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Research article

Modelling the effects of media coverage and quarantine on the COVID-19 infections in the UK

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Abstract: A new COVID-19 epidemic model with media coverage and quarantine is constructed. The model allows for the susceptibles to the unconscious and conscious susceptible compartment. First, mathematical analyses establish that the global dynamics of the spread of the COVID-19 infectious disease are completely determined by the basic reproduction number R_0. If $R_0 \leq 1$, then the disease free equilibrium is globally asymptotically stable. If $R_0 > 1$, the endemic equilibrium is globally asymptotically stable. Second, the unknown parameters of model are estimated by the MCMC algorithm on the basis of the total confirmed new cases from February 1, 2020 to March 23, 2020 in the UK. We also estimate that the basic reproduction number is $R_0 = 4.2816(95\%\text{CI} : (3.8882, 4.6750))$. Without the most restrictive measures, we forecast that the COVID-19 epidemic will peak on June 2 $(95\%\text{CI} : (\text{May } 23, \text{June } 13))$ (Figure 3a) and the number of infected individuals is more than 70% of UK population. In order to determine the key parameters of the model, sensitivity analysis are also explored. Finally, our results show reducing contact is effective against the spread of the disease. We suggest that the stringent containment strategies should be adopted in the UK.

Keywords: COVID-19; basic reproduction number; global stability; Lyapunov functional; parameter estimation

1. Introduction

Since December 2019, the outbreak of the novel coronavirus pneumonia firstly occurred in Wuhan, a central and packed city of China [1,2]. The World Health Organization(WHO) has named the virus as COVID-19 On January 12, 2020. Recently, COVID-19 has spread to the vast majority of countries, as United States, France, Iran, Italy and Spain etc. The outbreak of COVID-19 has been become a globally public health concern in medical community as the virus is spreading around the world. Initially,

the British government adopted a herd immunity strategy. As of March 27, cases of the COVID-19 coronavirus have been confirmed more than 11,000 mostly In the UK. The symptoms of COVID-19 most like SARS(Severe acute respiratory syndrome) and MERS(Middle East respiratory syndrome), include cough, fever, weakness and difficulty breath [3]. The period for such symptoms from mild to severe respiratory infections lasts 2–14 days. The transmission routes contain direct transmission, such as close touching and indirect transmission consist of the air by coughing and sneezing, even if contacting some contaminated things by virus particles. There are many mathematica models to discuss the dynamics of COVID-19 infection [4–8].

Additionally, coronaviruses can be extremely contagious and spread easily from person to person [9]. So a series of stringent control measures are necessary. For some diseases, such as influenza and tuberculosis, people often introduce the latent compartment (denoted by E), leading to an SEIR model. The latent compartment of COVID-19 is highly contagious [10, 11]. Such type of models have been widely discussed in recent decades [12–14].

Media coverage is a key factor in the transmission process of infectious disease. People know more about the COVID-19 and enhance their self-protecting awareness by the media reporting about the COVID-19. People will change their behaviours and take correct precautions such as frequent hand-washing, wearing masks, reducing the party, keeping social distances, and even quarantining themselves at home to avoid contacting with others. Zhou et al. [15] proposed a deterministic dynamical model to examine the interaction of the disease progression and the media reports and to investigate the effectiveness of media reporting on mitigating the spread of COVID-19. The result suggested that media coverage can be considered as an effective way to mitigate the disease spreading during the initial stage of an outbreak.

Quarantine is effective for the control of infectious disease. Chinese government advises all the Chinese citizens to isolate themselves at home, and people exposed to the virus have the medical observation for 14 days. In order to get closer to the reality, many scholars have introduced quarantine compartment into epidemic model. Amador and Gomez-Corral [16] studied extreme values in an SIQS model with two different states for quarantine, termed quarantined susceptible and quarantined infective, and limited carrying capacity for the quarantine compartment. Gao and Zhuang proposed a new VEIQS worm propagation model with saturated incidence and strategies of both vaccination and quarantine [17].

Motivated by the above, we consider a new COVID-19 epidemic model with media coverage and quarantine. The model assumes that the latent stage has certain infectivity. And we also introduce the quarantine compartment into the epidemic model, and the susceptible have consciousness to checking the spread of infectious diseases in the media coverage.

The organization of this paper is as follows. In the next section, the epidemic model with media coverage and quarantine is formulated. In section 3, the basic reproduction number and the existence of equilibria are investigated. In Section 4, the global stability of the disease free and endemic equilibria are proved. In Section 5, we use the MCMC algorithm to estimate the unknown parameters and initial values of the model. The basic reproduction number R_0 of the model and its confidence interval are solved by numerical methods. At the same time, we obtain the sensitivity of the unknown parameters of the model. In the last section, we give some discussions.

2. The model formulation

2.1. System description

In this section, we introduce a COVID-19 epidemic model with media coverage and quarantine. The total population is partitioned into six compartments: the unconscious susceptible compartment (S_1), the conscious susceptible compartment (S_2), the latent compartment (E), the infectious compartment (I), the quarantine compartment (Q) and the recovered compartment (R). The total number of population at time t is given by

$$N(t) = S_1(t) + S_2(t) + E(t) + I(t) + Q(t) + R(t).$$

The parameters are described in Table 1. The population flow among those compartments is shown in the following diagram (Figure 1).

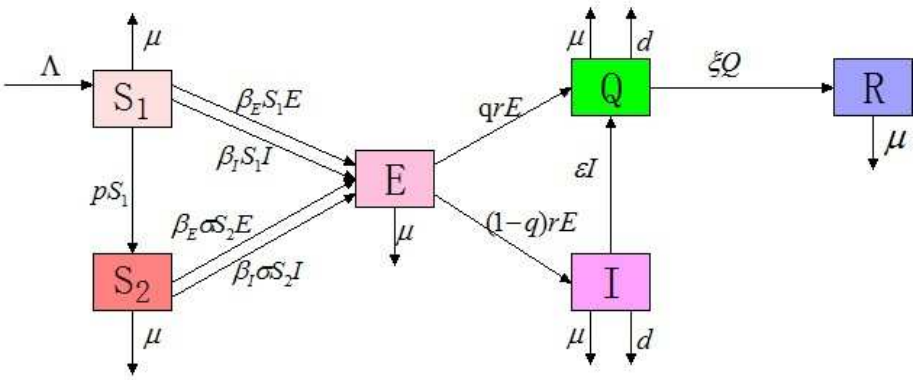


Figure 1. The transfer diagram for the model (2.1).

The transfer diagram leads to the following system of ordinary differential equations:

$$\begin{cases} S_1' = \Lambda - \beta_E S_1 E - \beta_I S_1 I - (p+\mu)S_1, \\ S_2' = pS_1 - \beta_E \sigma S_2 E - \beta_I \sigma S_2 I - \mu S_2, \\ E' = \beta_E E(S_1 + \sigma S_2) + \beta_I I(S_1 + \sigma S_2) - (r+\mu)E, \\ I' = (1-q)rE - (\mu + d + \varepsilon)I, \\ Q' = qrE + \varepsilon I - (\mu + d + \xi)Q, \\ R' = \xi Q - \mu R. \end{cases} \tag{2.1}$$

Table 1. Parameters of the model.

Parameter	Description
Λ	The birth rate of the population
β_E	Transmission coefficient of the latent compartment
β_I	Transmission coefficient of the infectious compartment
σ	The fraction of S_2 being infected and entering E
p	The migration rate to S_2 from S_1,reflecting the impact of media coverage
r	The rate coefficient of transfer from the latent compartment
q	The fraction of the latent compartment E jump into the quarantine compartment Q
$1-q$	The fraction of the latent compartment E jump into the infectious compartment I
ε	The transition rate to I for Q
ξ	The recovering rate of the quarantine compartment
μ	Naturally death rate
d	The disease-related death rate

Since the sixth equation in system (2.1) is independent of other equations, system (2.1) may be reduced to the following system:

$$\begin{cases} S_1' = \Lambda - \beta_E S_1 E - \beta_I S_1 I - (p+\mu) S_1, \\ S_2' = p S_1 - \beta_E \sigma S_2 E - \beta_I \sigma S_2 I - \mu S_2, \\ E' = \beta_E E(S_1 + \sigma S_2) + \beta_I I(S_1 + \sigma S_2) - (r+\mu) E, \\ I' = (1-q) r E - (\mu + d + \varepsilon) I, \\ Q' = q r E + \varepsilon I - (\mu + d + \xi) Q. \end{cases} \tag{2.2}$$

2.2. Basic properties

2.2.1. Positivity of solutions

It is important to show positivity for the system (2.1) as they represent populations. We thus state the following lemma.

Lemma 1. If the initial values $S_1(0) > 0$, $S_2(0) > 0$, $E(0) > 0$, $I(0) > 0$, $Q(0) > 0$ and $R(0) > 0$, the solutions $S_1(t)$, $S_2(t)$, $E(t)$, $I(t)$, $Q(t)$ and $R(t)$ of system (2.1) are positive for all $t > 0$.

Proof. Let $W(t) = \min\{S_1(t), S_2(t), E(t), I(t), Q(t), R(t)\}$, for all $t > 0$.

It is clear that $W(0) > 0$. Assuming that there exists a $t_1 > 0$ such that $W(t_1) = 0$ and $W(t) > 0$, for all $t \in [0, t_1)$.

If $W(t_1) = S_1(t_1)$, then $S_2(t) \geq 0,\ E(t) \geq 0,\ I(t) \geq 0,\ Q(t) \geq 0,\ R(t) \geq 0$ for all $t \in [0, t_1]$. From the first equation of model (2.1), we can obtain

$$S_1' \geq -\beta_E S_1 E - \beta_I S_1 I - (p+\mu) S_1, \qquad t \in [0, t_1]$$

Thus, we have

$$0 = S_1(t_1) \geq S_1(0) e^{-\int_0^{t_1} [\beta_E E + \beta_I I + (p+\mu)] dt} > 0,$$

which leads to a contradiction. Thus, $S_1(t) > 0$ for all $t \geq 0$.

Similarly, we can also prove that $S_2(t) > 0$, $E(t) > 0$, $I(t) > 0$, $Q(t) > 0$ and $R(t) > 0$ for all $t \geq 0$.

2.2.2. Invariant region

Lemma 2. The feasible region Ω defined by

$$\Omega = \{(S_1(t),\ S_2(t),\ E(t),\ I(t),\ Q(t),\ R(t)) \in R_+^6 : N(t) \le \frac{\Lambda}{\mu}\}$$

with initial conditions $S_1(0) \ge 0,\ S_2(0) \ge 0,\ E(0) \ge 0,\ I(0) \ge 0,\ Q(0) \ge 0,\ R(0) \ge 0$ is positively invariant for system (2.1).

Proof. Adding the equations of system (2.1) we obtain

$$\begin{aligned}\frac{dN}{dt} &= \Lambda - \mu N - d(I+Q)\\ &\le \Lambda - \mu N.\end{aligned}$$

It follows that

$$0 \le N(t) \le \frac{\Lambda}{\mu} + N(0)e^{-\mu t},$$

where $N(0)$ represents the initial values of the total population. Thus $\lim\limits_{t\to+\infty} \sup\ N(t) \le \frac{\Lambda}{\mu}$. It implies that the region $\Omega = \{(S_1(t),\ S_2(t),\ E(t),\ I(t),\ Q(t),\ R(t)) \in R_+^6 : N(t) \le \frac{\Lambda}{\mu}\}$ is a positively invariant set for system (2.1). So we consider dynamics of system (2.1) and (2.2) on the set Ω in this paper.

3. The basic reproduction number and existence of equilibria

The model has a disease free equilibrium $(S_1^0, S_2^0, 0, 0, 0)$, where

$$S_1^0 = \frac{\Lambda}{p+\mu}, \qquad S_2^0 = \frac{p\Lambda}{\mu(p+\mu)}.$$

In the following, the basic reproduction number of system (2.2) will be obtained by the next generation matrix method formulated in [18].

Let $x = (E,\ I,\ Q,\ S_1,\ S_2)^T$, then system (2.2) can be written as

$$\frac{dx}{dt} = \mathcal{F}(x) - \mathcal{V}(x),$$

where

$$\mathcal{F}(x) = \begin{pmatrix} \beta_E E(S_1+\sigma S_2) + \beta_I I(S_1+\sigma S_2) \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \mathcal{V}(x) = \begin{pmatrix} (r+\mu)E \\ (\mu+d+\varepsilon)I - (1-q)rE \\ (\mu+d+\xi)Q - qrE - \varepsilon I \\ \beta_E S_1 E + \beta_I S_1 I + (p+\mu)S_1 - \Lambda \\ \beta_E \sigma S_2 E + \beta_I \sigma S_2 I + \mu S_2 - pS_1 \end{pmatrix}. \tag{3.1}$$

The Jacobian matrices of $\mathcal{F}(x)$ and $\mathcal{V}(x)$ at the disease free equilibrium P^0 are, respectively,

$$D\mathcal{F}(P^0) = \begin{pmatrix} F_{3\times3} & 0 \\ 0 & 0 \end{pmatrix}, \qquad D\mathcal{V}(P^0) = \begin{pmatrix} V_{3\times3} & 0 & 0 \\ \beta_E S_1^0 & \beta_I S_1^0 & 0 & p+\mu & 0 \\ \beta_E \sigma S_2^0 & \beta_I \sigma S_2^0 & 0 & -p & \mu \end{pmatrix}, \tag{3.2}$$

where

$$F = \begin{pmatrix} \beta_E(S_1^0 + \sigma S_2^0) & \beta_I(S_1^0 + \sigma S_2^0) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad V = \begin{pmatrix} r+\mu & 0 & 0 \\ -(1-q)r & \mu+d+\varepsilon & 0 \\ -qr & -\varepsilon & \mu+d+\xi \end{pmatrix}.$$

The model reproduction number, denoted by R_0 is thus given by

$$\begin{aligned} R_0 &= \rho(FV^{-1}) = \frac{(S_1^0 + \sigma S_2^0)[\beta_E(\mu+d+\varepsilon) + \beta_I(1-q)r]}{(r+\mu)(\mu+d+\varepsilon)} \\ &= \frac{(S_1^0 + \sigma S_2^0)(\beta_E A + \beta_I B)}{(r+\mu)A}. \end{aligned} \tag{3.3}$$

where

$$A = \mu + d + \varepsilon, \qquad B = (1-q)r. \tag{3.4}$$

The endemic equilibrium $P^*(S_1^*, S_2^*, E^*, I^*, Q^*)$ of system (2.2) is determined by equations

$$\begin{cases} \Lambda - \beta_E S_1 E - \beta_I S_1 I - (p+\mu)S_1 = 0, \\ pS_1 - \beta_E \sigma S_2 E - \beta_I \sigma S_2 I - \mu S_2 = 0, \\ \beta_E E(S_1 + \sigma S_2) + \beta_I I(S_1 + \sigma S_2) - (r+\mu)E = 0, \\ (1-q)rE - (\mu+d+\varepsilon)I = 0, \\ qrE + \varepsilon I - (\mu+d+\xi)Q = 0. \end{cases} \tag{3.5}$$

The first two equations in (3.5) lead to

$$S_1 = \frac{\Lambda}{\beta_E E + \beta_I I + (p+\mu)}, \qquad S_2 = \frac{p\Lambda}{[\beta_E E + \beta_I I + (p+\mu)](\beta_E \sigma E + \beta_I \sigma I + \mu)}. \tag{3.6}$$

From the fourth equation in (3.5), we have

$$\begin{aligned} E &= \frac{\mu+d+\varepsilon}{(1-q)r} I \\ &= \frac{A}{B} I. \end{aligned} \tag{3.7}$$

Substituting (3.7) into the last equation in (3.5) gives

$$\begin{aligned} Q &= \frac{q(\mu+d+\varepsilon) + (1-q)\varepsilon}{(1-q)(\mu+d+\xi)} I \\ &= \frac{Aqr + B\varepsilon}{(\mu+d+\xi)B} I. \end{aligned} \tag{3.8}$$

For $I \neq 0$, substituting (3.7) into the third equation in (3.5) gives

$$S_1 + \sigma S_2 = \frac{(r+\mu)A}{\beta_E A + \beta_I B} \tag{3.9}$$

From (3.6) and (3.7), we have

$$\begin{aligned} S_1 + \sigma S_2 &= \frac{\Lambda}{\beta_E E + \beta_I I + (p+\mu)} + \frac{\sigma p \Lambda}{[\beta_E E + \beta_I I + (p+\mu)](\beta_E \sigma E + \beta_I \sigma I + \mu)} \\ &= \frac{\Lambda}{\beta_E E + \beta_I I + (p+\mu)} \cdot [1 + \frac{\sigma p}{\sigma(\beta_E E + \beta_I I) + \mu}] \\ &= \frac{\Lambda}{\beta_E E + \beta_I I + (p+\mu)} \cdot \frac{\sigma(\beta_E E + \beta_I I) + \mu + p\sigma}{\sigma(\beta_E E + \beta_I I) + \mu} \\ &= \frac{\Lambda}{\frac{\beta_E A + \beta_I B}{B} I + (p+\mu)} \cdot \frac{\sigma \frac{\beta_E A + \beta_I B}{B} I + \mu + p\sigma}{\sigma \frac{\beta_E A + \beta_I B}{B} I + \mu} \\ &= \frac{B\Lambda}{(\beta_E A + \beta_I B) I + (p+\mu) B} \cdot \frac{\sigma(\beta_E A + \beta_I B) I + (\mu + p\sigma) B}{\sigma(\beta_E A + \beta_I B) I + \mu B}. \end{aligned} \tag{3.10}$$

Substituting (3.9) into (3.10) yields

$$\begin{aligned} H(I) &:= \frac{B[\sigma(\beta_E A + \beta_I B) I + (\mu + p\sigma) B]}{[(\beta_E A + \beta_I B) I + (p+\mu) B][\sigma(\beta_E A + \beta_I B) I + \mu B]} - \frac{(r+\mu) A}{(\beta_E A + \beta_I B)\Lambda} \\ &= 0. \end{aligned} \tag{3.11}$$

Direct calculation shows

$$H^{'}(I) = \frac{H_1(I)}{\{[(\beta_E A + \beta_I B) I + (p+\mu) B][\sigma(\beta_E A + \beta_I B) I + \mu B]\}^2},$$

Denote $C = \beta_E A + \beta_I B$,

$$\begin{aligned} H_1(I) &= B\sigma(\beta_E A + \beta_I B)[(\beta_E A + \beta_I B) I + (p+\mu) B][\sigma(\beta_E A + \beta_I B) I + \mu B] - B[\sigma(\beta_E A + \beta_I B) I \\ &\quad + (\mu + p\sigma) B](\beta_E A + \beta_I B)\{[\sigma(\beta_E A + \beta_I B) I + \mu B] + \sigma[(\beta_E A + \beta_I B) I + (p+\mu) B]\} \\ &= \sigma BC[CI + (p+\mu) B][\sigma CI + \mu B] - BC[\sigma CI + (\mu + p\sigma) B]\{\sigma CI + \mu B + \sigma[CI + (p+\mu) B]\} \\ &= BC\{\sigma[CI + (p+\mu) B][\sigma CI + \mu B] - [\sigma CI + (\mu + p\sigma) B][\sigma CI + \mu B + \sigma[CI + (p+\mu) B]]\} \\ &= BC\{(\sigma\mu B - \mu B)(\sigma CI + \mu B) - [\sigma CI + (\mu + p\sigma) B]\sigma[CI + (p+\mu) B]\} \\ &= -BC\{\mu B(\sigma CI + \mu B) + \sigma[\sigma C^2 I^2 + \sigma pBCI + C(\mu + p\sigma) BI + [(\mu + p\sigma) p + p\sigma\mu] B^2]\} \\ &= -BC\{(\sigma C)^2 I^2 + 2\sigma BC(\mu + \sigma p) I + B^2[\mu^2 + \mu p\sigma + p(p+\mu)\sigma^2]\} < 0. \end{aligned}$$

then

$$H^{'}(I) < 0.$$

then function $H(I)$ is decreasing for $I > 0$. Since $[(\beta_E A + \beta_I B) I + (p+\mu) B][\sigma(\beta_E A + \beta_I B) I + \mu B] > (\beta_E A + \beta_I B) I[\sigma(\beta_E A + \beta_I B) I + (\mu + p\sigma) B]$, then

$$H(I) < \frac{B}{(\beta_E A + \beta_I B) I} - \frac{(r+\mu) A}{(\beta_E A + \beta_I B)\Lambda}.$$

Thus,

$$H(0) = \frac{\mu + p\sigma}{\mu(p+\mu)} - \frac{(r+\mu) A}{(\beta_E A + \beta_I B)\Lambda}$$

$$
\begin{aligned}
&= \frac{(S_1^0 + \sigma S_2^0)}{\Lambda} - \frac{(r+\mu)A}{(\beta_E A + \beta_I B)\Lambda} \\
&= \frac{(\beta_E A + \beta_I B)(S_1^0 + \sigma S_2^0) - (r+\mu)A}{(\beta_E A + \beta_I B)\Lambda} \\
&= \frac{(r+\mu)AR_0 - (r+\mu)A}{(\beta_E A + \beta_I B)\Lambda} \\
&= \frac{(r+\mu)A}{(\beta_E A + \beta_I B)\Lambda}(R_0 - 1),
\end{aligned}
$$

and

$$
\begin{aligned}
H(\frac{\Lambda}{\mu}) &< \frac{\mu B}{(\beta_E A + \beta_I B)\Lambda} - \frac{(r+\mu)A}{(\beta_E A + \beta_I B)\Lambda} \\
&= \frac{\mu B - (r+\mu)A}{(\beta_E A + \beta_I B)\Lambda} \\
&= \frac{\mu(1-q)r - (r+\mu)(\mu + d + \varepsilon)}{(\beta_E A + \beta_I B)\Lambda} \\
&= -\frac{(r+\mu)(d+\varepsilon) + (qr+\mu)\mu}{(\beta_E A + \beta_I B)\Lambda} \\
&< 0.
\end{aligned}
$$

Therefore, by the monotonicity of function $H(I)$, for (3.11) there exists a unique positive root in the interval $(0, \frac{\Lambda}{\mu})$ when $R_0 > 1$; there is no positive root in the interval $(0, \frac{\Lambda}{\mu})$ when $R_0 \leq 1$. We summarize this result in Theorem 3.1.

Theorem 3.1. For system (2.2), there is always the disease free equilibrium $P^0(S_1^0, S_2^0, 0, 0, 0)$. When $R_0 > 1$, besides the disease free equilibrium P^0, system (2.2) also has a unique endemic equilibrium $P^*(S_1^*, S_2^*, E^*, I^*, Q^*)$, where

$$
\begin{aligned}
S_1^* &= \frac{B\Lambda}{(\beta_E A + \beta_I B)I^* + (p+\mu)B}, \\
S_2^* &= \frac{p\Lambda B^2}{[(\beta_E A + \beta_I B)I^* + (p+\mu)B][(\beta_E A + \beta_I B)\sigma I^* + \mu B]}, \\
E^* &= \frac{A}{B} I^*, \\
Q^* &= \frac{Aqr + B\varepsilon}{(\mu + d + \xi)B} I^*.
\end{aligned}
$$

and I^* is the unique positive root of equation $H(I) = 0$.

4. Global stability of equilibria

Theorem 4.1. For system (2.2), the disease free equilibrium P^0 is globally stable if $R_0 \leq 1$; the endemic equilibrium P^* is globally stable if $R_0 > 1$.

4.1. Global stability of the disease free equilibrium

For the disease free equilibrium $P^0(S_1^0, S_2^0, 0, 0, 0)$, S_1^0 and S_2^0 satisfies equations

$$\begin{cases} \Lambda - \beta_E S_1 E - \beta_I S_1 I - (p+\mu) S_1 = 0, \\ p S_1 - \beta_E \sigma S_2 E - \beta_I \sigma S_2 I - \mu S_2 = 0, \end{cases} \tag{4.1}$$

then (2.2) can be rewritten as follows:

$$\begin{cases} {S_1}' = S_1[\Lambda(\frac{1}{S_1} - \frac{1}{S_1^0}) - \beta_E E - \beta_I I], \\ {S_2}' = S_2[p(\frac{S_1}{S_2} - \frac{S_1^0}{S_2^0}) - \beta_E \sigma E - \beta_I \sigma I], \\ E' = (\beta_E E + \beta_I I)[(S_1^0 + \sigma S_2^0) + (S_1 - S_1^0) + \sigma(S_2 - S_2^0)] - (r+\mu)E, \\ I' = (1-q)rE - (\mu + d + \varepsilon)I, \\ Q' = qrE + \varepsilon I - (\mu + d + \xi)Q \end{cases} \tag{4.2}$$

Define the Lyapunov function

$$V_1 = (S_1 - S_1^0 - S_1^0 \ln \frac{S_1}{S_1^0}) + (S_2 - S_2^0 - S_2^0 \ln \frac{S_2}{S_2^0}) + E + \frac{(r+\mu) - \beta_E(S_1^0 + \sigma S_2^0)}{(1-q)r} I. \tag{4.3}$$

The derivative of V_1 is given by

$$\begin{aligned} {V_1}' &= (S_1 - S_1^0)[\Lambda(\frac{1}{S_1} - \frac{1}{S_1^0}) - \beta_E E - \beta_I I] \\ &\quad + (S_2 - S_2^0)[p(\frac{S_1}{S_2} - \frac{S_1^0}{S_2^0}) - \beta_E \sigma E - \beta_I \sigma I] \\ &\quad + (\beta_E E + \beta_I I)[(S_1^0 + \sigma S_2^0) + (S_1 - S_1^0) + \sigma(S_2 - S_2^0)] - (r+\mu)E \\ &\quad + \frac{(r+\mu) - \beta_E(S_1^0 + \sigma S_2^0)}{(1-q)r}[(1-q)rE - (\mu + d + \varepsilon)I] \\ &= \beta_I(S_1^0 + \sigma S_2^0)I - \frac{[(r+\mu) - \beta_E(S_1^0 + \sigma S_2^0)](\mu + d + \varepsilon)}{(1-q)r} I + F(S, I) \\ &= \frac{(r+\mu)(\mu + d + \varepsilon)}{(1-q)r}(R_0 - 1)I + F(S, I), \\ &= \frac{(r+\mu)A}{B}(R_0 - 1)I + F(S, I), \end{aligned} \tag{4.4}$$

where

$$F(S, I) = \Lambda(S_1 - S_1^0)(\frac{1}{S_1} - \frac{1}{S_1^0}) + p(S_2 - S_2^0)(\frac{S_1}{S_2} - \frac{S_1^0}{S_2^0}).$$

Denote $x = \frac{S_1}{S_1^0}$, $y = \frac{S_2}{S_2^0}$, then

$$F(S, I) = \Lambda(x-1)(\frac{1}{x} - 1) + pS_1^0(y-1)(\frac{x}{y} - 1) =: \overline{F}(x, y). \tag{4.5}$$

Applying (4.1) to function $\overline{F}(x, y)$ yields

$$\begin{aligned}\overline{F}(x,y) &= \Lambda(2-x-\frac{1}{x})+pS_1^0(x-y-\frac{x}{y}+1)\\ &= (2\Lambda+pS_1^0)-(\Lambda-pS_1^0)x-\Lambda\frac{1}{x}-pS_1^0y-pS_1^0\frac{x}{y}\\ &= 3pS_1^0+2\mu S_1^0-\mu S_1^0x-(pS_1^0+\mu S_1^0)\frac{1}{x}-pS_1^0y-pS_1^0\frac{x}{y}\\ &= pS_1^0(3-\frac{1}{x}-y-\frac{x}{y})+\mu S_1^0(2-x-\frac{1}{x}).\end{aligned} \tag{4.6}$$

We have $\overline{F}(x, y) \leq 0$ for $x, y > 0$ and $\overline{F}(x, y) = 0$ if and only if $x = y = 1$. Since $R_0 \leq 1$, then ${V_1}' \leq 0$. It follows from LaSalle invariance principle [19] that the disease free equilibrium P^0 is globally asymptotically stable when $R_0 \leq 1$.

4.2. Global stability of the endemic equilibrium

For the endemic equilibrium $P^*(S_1^*, S_2^*, E^*, I^*, Q^*)$, S_1^*, S_2^*, E^*, I^*, and Q^* satisfies equations

$$\begin{cases}\Lambda-\beta_E S_1E-\beta_I S_1I-(p+\mu)S_1=0,\\ pS_1-\beta_E\sigma S_2E-\beta_I\sigma S_2I-\mu S_2=0,\\ \beta_E E(S_1+\sigma S_2)+\beta_I I(S_1+\sigma S_2)-(r+\mu)E=0,\\ (1-q)rE-(\mu+d+\varepsilon)I=0,\\ qrE+\varepsilon I-(\mu+d+\xi)Q=0.\end{cases} \tag{4.7}$$

By applying (4.7) and denoting

$$\frac{S_1}{S_1^*}=x,\qquad \frac{S_2}{S_2^*}=y,\qquad \frac{E}{E^*}=z,\qquad \frac{I}{I^*}=u,\qquad \frac{Q}{Q^*}=v$$

we have

$$\begin{cases}x'=x[\frac{\Lambda}{S_1^*}(\frac{1}{x}-1)-\beta_E E^*(z-1)-\beta_I I^*(u-1)],\\ y'=y[\frac{pS_1^*}{S_2^*}(\frac{x}{y}-1)-\beta_E\sigma E^*(z-1)-\beta_I\sigma I^*(u-1)],\\ z'=z\{\beta_E[S_1^*(x-1)+\sigma S_2^*(y-1)]+\frac{\beta_I I^*}{E^*}[S_1^*(\frac{xu}{z}-1)+\sigma S_2^*(\frac{yu}{z}-1)]\},\\ u'=u\frac{(1-q)rE^*}{I^*}(\frac{z}{u}-1),\\ v'=v[\frac{qrE^*}{Q^*}(\frac{z}{v}-1)+\frac{\varepsilon I^*}{Q^*}(\frac{u}{v}-1)].\end{cases} \tag{4.8}$$

Define the Lyapunov function

$$V_2 = S_1^*(x-1-\ln x)+S_2^*(y-1-\ln y)+E^*(z-1-\ln z)+\frac{\beta_I(r+\mu)}{\beta_E A+\beta_I B}I^*(u-1-\ln u). \tag{4.9}$$

The derivative of V_2 is given by

$${V_2}' = S_1^*\frac{x-1}{x}x'+S_2^*\frac{y-1}{y}y'+E^*\frac{z-1}{z}z'+\frac{\beta_I(r+\mu)}{\beta_E A+\beta_I B}I^*\frac{u-1}{u}u'$$

$$
\begin{aligned}
&= (x-1)[\Lambda(\frac{1}{x}-1)-\beta_E S_1^* E^*(z-1)-\beta_I S_1^* I^*(u-1)]\\
&\quad +(y-1)[pS_1^*(\frac{x}{y}-1)-\beta_E\sigma S_2^* E^*(z-1)-\beta_I\sigma S_2^* I^*(u-1)]\\
&\quad +(z-1)\{\beta_E E^*[S_1^*(x-1)+\sigma S_2^*(y-1)]+\beta_I I^*[S_1^*(\frac{xu}{z}-1)+\sigma S_2^*(\frac{yu}{z}-1)]\}\\
&\quad +\frac{\beta_I(r+\mu)}{\beta_E A+\beta_I B}(u-1)(1-q)rE^*(\frac{z}{u}-1)\\
&= \Lambda(x-1)(\frac{1}{x}-1)-\beta_I S_1^* I^*(x-1)(u-1)\\
&\quad +pS_1^*(y-1)(\frac{x}{y}-1)-\beta_I\sigma S_2^* I^*(y-1)(u-1)\\
&\quad +\beta_I S_1^* I^*(z-1)(\frac{xu}{z}-1)+\beta_I\sigma S_2^* I^*(z-1)(\frac{yu}{z}-1)\\
&\quad +\frac{\beta_I(r+\mu)}{\beta_E A+\beta_I B}(1-q)rE^*(u-1)(\frac{z}{u}-1)\\
&= \Lambda(2-x-\frac{1}{x})-\beta_I S_1^* I^*(xu-x-u-1)+pS_1^*(x-y-\frac{x}{y}+1)\\
&\quad -\beta_I\sigma S_2^* I^*(yu-y-u+1)+\beta_I S_1^* I^*(xu-z-\frac{xu}{z}+1)+\beta_I\sigma S_2^* I^*(yu-z-\frac{yu}{z}+1)\\
&\quad +\frac{\beta_I(r+\mu)}{\beta_E A+\beta_I B}(1-q)rE^*(z-u-\frac{z}{u}+1)\\
&= 2\Lambda+pS_1^*+\frac{\beta_I(r+\mu)}{\beta_E A+\beta_I B}(1-q)rE^*-x(\Lambda-\beta_I S_1^* I^*-pS_1^*)-\Lambda\frac{1}{x}-y(pS_1^*-\beta_I\sigma S_2^* I^*)\\
&\quad -pS_1^*\frac{x}{y}-\beta_I S_1^* I^*\frac{xu}{z}-\beta_I\sigma S_2^* I^*\frac{yu}{z}-\frac{\beta_I(r+\mu)}{\beta_E A+\beta_I B}(1-q)rE^*\frac{z}{u}\\
&= (\Lambda-\beta_I S_1^* I^*-pS_1^*)(2-x-\frac{1}{x})+(pS_1^*-\beta_I\sigma S_2^* I^*)(3-\frac{1}{x}-y-\frac{x}{y})\\
&\quad +\beta_I S_1^* I^*(3-\frac{1}{x}-\frac{xu}{z}-\frac{z}{u})+\beta_I\sigma S_2^* I^*(4-\frac{1}{x}-\frac{x}{y}-\frac{yu}{z}-\frac{z}{u}) \qquad (4.10)
\end{aligned}
$$

Since the arithmetical mean is greater than, or equal to the geometrical mean, then, $2-x-\frac{1}{x}\le 0$ for $x>0$ and $2-x-\frac{1}{x}=0$ if and only if $x=1$; $3-\frac{1}{x}-y-\frac{x}{y}\le 0$ for $x,y>0$ and $3-\frac{1}{x}-y-\frac{x}{y}=0$ if and only if $x=y=1$; $3-\frac{1}{x}-\frac{xu}{z}-\frac{z}{u}\le 0$ for $x,z,u>0$ and $3-\frac{1}{x}-\frac{xu}{z}-\frac{z}{u}=0$ if and only if $x=1,z=u$; $4-\frac{1}{x}-\frac{x}{y}-\frac{yu}{z}-\frac{z}{u}\le 0$ for $x,y,z,u>0$ and $4-\frac{1}{x}-\frac{x}{y}-\frac{yu}{z}-\frac{z}{u}=0$ if and only if $x=y=1,z=u$. Therefore, $V_2'\le 0$ for $x,y,z,u>0$ and $V_2'=0$ if and only if $x=y=1,z=u$, the maximum invariant set of system (2.2) on the set $\{(x,y,z,u): V_2'=0\}$ is the singleton $(1,1,1,1)$. Thus, for system (2.2), the endemic equilibrium P^* is globally asymptotically stable if $R_0>1$ by LaSalle Invariance Principle [19].

5. A case study

In this section, we estimate the unknown parameters of model (2.2) on the basis of the total confirmed new cases in the UK from February 1, 2020 to March 23, 2020 by using MCMC algorithm. By estimating the unknown parameters, we estimate the mean and confidence interval of the basic reproduction number R_0.

5.1. Parameter estimation and model fitting

The total confirmed cases can be expressed as follows

$$\frac{dC}{dt} = qrE + \varepsilon I,$$

where $C(t)$ indicates the total confirmed cases.

As for the total confirmed new cases, it can be expressed as following

$$NC = C(t) - C(t-1), \tag{5.1}$$

where NC represents the total confirmed new cases.

We use the MCMC method [20–22] for 20000 iterations with a burn-in of 5000 iterations to fit the Eq (5.1) and estimate the parameters and the initial conditions of variables (see Table 2). Figure 2 shows a good fitting between the model solution and real data, well suggesting the epidemic trend in the United Kingdom. According to the estimated parameter values and initial conditions as given in Table 2, we estimate the mean value of the reproduction number $R_0 = 4.2816$ (95%CI : (3.8882, 4.6750)).

Table 2. The parameters values and initial values of the model (2.2).

Parameters	Mean value	Std	95% CI	Reference
Λ	0	–	–	Estimated
μ	0	–	–	Estimated
d	1/18	–	–	[23]
γ	1/7	–	–	[24]
ξ	1/18	–	–	[23]
β_E	5.1561×10^{-9}	5.1184×10^{-10}	$[0.4153 \times 10^{-8}, 0.6159 \times 10^{-8}]$	MCMC
β_I	2.3204×10^{-8}	3.1451×10^{-9}	$[0.1704 \times 10^{-7}, 0.2937 \times 10^{-7}]$	MCMC
p	0.5961	0.0377	[0.5223, 0.6700]	MCMC
q	0.2250	0.0787	[0.0707, 0.3793]	MCMC
σ	0.3754	0.0135	[0.3490, 0.4018]	MCMC
ε	0.2642	0.0598	[0.1470, 0.3814]	MCMC
$S_1(0)$	3.3928×10^7	2.9130×10^6	$[2.8219 \times 10^7, 3.9638 \times 10^7]$	Calculated
$S_2(0)$	3.2645×10^7	2.9130×10^6	$[2.6936 \times 10^7, 3.8355 \times 10^7]$	MCMC
$E(0)$	12.8488	3.2238	[6.5302, 19.1674]	MCMC
$I(0)$	0.8070	0.7739	[0, 2.3239]	MCMC
$Q(0)$	2	–	–	[25]
$R(0)$	0	–	–	Estimated
$N(0)$	66573504	–	–	[26]

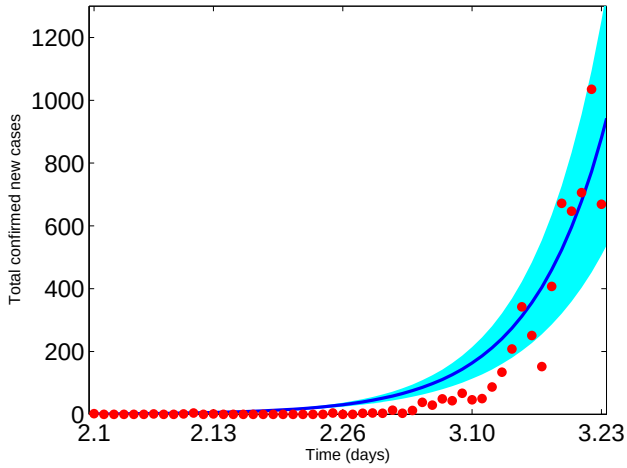
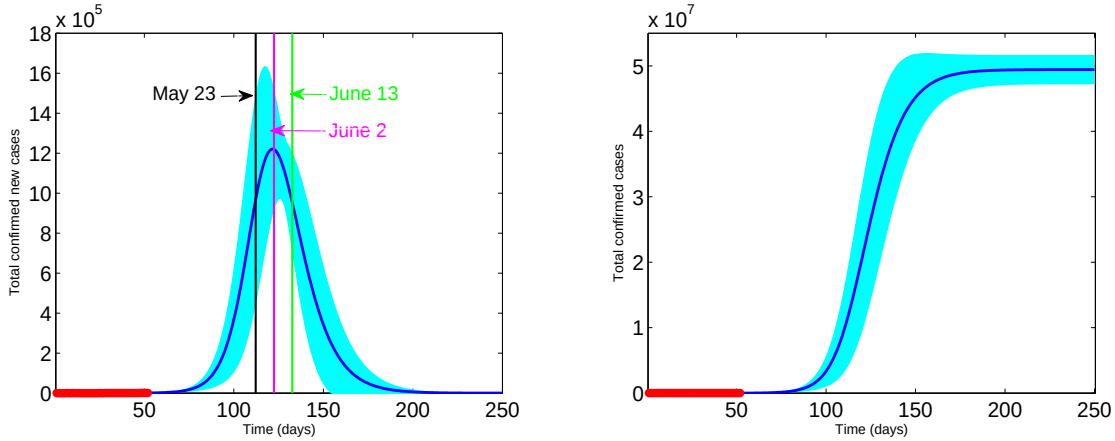


Figure 2. The fitting results of the total confirmed new cases from February 1, 2020 to March 23, 2020. The blue line is the simulated curve of model (2.2). The red dots represent the actual data. The light blue area is the 95% confidence interval (CI) for all 5000 simulations.

5.2. Prediction of epidemiological quantities

Applying the estimated parameter values, without the most restrictive measures in UK, we forecast that the peak size is 1.2902×10^6 (95%CI : $(1.1429 \times 10^6, 1.4374 \times 10^6)$), the peak time is June 2 (95%CI : (May 23, June 13)) (Figure 3a), and the final size is 4.9437×10^7 (95%CI : $(4.7199 \times 10^7, 5.1675 \times 10^7)$) in the UK (Figure 3b).



(a) (b)

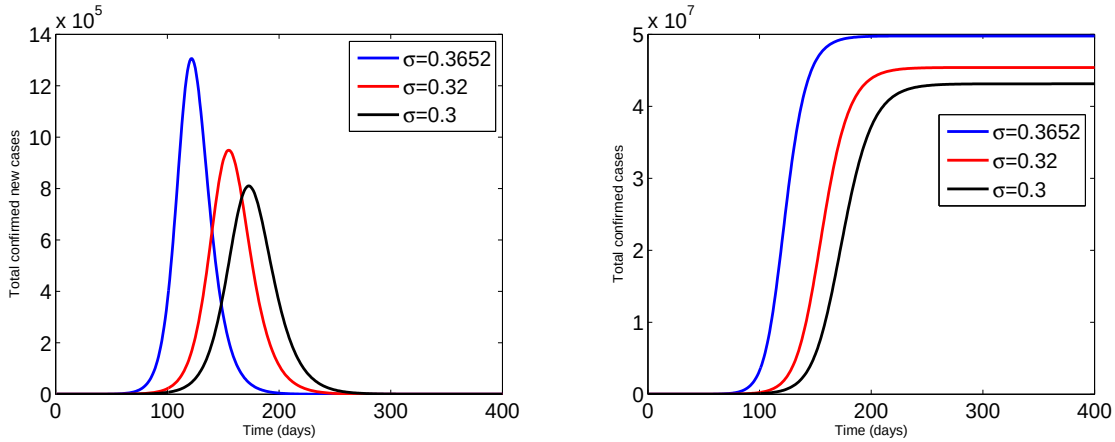
Figure 3. (a) Forecasting trends of total confirmed new cases. (b) Forecasting trends of total confirmed cases. The blue line is the simulated curve of model (2.2). The red dots represent the actual data. The light blue area is the 95% confidence interval (CI) for all 5000 simulations.

5.3. *Sensitivity analysis*

In this section, we do the sensitivity analysis for four vital model parameters σ, p, q and ε, which reflect the intensity of contact, media coverage and isolation, respectively.

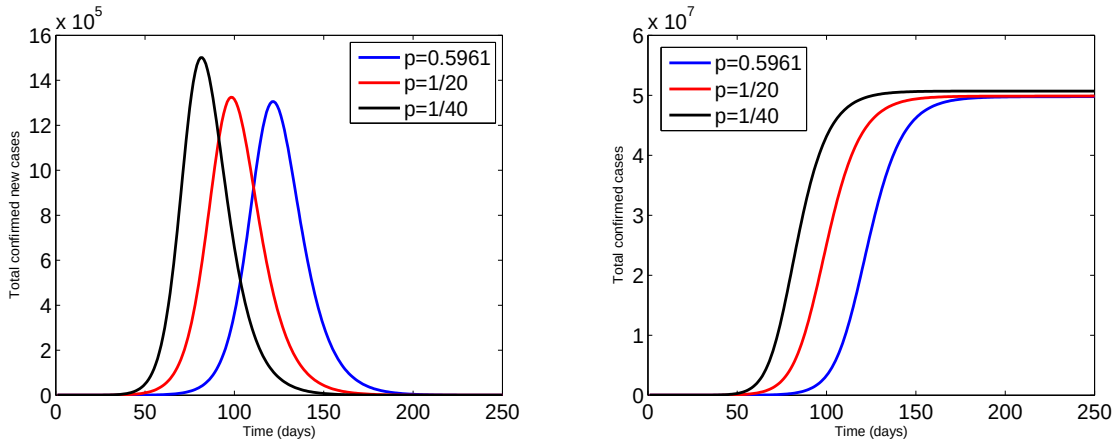
Figure 4 and Table 3 show that reducing the fraction σ of the conscious susceptible S_2 contacting with the latent compartment (E) and the infectious compartment (I) delays the peak arrival time, decreases the peak size of confirmed cases and decreases the final size. Reducing the fraction σ is in favor of controlling COVID-19 transmission.

Figure 5 and Table 3 show that reducing migration rate p to S_2 from S_1,reflecting the impact of media coverage advances the peak arrival time, increase the peak size of confirmed cases and the final size.



(a) (b)

Figure 4. (a) Effects of different Parameter σ on The numbers of total confirmed new cases. (b) Effects of different Parameter σ on The numbers of total confirmed cases.



(a) (b)

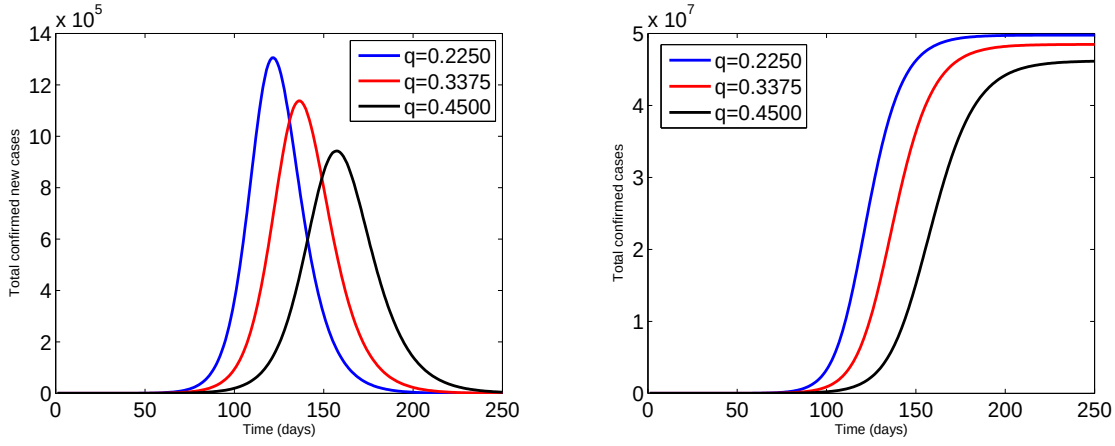
Figure 5. (a) Effects of different Parameter p on The numbers of total confirmed new cases. (b) Effects of different Parameter p on The numbers of total confirmed cases.

Table 3. Epidemic quantities for the UK.

Parameter	Peak size	Peak time	Final size
$\sigma = 0.3652$	1.2056×10^6	May 23	4.9766×10^7
$\sigma = 0.32$	9.4930×10^5	June 25	4.5390×10^7
$\sigma = 0.3$	8.1063×10^5	July 13	4.3121×10^7
$p = 0.5961$	1.3056×10^6	May 23	4.9766×10^7
$p = 1/20$	1.3248×10^6	May 19	4.9864×10^7
$p = 1/40$	1.5009×10^6	May 3	5.0699×10^7
$q = 0.2250$	1.3056×10^6	May 23	4.9766×10^7
$q = 0.3375$	1.1379×10^6	June 6	4.8471×10^7
$q = 0.4500$	9.4266×10^5	June 27	4.6141×10^7
$\varepsilon = 0.2642$	1.3056×10^6	May 23	4.9766×10^7
$\varepsilon = 0.3170$	1.1766×10^6	June 2	4.8598×10^7
$\varepsilon = 0.3963$	9.9640×10^5	June 18	4.6145×10^7

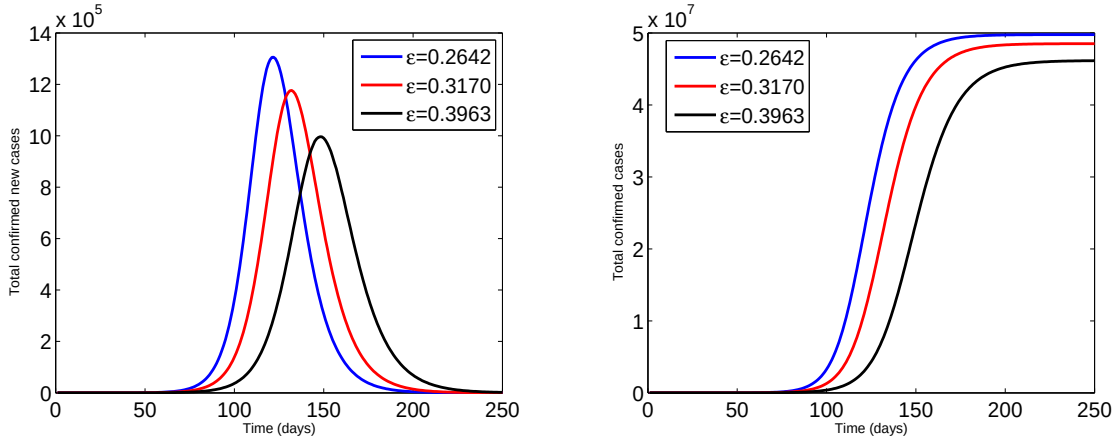
Figures 6,7 and Table 3 show that, with increase the fraction q (Individuals in the latent compartment E jump into the quarantine compartment Q) and the transition rate ε (the infectious compartment I jump into the quarantine compartment Q), the peak time delays, the the peak size and the final size decrease. This show that Increasing the intensity of detection and isolation may affect the spread of COVID-19.

Then the test set data is used to verify the short-term prediction effect of the model (Figure 8). we fit the model with the total confirmed new cases from February 1, 2020 to March 23, 2020, and verify the fitting results with the new cases from March 24, 2020 to April 12, 2020. The model has a good fit to the trajectory of the coronavirus prevalence for a short time in the UK.



(a) (b)

Figure 6. (a) Effects of different Parameter q on The numbers of total confirmed new cases. (b) Effects of different Parameter q on The numbers of total confirmed cases.



(a) (b)

Figure 7. (a) Effects of different Parameter ε on The numbers of total confirmed new cases. (b) Effects of different Parameter ε on The numbers of total confirmed cases.

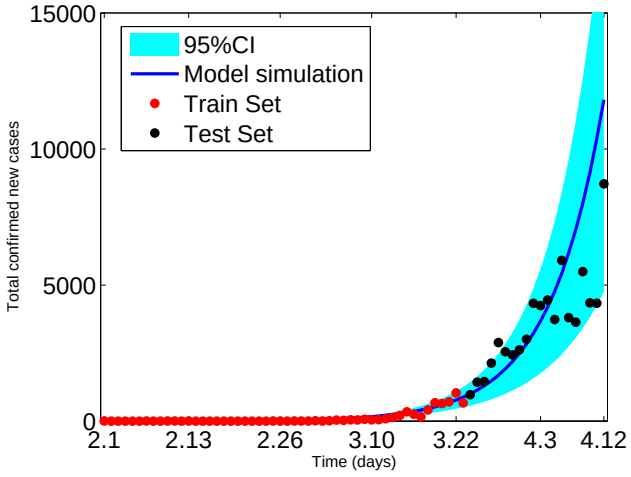


Figure 8. The test set data is used to verify the short-term prediction effect of the model.

6. Discussions

We have formulated the COVID-19 epidemic model with media coverage and quarantine and investigated their dynamical behaviors. By means of the next generation matrix, we obtained their basic reproduction number, R_0, which play a crucial role in controlling the spread of COVID-19. By constructing Lyapunov function, we proved the global stability of their equilibria: when the basic reproduction number is less than or equal to one, all solutions converge to the disease free equilibrium, that is, the disease dies out eventually; when the basic reproduction number exceeds one, the unique endemic equilibrium is globally stable, that is, the disease will persist in the population and the number of infected individuals tends to a positive constant. We use the MCMC algorithm to estimate the unknown parameters and initial values of the model (2.2) on the basis of the total confirmed new cases in the UK. The sensitivity of all parameters are evaluated.

Through the mean and confidence intervals of the parameters in Table 2, we obtain the basic reproduction number $R_0 = 4.2816(95\%\text{CI} : (3.8882, 4.6750))$, which means that the novel coronavirus pneumonia is still pandemic in the crowd. The sensitivity of the parameters provides a possible intervention to reduce COVID-19 infection. People should wear masks, avoid contact or reduce their outings, take isolation measure to reduce the spread of virus during COVID-19 outbreaks.

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Conflict of interest

The authors declare there is no conflict of interest.

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