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Research article

Discovering AI tokens in the fractal markets hypothesis and their time-frequency co-movements with the leading high-carbon cryptocurrency

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Appendix.

Multifractal Fluctuation Analysis (MFA) is a technique used to analyze the scaling properties and multifractal characteristics of time series data. When applied to AI tokens, it helps identify the presence of multifractality, which can provide insights into the complexity and heterogeneity of the market dynamics. Below is the mathematical framework and estimation procedure for conducting a Multifractal Fluctuation Analysis (MFA) for AI tokens:

1. Time Series Preparation:
   Let $\{x(t)\}$ be the time series of AI token prices or returns, where t=1,2,… ,N.

2. Profile Construction:
   Compute the cumulative deviation (profile) of the time series:

$$Y_i = \sum_{t=1}^{i}(X_t - m), i=1, 2, \ldots, N. \quad \text{(A1)}$$

   where m is the mean of the time series.

3. Division into Segments:
   Divide the profile Y_i into $Ns = \text{int}(N/s)$ non-overlapping segments of length s. To account for the possibility of the time series length not being a multiple of s, repeat the process starting from the end of the series, resulting in $2Ns$ segments.

4. Local Trend Removal:
   For each segment v, fit a polynomial y_i^v (typically of order 2) to the profile and compute the variance:

$$F^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}(Y((v-1)s+i-y_i^v)^2 \tag{A2}$$

   For v=1,2, …,Ns, and

$$F^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}(Y(N-(v-N_s)s+i-y_i^v)^2 \tag{A3}$$

   For v=Ns+1, …,2Ns.
5. q-th Order Fluctuation Function:
   Compute the q-th order fluctuation function:

$$F_q(s) = \left(\frac{1}{2N_s}\sum_{v=1}^{2N_s}[F^2(v,s)]^{q/2}\right)^{1/q} \tag{A4}$$

   for q≠0. For q=0, use the logarithmic averaging:

$$F_0(s) = \exp\left(\frac{1}{4N_s}\sum_{v=1}^{2N_s}\ln[F^2(v,s)]\right) \tag{A5}$$

6. Scaling Behavior:
   The fluctuation function $Fq(s)$ scales with the segment size ss as:

$$F_q(s) \sim s^{H(q)} \tag{A6}$$

   where $H(q)$ is the generalized Hurst exponent. For a monofractal series, $H(q)$ is constant; for a multifractal series, $H(q)$ varies with q.

7. Estimate the Scaling Exponent:
   - Plot log$Fq(s)$ against $logs$ for each q.
   - Fit a linear regression to estimate $H(q)$.

8. Multifractal Spectrum:
   The multifractal spectrum $f(\alpha)$ is obtained via the Legendre transform:

$$\alpha = H(q) + q \cdot H'(q)$$
$$f(\alpha) = q \cdot \alpha - \tau(q) \tag{A7}$$