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Research article

A class of weighted Tchebycheff preference relations and multi-objective optimization

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Appendix

The important notations used in this paper are summarized below.

Notations	Description
(1) Models	
(MOP)	multi-objective optimization problem
$(\text{WTSOP})_\beta$	the weighted Tchebycheff scalarization method
(2) Constants	
ε	an arbitrary small positive constant
λ	any positive scalar
(3) Vectors	
$u(x; i)$	$(x_1, x_2, \cdots, x_i)^\top, i \in [p]$, represents the vector formed by the first i components of x
$\mathbf{e}^i$	the p-dimensional vector with the i-th component being 1 and all other components being 0
$x \circ y$	$(x_1y_1, x_2y_2, \cdots, x_py_p)^\top, \forall x, y \in \mathbb{R}^p$
β	weight vector

Notations	Description
f^*	utopia point, $f^* = \left(\min\limits_{x \in X} f_1(x) - \varepsilon, \min\limits_{x \in X} f_2(x) - \varepsilon, \cdots, \min\limits_{x \in X} f_p(x) - \varepsilon\right)$
$\mathbf{0}$	the p-dimensional zero vector
(4) Sets	
$\mathbb{R}$	the set of all real numbers
$\mathbb{N}^+$	the set of all positive integers
$[p]$	$\{1, 2, \cdots, p\}, p \in \mathbb{N}^+$
$\mathbb{R}^p$	the p-dimensional Euclidean space
$\mathbb{R}^p_{\geqq}$	$\{y \in \mathbb{R}^p \mid y_i \geq 0, i \in [p]\}$
$\mathbb{R}^p_{>}$	$\{y \in \mathbb{R}^p \mid y_i > 0, i \in [p]\}$
$\mathbb{R}^p_{\geq}$	$\mathbb{R}^p_{\geqq} \setminus \{\mathbf{0}\}$
W_e	$\{\mathbf{e}^1, \mathbf{e}^2, \cdots, \mathbf{e}^p\}$
X	the feasible set of the (MOP)
Y	$f(X)$, represents the image of the feasible set X under the objective function mapping f
(5) Orders	
$y < y'$	$y' - y \in \mathbb{R}^p_{>} \Leftrightarrow y_i < y'_i,\ \forall i \in [p]$
$y \leqq y'$	$y' - y \in \mathbb{R}^p_{\geqq} \Leftrightarrow y_i \leq y'_i,\ \forall i \in [p]$
$y \leq y'$	$y' - y \in \mathbb{R}^p_{\geq} \Leftrightarrow y \leqq y', y \neq y'$
$y \leqq_{ws} y'$	$\sum_{i=1}^{t} \beta_i y_i \leq \sum_{i=1}^{t} \beta_i y'_i,\ \forall t \in [p]$
$y <_{ws} y'$	$\sum_{i=1}^{t} \beta_i y_i < \sum_{i=1}^{t} \beta_i y'_i,\ \forall t \in [p]$
$y \leq_{ws} y'$	$y \leqq_{ws} y', y \neq y'$
$y \leqq_{\Vert\cdot\Vert_{\infty,\beta,f^*}} y'$	$\Vert u(\beta; i) \circ u(y - f^*; i)\Vert_\infty \leq \Vert u(\beta; i) \circ u(y' - f^*; i)\Vert_\infty,\ \forall i \in [p]$
$y <_{\Vert\cdot\Vert_{\infty,\beta,f^*}} y'$	$\Vert u(\beta; i) \circ u(y - f^*; i)\Vert_\infty < \Vert u(\beta; i) \circ u(y' - f^*; i)\Vert_\infty,\ \forall i \in [p]$
$y \leq_{\Vert\cdot\Vert_{\infty,\beta,f^*}} y'$	$y \leqq_{\Vert\cdot\Vert_{\infty,\beta,f^*}} y', y \neq_{\Vert\cdot\Vert_{\infty,\beta,f^*}} y'$

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