_8 - \mu_2 = -\frac{(\eta^2 - \beta^2\sigma^2)[U_3 + 2\beta\eta(\beta\xi + 2)]}{2\eta U_3}$. Thus, when $\eta \geq \beta\sigma$, we can obtain that $\frac{\partial P_1}{\partial \eta}, \frac{\partial P_2}{\partial \eta} < 0$ and $\frac{\partial Q_1}{\partial \eta}, \frac{\partial \pi_1}{\partial \eta} > 0$; when $\eta < \beta\sigma$, $\frac{\partial P_1}{\partial \eta}, \frac{\partial P_2}{\partial \eta} > 0$ and $\frac{\partial Q_1}{\partial \eta}, \frac{\partial \pi_1}{\partial \eta} < 0$ hold if $\mu \in (\mu_1, \mu_8)$, and $\frac{\partial P_1}{\partial \eta}, \frac{\partial P_2}{\partial \eta} \leq 0$ and $\frac{\partial Q_1}{\partial \eta}, \frac{\partial \pi_1}{\partial \eta} \geq 0$ hold if $\mu \in [\mu_8, +\infty)$. Letting $\mu_9 = \frac{\beta(2+\beta\xi)(\eta + \beta\sigma)^3}{\beta\sigma U_3 + \eta U_4}$, it is obtained that $\mu_9 - \mu_1 = -\frac{\gamma^2(1+\beta\xi)(\eta - \beta\sigma)(\eta + \beta\sigma)^2}{2\beta(\beta\sigma U_3 + \eta U_4)}$ and $\mu_9 - \mu_2 = -\frac{\gamma^2(1+\beta\xi)[2\beta\eta(2+\beta\xi) + \sigma U_3](\eta^2 - \beta^2\sigma^2)}{U_3(\beta\sigma U_3 + \eta U_4)}$. Thus, when $\eta \geq \beta\sigma$, we can obtain that $\frac{\partial \pi_2}{\partial \eta} > 0$; when $\eta < \beta\sigma$, $\frac{\partial \pi_2}{\partial \eta} < 0$ holds if $\mu \in (\mu_1, \mu_9)$, and $\frac{\partial \pi_2}{\partial \eta} \geq 0$ holds if $\mu \in [\mu_9, +\infty)$. Therefore, **Proposition 3** is proved.

Proof of Proposition 4: Calculating the partial derivatives of results with respect to parameter σ , it is obtained that $\frac{\partial P_1}{\partial \sigma} = \frac{\gamma(1+\beta\xi)U_1[2\beta^2\sigma\mu - \eta(\eta + \beta\sigma)^2]}{M^2}$, $\frac{\partial P_2}{\partial \sigma} = \frac{\beta(2+\beta\xi)U_1[2\beta^2\sigma\mu - \eta(\eta + \beta\sigma)^2]}{M^2}$, $\frac{\partial e}{\partial \sigma} = \frac{U_1[\beta\mu U_3 + (2\beta^2 - \gamma^2 + \beta^3\xi - \beta\gamma^2\xi)(\eta + \beta\sigma)^2]}{M^2}$, $\frac{\partial Q_1}{\partial \sigma} = -\frac{\beta\gamma U_1[2\beta^2\sigma\mu - \eta(\eta + \beta\sigma)^2]}{M^2}$, $\frac{\partial Q_2}{\partial \sigma} = \frac{\beta\mu U_1[\eta U_3 + 2\beta\sigma(\beta^3\xi - \beta\gamma^2\xi + 2\beta^2 - \gamma^2)]}{M^2}$, $\frac{\partial \pi_1}{\partial \sigma} = -\frac{\beta\gamma(2+\beta\xi)U_1 U_2[2\beta^2\sigma\mu - \eta(\eta + \beta\sigma)^2]}{M^3}$, and $\frac{\partial \pi_2}{\partial \sigma} = \frac{\mu U_1^2[\beta\mu[\eta U_3 + \beta\sigma(4\beta^2 - 3\gamma^2 + 2\beta^3\xi - 3\beta\gamma^2\xi)] - (2\beta^2 - \gamma^2 + \beta^3\xi - \beta\gamma^2\xi)(\eta + \beta\sigma)^3]}{M^3}$. It is clear that $\frac{\partial e}{\partial \sigma}, \frac{\partial Q_2}{\partial \sigma} > 0$. Letting $\mu_{10} = \frac{\eta(\eta + \beta\sigma)^2}{2\beta^2\sigma}$, we can obtain that $\mu_{10} - \mu_1 = \frac{(\eta - \beta\sigma)(\eta + \beta\sigma)^2}{2\beta^2\sigma}$ and $\mu_{10} - \mu_2 = \frac{(\eta^2 - \beta^2\sigma^2)(\beta\sigma U_3 + \eta U_4)}{2\beta^2 U_3 \sigma}$. Thus, when $\eta \leq \beta\sigma$, we can obtain that $\frac{\partial P_1}{\partial \sigma}, \frac{\partial P_2}{\partial \sigma} > 0$ and $\frac{\partial Q_1}{\partial \sigma}, \frac{\partial \pi_1}{\partial \sigma} < 0$ always hold; when $\eta > \beta\sigma$, $\frac{\partial P_1}{\partial \sigma}, \frac{\partial P_2}{\partial \sigma} < 0$ and $\frac{\partial Q_1}{\partial \sigma}, \frac{\partial \pi_1}{\partial \sigma} > 0$ hold if $\mu \in (\mu_2, \mu_{10})$, and $\frac{\partial P_1}{\partial \sigma}, \frac{\partial P_2}{\partial \sigma} \geq 0$ and $\frac{\partial Q_1}{\partial \sigma}, \frac{\partial \pi_1}{\partial \sigma} \leq 0$ hold if $\mu \in [\mu_{10}, +\infty)$. Let $\mu_{11} = \frac{(2\beta^2 - \gamma^2 + \beta^3\xi - \beta\gamma^2\xi)(\eta + \beta\sigma)^3}{\beta[\eta U_3 + \beta\sigma(4\beta^2 - 3\gamma^2 + 2\beta^3\xi - 3\beta\gamma^2\xi)]}$. Unlike the preceding thresholds, the relative position of μ_{11} with respect to μ_1 and μ_2 depends on the parameter configuration. Consequently, the effect of σ on the formal recycler's profit is conditional rather than uniform. Therefore, **Proposition 4** is proved.



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