



Research article

A hybrid relaxation method for solving generalized linear fractional programs

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Appendix A

A.1. References [18,19]

$$\left\{ \begin{array}{l} \min (x_1 + x_2)(x_1 - x_2 + 7) \\ \text{s.t. } 2x_1 + x_2 \leq 14, x_1 + x_2 \leq 10, \\ \quad -4x_1 + x_2 \leq 0, 2x_1 + x_2 \geq 6, x_1 + 2x_2 \geq 6, \\ \quad x_1 - x_2 \leq 3, x_1 + x_2 \geq 0, \\ \quad x_1 - x_2 + 7 \geq 0, x_1, x_2 \geq 0. \end{array} \right.$$

A.2. References [10,19]

$$\left\{ \begin{array}{l} \min (x_1 + \frac{1}{9}x_3)(x_2 + \frac{1}{9}x_3) \\ \text{s.t. } 9x_1 + 9x_2 + 2x_3 \leq 81, 8x_1 + x_2 + 8x_3 \leq 72, \\ \quad x_1 + 8x_2 + 8x_3 \leq 72, 7x_1 + x_2 + x_3 \geq 9, \\ \quad x_1 + 7x_2 + x_3 \geq 9, x_1 + x_2 + 7x_3 \geq 9, \\ \quad 0 \leq x_1 \leq 8, 0 \leq x_2 \leq 8, 0 \leq x_3 \leq 9. \end{array} \right.$$

A.3. References [10,19]

$$\begin{cases} \min (x_1 + x_2 + 1)^{2.5}(2x_1 + x_2 + 1)^{1.1}(x_1 + 2x_2 + 1)^{1.9} \\ \text{s.t. } x_1 + 2x_2 \leq 6, 2x_1 + 2x_2 \leq 8, 1 \leq x_1 \leq 3, 1 \leq x_2 \leq 3. \end{cases}$$

A.4. References [10,19]

$$\begin{cases} \min (-4x_1 - 2x_4 + 3x_5 + 21)(4x_1 + 2x_2 + 3x_3 - 4x_4 + 4x_5 - 3) \\ \quad \times (3x_1 + 4x_2 + 2x_3 - 2x_4 + 2x_5 - 7)(-2x_1 + x_2 - 2x_3 + 2x_5 + 11) \\ \text{s.t. } 4x_1 + 4x_2 + 5x_3 + 3x_4 + x_5 \leq 25, -x_1 - 5x_2 + 2x_3 + 3x_4 + x_5 \leq 2, \\ \quad x_1 + 2x_2 + x_3 - 2x_4 + 2x_5 \geq 6, 4x_2 + 3x_3 - 8x_4 + 11x_5 \geq 8, \\ \quad x_1 + x_2 + x_3 + x_4 + x_5 \leq 6, x_1 \geq 1, x_2 \geq 1, x_3 \geq 1, x_4 \geq 1, x_5 \geq 1. \end{cases}$$

A.5. References [10,19]

$$\begin{cases} \min (3x_1 - 4x_2 + 5)(x_1 + 2x_2 - 1)^{0.5}(2x_1 - x_2 + 4) \\ \quad \times (x_1 - 2x_2 + 8)^{0.5}(2x_1 + x_2 - 1) \\ \text{s.t. } 5x_1 - 8x_2 \geq 24, 5x_1 + 8x_2 \leq 44, 6x_1 - 3x_2 \leq 15, \\ \quad 4x_1 + 5x_2 \geq 10, 1 \leq x_1 \leq 3, 0 \leq x_2 \leq 1. \end{cases}$$

A.6. References [10,14,22]

$$\begin{cases} \min \frac{-x_1+2x_2+2}{3x_1-4x_2+5} \times \frac{4x_1-3x_2+4}{-2x_1+x_2+3} \\ \text{s.t. } x_1 + x_2 \leq 1.5, x_1 \leq x_2, 0 \leq x_1, x_2 \leq 1. \end{cases}$$

A.7. References [10]

$$\begin{cases} \max \frac{37x_1+73x_2+13}{13x_1+13x_2+13} \times \frac{63x_1-18x_2+39}{13x_1+26x_2+13} \\ \text{s.t. } 5x_1 - 3x_2 = 3, 1.5 \leq x_1 \leq 3. \end{cases}$$

A.8. References [14,22]

$$\begin{cases} \min \left(\frac{x_1+x_2+1}{x_1+x_2+2}\right)^{1.5} \times \left(\frac{x_1+x_2+3}{x_1+x_2+4}\right)^{2.1} \times \left(\frac{x_1+x_2+5}{x_1+x_2+6}\right)^{1.2} \times \left(\frac{x_1+x_2+7}{x_1+x_2+8}\right)^{1.1} \\ \text{s.t. } x_1 = x_2, 1.0 \leq x_1, x_2 \leq 2.0. \end{cases}$$

A.9. References [22]

$$\begin{cases} \max \frac{3x_1+5x_2+3x_3+50}{3x_2+4x_3+5x_3+50} + \frac{3x_1+4x_2+50}{4x_1+3x_2+2x_3+50} + \frac{4x_1+2x_2+4x_3+50}{5x_1+4x_2+3x_3+50} \\ \text{s.t. } 6x_1 + 3x_2 + 3x_3 \leq 10, 10x_1 + 3x_2 + 8x_3 \leq 10, \\ \quad 0 \leq x_1 \leq 1, 0 \leq x_2 \leq 3.333333, 0 \leq x_3 \leq 1. \end{cases}$$

A.10. References [14,22]

$$\left\{ \begin{array}{l} \max \frac{3x_1+5x_2+3x_3+50}{3x_2+4x_3+5x_3+50} + \frac{3x_1+5x_2+50}{3x_1+5x_2+3x_3+50} + \frac{4x_1+2x_2+4x_3+50}{5x_1+4x_2+3x_3+50} \\ \text{s.t. } 6x_1 + 3x_2 + 3x_3 \leq 10, 10x_1 + 3x_2 + 8x_3 \leq 10, \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0. \end{array} \right.$$

A.11. References [22]

$$\left\{ \begin{array}{l} \max \frac{4x_1+3x_2+3x_3+50}{3x_2+3x_3+50} + \frac{3x_1+4x_2+50}{4x_1+4x_2+5x_3+50} + \frac{x_1+2x_2+5x_3+50}{x_1+5x_2+5x_3+50} \\ \quad + \frac{x_1+2x_2+4x_3+50}{5x_2+4x_3+50} \\ \text{s.t. } 2x_1 + x_2 + 5x_3 \leq 10, x_1 + 6x_2 + 3x_3 \leq 10, \\ 5x_1 + 9x_2 + 2x_3 \leq 10, 9x_1 + 7x_2 + 3x_3 \leq 10, \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0. \end{array} \right.$$

A.12. References [22]

$$\left\{ \begin{array}{l} \min \frac{4x_1+3x_2+3x_3+50}{3x_2+3x_3+50} + \frac{3x_1+4x_3+50}{4x_1+4x_2+5x_3+50} + \frac{x_1+2x_2+4x_3+50}{x_1+5x_2+5x_3+50} \\ \quad + \frac{x_1+2x_2+4x_3+50}{5x_2+4x_3+50} \\ \text{s.t. } 2x_1 + x_2 + 5x_3 \leq 10, x_1 + 6x_2 + 2x_3 \leq 10, \\ 9x_1 + 7x_2 + 3x_3 \geq 10, x_1 \geq 0, x_2 \geq 0, x_3 \geq 0. \end{array} \right.$$



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