



Research article

Should multinational firms participate in blockchain to provide quality verification under different market structures?

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Appendix

Proof of Corollary 1.

(i) Because $0 < \vartheta < \rho < \alpha < 1$, $0 < \tau < \frac{1}{3}$, then we can obtain that

$$\frac{\partial w^N}{\partial a} = \frac{\rho(\alpha\rho(5-7\tau) - \rho^2(1-\tau) - 4\alpha^2(2-3\tau))}{2\alpha(\rho(3-2\tau) - 8\alpha(1-\tau))} > 0, \quad \frac{\partial w^P}{\partial a} = \frac{\alpha(\alpha(5-7\tau) - \alpha^2(1-\tau) + 12\tau - 8)}{2(\alpha^2 - 4 + 8\tau - 2\alpha(1+\tau))} > 0,$$

$$\frac{\partial p_R^N}{\partial a} = \frac{2\alpha\rho^2(2-3\tau) - 8\alpha^3(1-\tau) - \rho^3(1-\tau) - \alpha^2\rho(1-5\tau)}{2\alpha(\rho(3-2\tau) - 8\alpha(1-\tau))} > 0, \quad \frac{\partial p_R^P}{\partial a} = \frac{\alpha(3 - (5-\alpha)\tau)}{4 - \alpha^2 - 8\tau + 2\alpha(1+\tau)} > 0,$$

$$\frac{\partial p_{E1}^N}{\partial a} = \frac{\rho(\alpha(4-5\tau) - \rho(1-\tau))}{8\alpha(1-\tau) - \rho(3-2\tau)} > 0, \quad \frac{\partial p_{E1}^P}{\partial a} = \frac{6\alpha^2(1-\tau) - \alpha^3(1-\tau) + 8\tau - \alpha(7-5\tau) - 4}{2(\alpha^2 + 8\tau - 2\alpha(1+\tau) - 4)} > 0,$$

$$\frac{\partial \pi_R^N}{\partial a} = \frac{(8\alpha^2 - 7\alpha\rho + \rho^2)(1-\tau)^2}{4\alpha(\rho(3-2\tau) - 8\alpha(1-\tau))^2} > 0, \quad \frac{\partial \pi_R^P}{\partial a} = \frac{\alpha(2-5\alpha+\alpha^2)(1-\tau)^2}{4(\alpha^2+8\tau-2\alpha(1+\tau)-4)^2} > 0,$$

$$\frac{\partial \pi_{E1}^N}{\partial a} = \frac{\rho(\alpha\rho(3-5\tau) - \rho^2(1-\tau) + 2\alpha^2\tau)^2}{4\alpha^2(\rho(3-2\tau) + 8\alpha(1-\tau))^2} > 0, \quad \frac{\partial \pi_{E1}^P}{\partial a} = \frac{(\alpha+8\tau-7\alpha\tau+\alpha^2(1+\tau)-4)^2}{4(\alpha^2+8\tau-2\alpha(1+\tau)-4)^2} > 0,$$

$$\frac{\partial \pi_F^N}{\partial a} = \frac{3\alpha^2\rho(1+2\tau-3\tau^2) - 8\alpha^3(1-\tau) - \rho^3(1-\tau)^2 + 2\alpha\rho^2(1-4\tau+3\tau^2)}{4\alpha(\rho(3-2\tau) - 8\alpha(1-\tau))} > 0,$$

$$\frac{\partial \pi_F^P}{\partial a} = \frac{\alpha(1-\tau)(\alpha(2-6\tau) - \alpha^2(1-\tau) + 9\tau - 5)}{4(\alpha^2+8\tau-2\alpha(1+\tau)-4)} > 0.$$

(ii) Because $0 < \vartheta < \rho < \alpha < 1$, $0 < \tau < \frac{1}{3}$, then we can obtain that

$$\frac{\partial \pi_R^N}{\partial a} - \frac{\partial \pi_{E1}^N}{\partial a} = \frac{(\alpha-\rho)(64\alpha^4(1-\tau)^2 + \rho^4(1-\tau)^2 - 2\alpha\rho^3(3-8\tau+5\tau^2) - 4\alpha^3\rho(12-24\tau+13\tau^2) + \alpha^2\rho^2(17-46\tau+33\tau^2))}{4\alpha^2(\rho(3-2\tau) - 8\alpha(1-\tau))^2} > 0,$$

$$\frac{\partial \pi_R^P}{\partial a} - \frac{\partial \pi_{E1}^P}{\partial a} = \frac{-(1-\alpha)(16(1-2\tau)^2 + \alpha^4(1-\tau)^2 + \alpha(4+16\tau-52\tau^2) - 2\alpha^3(5-8\tau+5\tau^2) + \alpha^2(17-30\tau+33\tau^2))}{4(\alpha^2+8\tau-2\alpha(1+\tau)-4)^2} < 0.$$

Proof of Corollary 2.

Because $0 < \vartheta < \rho < \alpha < 1$, $0 < \tau < \frac{1}{3}$, $1 < a < 7$, then we can obtain that

$$(i) p_R^N - p_{E1}^N = \frac{a(\alpha-\rho)(\rho^2(1-\tau) - 8\alpha^2(1-\tau) - \alpha\rho(1-3\tau))}{2\alpha(\rho(3-2\tau) - 8\alpha(1-\tau))} > 0, \quad q_R^N - q_{E1}^N = \frac{(\alpha-\rho)(\rho-\rho\tau - \alpha(4-5\tau))}{\alpha(\rho(3-2\tau) - 8\alpha(1-\tau))} > 0,$$

$$(ii) p_R^P - p_{E1}^P = \frac{a(1-\alpha)(4+\alpha(5-3\tau) - \alpha^2(1-\tau) - 8\tau)}{2(\alpha^2+8\tau-2\alpha(1+\tau)-4)} < 0, \quad q_R^P - q_{E1}^P = \frac{(1-\alpha)(3-(5-\alpha)\tau)}{\alpha^2+8\tau-2\alpha(1+\tau)-4} < 0.$$

Proof of Corollary 3.

Because $0 < \vartheta < \rho < \alpha < 1$, $0 < \tau < \frac{1}{3}$, $1 < a < 7$, then we can obtain that

$$(i) \frac{\partial w^P}{\partial \tau} = \frac{a\alpha(16-52\alpha+40\alpha^2-11\alpha^3+\alpha^4)}{2(\alpha^2+8\tau-2\alpha(1+\tau)-4)^2} < 0, \quad \frac{\partial w^N}{\partial \tau} = \frac{a\rho(36\alpha^2\rho-32\alpha^3-11\alpha\rho^2+\rho^3)}{2\alpha(\rho(3-2\tau) - 8\alpha(1-\tau))^2} < 0,$$

$$\frac{\partial w^N}{\partial \tau} - \frac{\partial w^P}{\partial \tau} = \frac{a\rho(36\alpha^2\rho-32\alpha^3-11\alpha\rho^2+\rho^3)}{2\alpha(\rho(3-2\tau) - 8\alpha(1-\tau))^2} - \frac{a\alpha(16-52\alpha+40\alpha^2-11\alpha^3+\alpha^4)}{2(4-\alpha^2-8\tau+2\alpha(1+\tau))^2} > 0.$$

$$(ii) \frac{\partial p_R^N}{\partial \tau} = \frac{a(3\alpha - \rho)\rho(8\alpha^2 - 7\alpha\rho + \rho^2)}{2\alpha(8\alpha(1-\tau) - \rho(3-2\tau))^2} < 0, \quad \frac{\partial p_R^P}{\partial \tau} = \frac{a\alpha(12\alpha - 7\alpha^2 + \alpha^3 - 4)}{(4 - \alpha^2 - 8\tau + 2\alpha(1+\tau))^2} < 0,$$

$$\frac{\partial p_{E1}^N}{\partial \tau} = \frac{a\rho(8\alpha^2 - 7\alpha\rho + \rho^2)}{(8\alpha(1-\tau) - \rho(3-2\tau))^2} < 0, \quad \frac{\partial p_{E1}^P}{\partial \tau} = \frac{a\alpha(12 - 40\alpha + 33\alpha^2 - 10\alpha^3 + \alpha^4)}{2(\alpha^2 + 8\tau - 2\alpha(1+\tau) - 4)^2} < 0,$$

$$\frac{\partial p_R^N}{\partial \tau} - \frac{\partial p_{E1}^N}{\partial \tau} = \frac{a\rho(15\alpha^2\rho - 8\alpha\rho^2 + \rho^3 - 8\alpha^3)}{2\alpha(\rho(3-2\tau) - 8\alpha(1-\tau))^2} > 0, \quad \frac{\partial p_R^P}{\partial \tau} - \frac{\partial p_{E1}^P}{\partial \tau} = \frac{a\alpha(4 - 16\alpha + 19\alpha^2 - 8\alpha^3 + \alpha^4)}{2(4 - \alpha^2 - 8\tau + 2\alpha(1+\tau))^2} < 0,$$

$$\frac{\partial q_R^N}{\partial \tau} = \frac{\rho(8\alpha^2 - 7\alpha\rho + \rho^2)}{2\alpha(\rho(3-2\tau) - 8\alpha(1-\tau))^2} > 0, \quad \frac{\partial q_R^P}{\partial \tau} = \frac{(2-\alpha)^2(2-5\alpha+\alpha^2)}{2(4-\alpha^2-8\tau+2\alpha(1+\tau))^2} > 0,$$

$$\frac{\partial q_{E1}^N}{\partial \tau} = \frac{(2\alpha - \rho)(8\alpha^2 - 7\alpha\rho + \rho^2)}{2\alpha(\rho(3-2\tau) - 8\alpha(1-\tau))^2} > 0, \quad \frac{\partial q_{E1}^P}{\partial \tau} = \frac{\alpha(4 - 12\alpha + 7\alpha^2 - \alpha^3)}{2(4 - \alpha^2 - 8\tau + 2\alpha(1+\tau))^2} > 0,$$

$$\frac{\partial q_R^N}{\partial \tau} - \frac{\partial q_{E1}^N}{\partial \tau} = \frac{15\alpha^2\rho - 8\alpha\rho^2 + \rho^3 - 8\alpha^3}{\alpha(\rho(3-2\tau) - 8\alpha(1-\tau))^2} > 0, \quad \frac{\partial q_R^P}{\partial \tau} - \frac{\partial q_{E1}^P}{\partial \tau} = \frac{4 - 16\alpha + 19\alpha^2 - 8\alpha^3 + \alpha^4}{(4 - \alpha^2 - 8\tau + 2\alpha(1+\tau))^2} < 0.$$

Proof of Corollary 4.

According to the assumptions in the original paper in 5.5.1, because $0 < \tau < \frac{1}{3}$, $\rho = \theta$, $\alpha \rightarrow 1$, then we can obtain that

$$(i) \frac{\partial w^{PN/NP}}{\partial \rho} = \frac{-4\rho^3(7-13\tau+6\tau^2) - 36\rho(10-21\tau+11\tau^2) + 18(13-28\tau+15\tau^2) + \rho^2(178-353\tau+174\tau^2)}{(18-7\rho-18\tau+6\rho\tau)^2} > 0,$$

$$\frac{\partial w^{NN}}{\partial \rho} = \frac{4\rho^4(-1+\tau)^2 + 72(4-9\tau+5\tau^2) - 48\rho(9-20\tau+11\tau^2) - 4\rho^3(15-29\tau+14\tau^2) + \rho^2(255-525\tau+268\tau^2)}{(\rho(15-14\tau) + 24(-1+\tau) + 2\rho^2(-1+\tau))^2} > 0,$$

$$\begin{aligned} \frac{\partial w^{PN/NP}}{\partial \rho} - \frac{\partial w^{NN}}{\partial \rho} = & \frac{-2(-3+\rho)(-3+2\rho)(-13+7\rho) + (-504 + \rho(756 + \rho(-353+52\rho)))\tau - 6(-3+\rho)^2(-5+4\rho)\tau^2}{(18-7\rho+6(-3+\rho)\tau)^2} + \\ & \frac{\rho(432 + \rho(-255-4(-15+\rho)\rho)) + \rho(-960 + \rho(525+4\rho(-29+2\rho)))\tau - 4(-3+\rho)^2(10+(-8+\rho)\rho)\tau^2 + 72(-4+9\tau)}{(24(-1+\tau) + \rho(15-2\rho+2(-7+\rho)\tau))^2} \end{aligned}$$

$$\text{when } 0 < \rho < A, \quad \frac{\partial w^{PN/NP}}{\partial \rho} - \frac{\partial w^{NN}}{\partial \rho} > 0; \text{ when } A < \rho < \alpha, \quad \frac{\partial w^{PN/NP}}{\partial \rho} - \frac{\partial w^{NN}}{\partial \rho} < 0.$$

Plot3D $\left[\frac{\partial w^{PN/NP}}{\partial \rho} - \frac{\partial w^{NN}}{\partial \rho} > 0, \{0 < \rho < 0.8\}, \{0 < \tau < \frac{1}{3}\}\right]$, we can get Figure a:

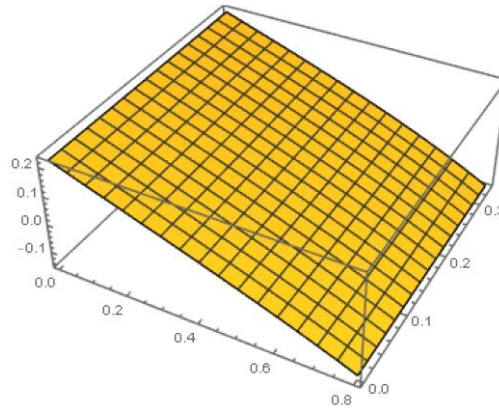


Figure. a.

$$(ii) \quad \frac{\partial w^{PP}}{\partial \tau} = -\frac{6}{(11-12\tau)^2} < 0, \quad \frac{\partial w^{NN}}{\partial \tau} = \frac{3\rho(-24+35\rho-15\rho^2+2\rho^3)}{(\rho(15-14\tau)+24(-1+\tau)+2\rho^2(-1+\tau))^2} < 0,$$

$$\frac{\partial w^{NP/PN}}{\partial \tau} = \frac{\rho(-36+45\rho-17\rho^2+2\rho^3)}{(18-7\rho-18\tau+6\rho\tau)^2} < 0,$$

$$\frac{\partial w^{NN}}{\partial \tau} - \frac{\partial w^{NP/PN}}{\partial \tau} = -\frac{\rho(-36+45\rho-17\rho^2+2\rho^3)}{(18-7\rho-18\tau+6\rho\tau)^2} + \frac{3(-3+\rho)\rho(8+\rho(-9+2\rho))}{(24(-1+\tau)+\rho(15-2\rho+2(-7+\rho)\tau))^2} < 0,$$

$$\frac{\partial w^{NN}}{\partial \tau} - \frac{\partial w^{PP}}{\partial \tau} = \frac{3(-1+\rho)(-1152(-1+\tau)^2 + \rho^2(-1685+3648\tau-1976\tau^2) + 24\rho(133-284\tau+152\tau^2) + \rho^3(250-544\tau+296\tau^2))}{(\rho(15-14\tau)+24(-1+\tau)+2\rho^2(-1+\tau))^2(11-12\tau)^2} > 0,$$

$$\frac{\partial w^{PN}}{\partial \tau} - \frac{\partial w^{PP}}{\partial \tau} = \frac{(-1+\rho)(-15\rho^2(11-12\tau)^2 + 2\rho^3(11-12\tau)^2 - 1944(-1+\tau)^2 + 36\rho(109-234\tau+126\tau^2))}{(11-12\tau)^2(18-7\rho-18\tau+6\rho\tau)^2} > 0,$$

Then we can easily yield $\frac{\partial w^{PP}}{\partial \tau} < \frac{\partial w^{NN}}{\partial \tau} < \frac{\partial w^{NP/PN}}{\partial \tau} < 0$.

$$(iii) \quad w^{NN} - w^{PN/NP} = \frac{(-1+\rho)\rho(-96+4\rho^3(-1+\tau)^2 + 186\tau-90\tau^2 + \rho^2(-32+66\tau-34\tau^2) + \rho(93-188\tau+96\tau^2))}{(\rho(15-14\tau)+24(-1+\tau)+2\rho^2(-1+\tau))(18-7\rho-18\tau+6\rho\tau)} < 0,$$

$$w^{NN} - w^{PP} = \frac{(-1+\rho)(\rho(-87+205\tau-120\tau^2) + 24(5-11\tau+6\tau^2) + \rho^2(22-46\tau+24\tau^2))}{(\rho(15-14\tau)+24(-1+\tau)+2\rho^2(-1+\tau))(-11+12\tau)} < 0,$$

$$w^{PN} - w^{PP} = -\frac{(-1+\rho)(\rho(-88+195\tau-108\tau^2) + 18(5-11\tau+6\tau^2) + \rho^2(22-46\tau+24\tau^2))}{(-11+12\tau)(18-7\rho-18\tau+6\rho\tau)} < 0,$$

Then we can easily yield $w^{NN} < w^{PN/NP} < w^{PP}$.

Corollary 5. Because $0 < \tau < \frac{1}{3}$, $\rho = \theta$, $\alpha \rightarrow 1$, then we can obtain that

$$(i) \quad \frac{\partial p_R^{PN/PP}}{\partial \rho} = \frac{-4\rho^3(7-13\tau+6\tau^2)-36\rho(9-19\tau+10\tau^2)+3(59-128\tau+69\tau^2)+\rho^2(171-340\tau+168\tau^2)}{(18-7\rho-18\tau+6\rho\tau)^2} > 0,$$

$$\frac{\partial p_R^{NN}}{\partial \rho} = \frac{4\rho^4(-1+\tau)^2-48\rho(7-16\tau+9\tau^2)-4\rho^3(15-29\tau+14\tau^2)+12(12-29\tau+17\tau^2)+\rho^2(237-492\tau+253\tau^2)}{(\rho(15-14\tau)+24(-1+\tau)+2\rho^2(-1+\tau))^2} > 0,$$

$$\frac{\partial p_R^{NN}}{\partial \rho} = \frac{4\rho^4(-1+\tau)^2+96(6-13\tau+7\tau^2)-4\rho^3(15-29\tau+14\tau^2)-48\rho(13-28\tau+15\tau^2)+\rho^2(291-591\tau+298\tau^2)}{2(\rho(15-14\tau)+24(-1+\tau)+2\rho^2(-1+\tau))^2} > 0,$$

$$\frac{\partial p_{E1}^{PN/PP}}{\partial \rho} = \frac{-4\rho^3(7-13\tau+6\tau^2)-36\rho(9-19\tau+10\tau^2)+3(59-128\tau+69\tau^2)+\rho^2(171-340\tau+168\tau^2)}{(18-7\rho-18\tau+6\rho\tau)^2} > 0;$$

$$(ii) \quad \frac{\partial p_R^{PP}}{\partial \tau} = -\frac{9}{2(11-12\tau)^2} < 0, \quad \frac{\partial p_R^{NN}}{\partial \tau} = \frac{3\rho(-40+61\rho-28\rho^2+4\rho^3)}{2(\rho(15-14\tau)+24(-1+\tau)+2\rho^2(-1+\tau))^2} < 0, \quad \frac{\partial p_R^{NP/PN}}{\partial \tau} = \frac{\rho(-60+79\rho-32\rho^2+4\rho^3)}{2(18-7\rho-18\tau+6\rho\tau)^2} < 0,$$

$$\frac{\partial p_R^{NN}}{\partial \tau} - \frac{\partial p_R^{NP/PN}}{\partial \tau} = -\frac{\rho(-60+79\rho-32\rho^2+4\rho^3)}{2(18-7\rho-18\tau+6\rho\tau)^2} + \frac{3\rho(-5+2\rho)(8+\rho(-9+2\rho))}{2(24(-1+\tau)+\rho(15-2\rho+2(-7+\rho)\tau))^2} < 0,$$

$$\frac{\partial p_R^{NN}}{\partial \tau} - \frac{\partial p_R^{PP}}{\partial \tau} = \frac{3}{2} \left(\frac{3}{(11-12\tau)^2} + \frac{\rho(-5+2\rho)(8+\rho(-9+2\rho))}{(24(-1+\tau)+\rho(15-2\rho+2(-7+\rho)\tau))^2} \right) > 0,$$

$$\frac{\partial p_R^{NP/PN}}{\partial \tau} - \frac{\partial p_R^{PP}}{\partial \tau} = \frac{1}{2} \left(\frac{9}{(11-12\tau)^2} + \frac{(-4+\rho)\rho(-5+2\rho)(-3+2\rho)}{(18-7\rho+6(-3+\rho)\tau)^2} \right) > 0,$$

Then we can easily yield $\frac{\partial p_R^{PP}}{\partial \tau} < \frac{\partial p_R^{NN}}{\partial \tau} < \frac{\partial p_R^{NP/PN}}{\partial \tau} < 0$; accordingly, we can also easily yield

$\frac{\partial p_{E1}^{PP}}{\partial \tau} < \frac{\partial p_{E1}^{NN}}{\partial \tau} < \frac{\partial p_{E1}^{PN/PP}}{\partial \tau} < 0$, and we omit the proof process.

$$(iii) \quad p_R^{NN}, p_R^{PP} > p_R^{PN/PP}; \text{ when } \rho \text{ and } \tau \text{ are larger, there exists } p_R^{NN} > p_R^{PP}; \text{ when } \rho \text{ and } \tau$$

are smaller, there exists $p_R^{NN} < p_R^{PP}$.

We can easily get it from Figure b by Plot3D $\{p_{RNN}, p_{RPN}, p_{RPP}\}, \{\rho, 0, 0.8\}, \{\tau, 0, \frac{1}{3}\}$.

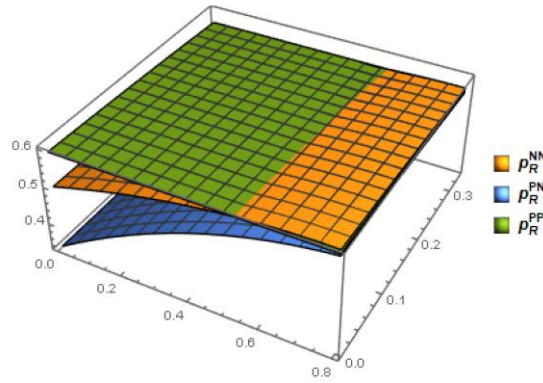


Figure. b.

Corollary 6. Because $0 < \tau < \frac{1}{3}$, $\rho = \theta$, $\alpha \rightarrow 1$, then we can obtain that,

$$(i) \quad \frac{\partial q_R^{NP/PN}}{\partial \rho} = -\frac{(-1+\tau)(-57-36\rho(-1+\tau)+63\tau+\rho^2(-7+6\tau))}{(18-7\rho-18\tau+6\rho\tau)^2} < 0,$$

$$\frac{\partial q_R^{NN}}{\partial \rho} = -\frac{3(-1+\tau)(-48-32\rho(-1+\tau)+52\tau+\rho^2(-6+5\tau))}{(\rho(15-14\tau)+24(-1+\tau)+2\rho^2(-1+\tau))^2} < 0,$$

$$\frac{\partial q_{E1}^{NP}}{\partial \rho} = \frac{203-144\rho(-1+\tau)^2-435\tau+234\tau^2+4\rho^2(7-13\tau+6\tau^2)}{2(18-7\rho-18\tau+6\rho\tau)^2} > 0,$$

$$\frac{\partial q_{E1}^{NN}}{\partial \rho} = \frac{60-129\tau+70\tau^2+\rho(-48+92\tau-44\tau^2)+\rho^2(10-17\tau+7\tau^2)}{(\rho(15-14\tau)+24(-1+\tau)+2\rho^2(-1+\tau))^2} > 0.$$

$$(ii) \quad \frac{\partial q_R^{PP}}{\partial \tau} = \frac{3}{2(11-12\tau)^2} > 0, \quad \frac{\partial q_R^{NN}}{\partial \tau} = \frac{3\rho(8-9\rho+2\rho^2)}{2(\rho(15-14\tau)+24(-1+\tau)+2\rho^2(-1+\tau))^2} > 0, \quad \frac{\partial q_R^{NP/PN}}{\partial \tau} = \frac{\rho(12-11\rho+2\rho^2)}{2(18-7\rho-18\tau+6\rho\tau)^2} > 0,$$

$$\frac{\partial q_R^{NN}}{\partial \tau} - \frac{\partial q_R^{NP/PN}}{\partial \tau} = -\frac{\rho(12-11\rho+2\rho^2)}{2(18-7\rho-18\tau+6\rho\tau)^2} + \frac{3\rho(8+\rho(-9+2\rho))}{2(24(-1+\tau)+\rho(15-2\rho+2(-7+\rho)\tau))^2} > 0,$$

$$\frac{\partial q_R^{NN}}{\partial \tau} - \frac{\partial q_R^{PP}}{\partial \tau} = -\frac{3(-1+\rho)(-288(-1+\tau)^2+2\rho^3(-1+\tau)^2+\rho^2(-149+318\tau-170\tau^2)+4\rho(139-294\tau+156\tau^2))}{(\rho(15-14\tau)+24(-1+\tau)+2\rho^2(-1+\tau))^2(11-12\tau)^2} < 0,$$

$$\frac{\partial q_R^{NP/PN}}{\partial \tau} - \frac{\partial q_R^{PP}}{\partial \tau} = \frac{(-1+\rho)(\rho^2(11-12\tau)^2+486(-1+\tau)^2-6\rho(103-219\tau+117\tau^2))}{(11-12\tau)^2(18-7\rho-18\tau+6\rho\tau)^2} < 0,$$

Then we can easily yield $\frac{\partial q_R^{PP}}{\partial \tau} > \frac{\partial q_R^{NN}}{\partial \tau} > \frac{\partial q_R^{NP/PN}}{\partial \tau} > 0$; accordingly, we can also easily yield $\frac{\partial q_{E1}^{PP}}{\partial \tau} > 0$; $\frac{\partial q_{E1}^{NN}}{\partial \tau} > 0$; $\frac{\partial q_{E1}^{PN}}{\partial \tau} > \frac{\partial q_{E1}^{NP}}{\partial \tau} > 0$, and we omit the proof process.

$$(iii) \quad q_R^{NN} - q_R^{PN/NP} = \frac{(3-4\rho+\rho^2)(\rho(-3+\tau)-24(-1+\tau)+2\rho^2(-1+\tau))(-1+\tau)}{(\rho(15-14\tau)+24(-1+\tau)+2\rho^2(-1+\tau))(18-7\rho-18\tau+6\rho\tau)} > 0,$$

$$q_R^{NN} - q_R^{PP} = \frac{3(-1+\rho)(-1+\tau)(32-36\tau+\rho(-10+11\tau))}{(\rho(15-14\tau)+24(-1+\tau)+2\rho^2(-1+\tau))(-11+12\tau)} > 0, \quad q_R^{PN/NP} - q_R^{PP} = \frac{(1-\rho)(1-\tau)(39-45\tau+\rho(-11+12\tau))}{(11-12\tau)(18-7\rho-18\tau+6\rho\tau)} > 0,$$

Then we can easily yield $q_R^{NN} > q_R^{PN/NP} > q_R^{PP}$.



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