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Research article

An ODE model of LPS-induced neuroinflammation with bistability and dose-dependent switching

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Supplementary materials

Table S1. Structural sensitivity of the bifurcation and dose threshold to the Hill coefficient n.

n	ρ_c	Bistability at $\rho = 2.30$	D_c (mg/kg)
1.6	1.0869	Yes	0.223
1.8	1.3040	Yes	0.448
2.0	1.4761	Yes	0.860
2.2	1.6122	Yes	1.647
2.4	1.7201	Yes	3.170

Table S2. Effect of saturation of the production terms on the bifurcation and on the critical dose.

Model variant	ρ_c	M_1^* (saddle)	M_2^* (persistent)	D_c, mg/kg
Linear (reference)	1.476	0.0666	0.5621	0.860
Saturating, $K_{sat} = 5$	1.567	0.0690	0.5348	0.949
Saturating, $K_{sat} = 2$	1.693	0.0731	0.4914	1.122
Saturating, $K_{sat} = 1$	1.886	0.0816	0.4145	1.610
Saturating, $K_{sat} = 0.6$	2.117	0.1003	0.3054	3.836

Note: The three linear production terms of Eqs. (2) and (4) were replaced by Michaelis–Menten functions with the same slope at the origin: $\beta_C \cdot M \to \beta_C \cdot K_{sat} \cdot M/(K_{sat} + M)$, $k_{C,D} \cdot C \to k_{C,D} \cdot K_{sat} \cdot C/(K_{sat} + C)$, and $k_{M,D} \cdot M \to k_{M,D} \cdot K_{sat} \cdot M/(K_{sat} + M)$. The factor K_{sat} preserves the slope at the origin, so that the reference model is recovered as $K_{sat} \to \infty$ and smaller K_{sat} corresponds to stronger saturation. All remaining parameters were fixed at their reference values (Table 1). For each variant, the equilibria, the saddle-node value ρ_c and the critical dose D_c were recomputed using the same procedures as for the reference model. Bistability at $\rho = 2.30$ was retained down to $K_{sat} \approx 0.45$, the threshold value being located by bisection.

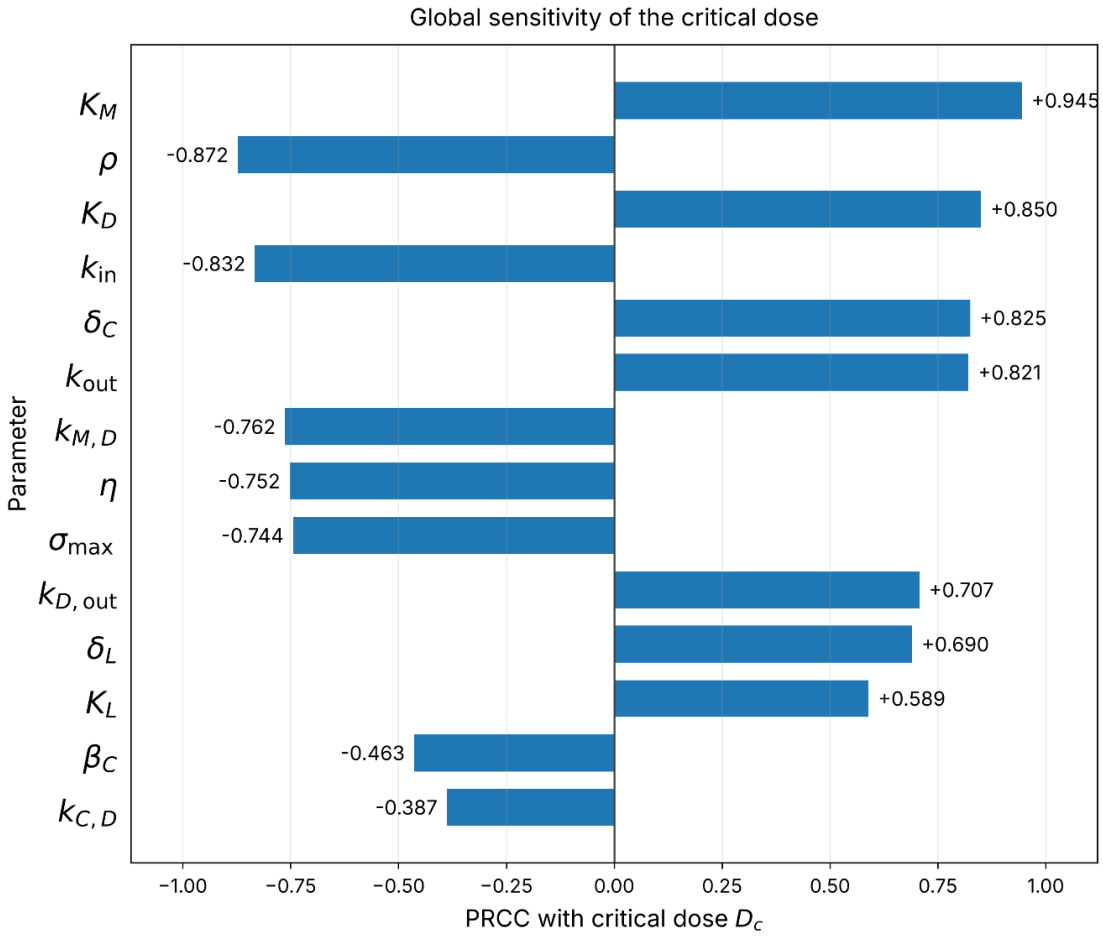


Figure S1. Partial rank correlation coefficients between the varied model parameters and the critical dose D_c. Positive values increase the dose required for switching, whereas negative values decrease it.

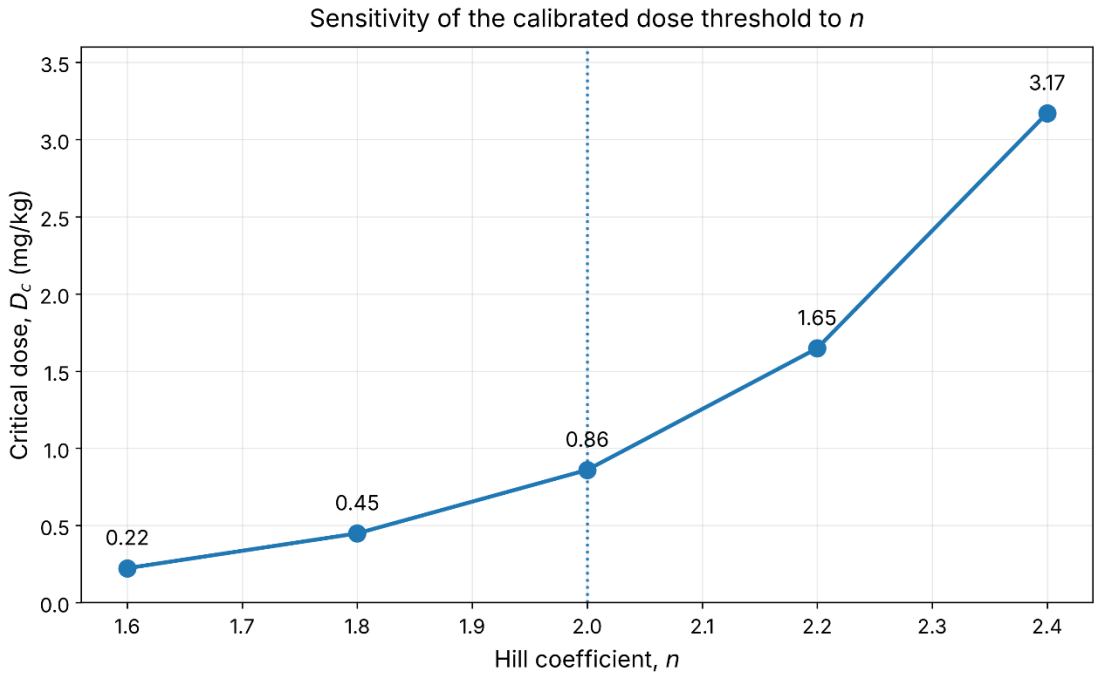


Figure S2. Sensitivity of the critical dose D_c to the Hill coefficient n with all other parameters fixed at their reference values. Bistability is preserved throughout the investigated range, whereas the quantitative switching threshold shifts substantially.

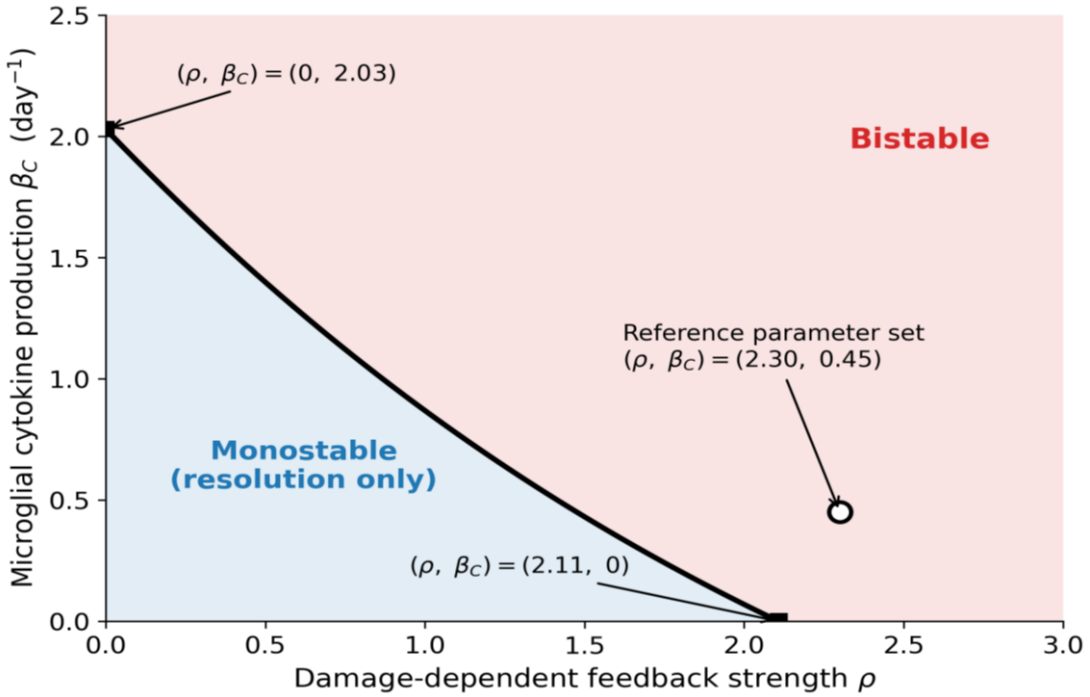


Figure S3. Saddle-node bifurcation boundary in the (ρ, β_C) plane, obtained by bisection on the maximum of the scalar equilibrium function with all remaining parameters at their reference values (Table 1). Above the boundary the model is bistable; below it only E_0 exists.

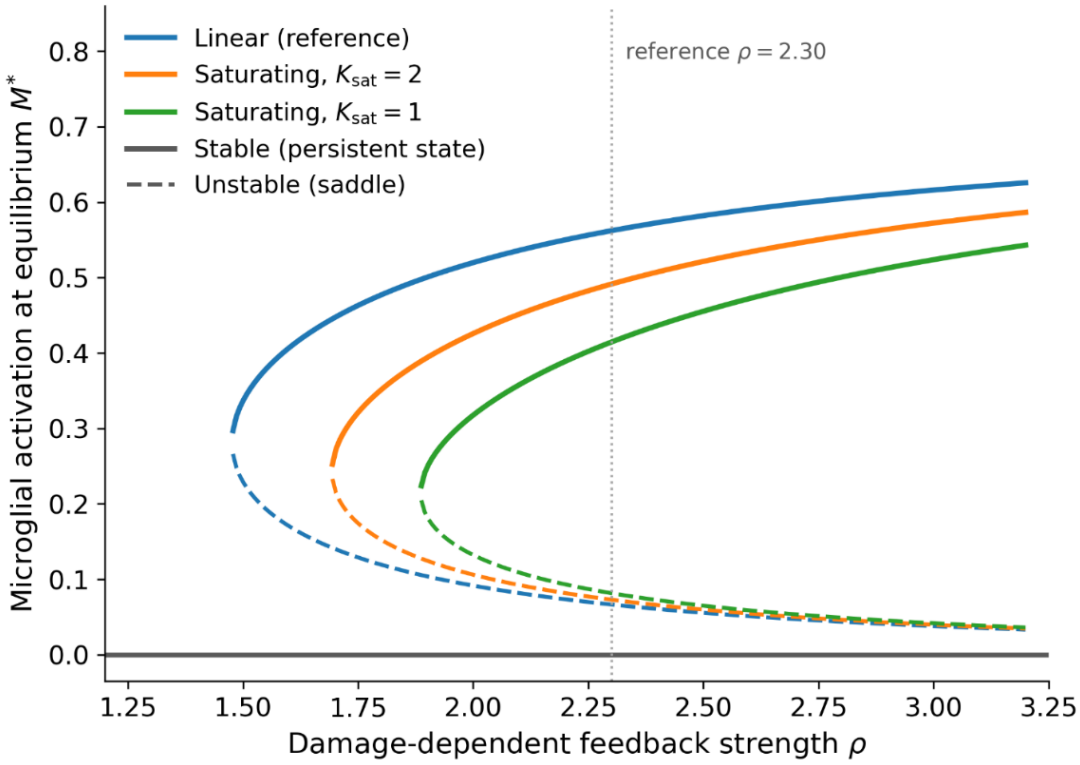


Figure S4. Branches of the steady-state values of M^* as a function of the damage-dependent feedback strength ρ for the reference model and for the slope-matched saturating variants at $K_{sat} = 2$ and $K_{sat} = 1$ (Supplementary Table S2). Strengthening saturation shifts the saddle-node bifurcation towards higher values of ρ and lowers the persistent equilibrium, but does not eliminate the bifurcation.

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