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Research article

Government bond market risk-return trade-off

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Supplementary

Appendix

Conditional Higher Moments Calculation

We assume that the log-returns are governed by the very general SGED distribution (Theodossiou, 2015)

$$f_r\left(r_t\,|\,\Phi_{t-1}\right)=\frac{1}{2\theta_t\sigma_t}k_t^{1-\frac{1}{k_t}}\Gamma\left(\frac{1}{k_t}\right)^{-1}\exp\left(-\frac{1}{k_t}\left|\frac{u_t}{\left(1+sgn(u_t)\lambda_t\right)\theta_t\sigma_t}\right|^{k_t}\right), \qquad \text{(A1)}$$

where $\Gamma(.)$ is the Gamma function and u_t comes from the risk-return equation

$$r_t = m_0 + br_{t-1} + \left(\varphi+\delta_t\right)\sigma_t + u_t \qquad \text{(A2)}$$

where the risk is decomposed into two components the pure price of risk (φ) and the time varying price of risk (δ_t) attributed to skewness and kurtosis in the data (see Theodossiou and Savva, 2016; and Delis et al., 2020 for more details).
δ_t is defined as

$$\delta_t = \frac{2\lambda_t G_1}{\sqrt{\left(3\lambda_t^2 + 1\right)G_2 - 4\lambda_t^2 G_1^2}} \tag{A3}$$

with

$$G_s = k_t^{\frac{s}{k_t}} \Gamma\left(\frac{s+1}{k_t}\right)\Gamma\left(\frac{1}{k_t}\right)^{-1}, \tag{A4}$$

for $s = 1, 2, 3, 4$.
θ_t is defined as

$$\theta_t = 1 \Big/ \sqrt{\left(1 + 3\lambda_t^2\right)G_2 - 4\lambda_t^2 G_1^2} \tag{A5}$$

while σ_t, λ_t and k_t are the conditional variance, asymmetry (skewness) and shape (kurtosis) parameters/variables respectively, defined as follows:
The conditional variance of returns is specified as

$$\sigma_t^2 = v_0 + \left(\alpha + \alpha_N N_{t-1}\right)u_{t-1}^2 + \beta\,\sigma_{t-1}^2 \tag{A6}$$

where $N_{t-1} = 1$ for $u_{t-1} < 0$, and $N_{t-1} = 0$ for $u_{t-1} > 0$ and v_0, α, α_N, β are interpreted as usual parameters of a GJR-GARCH specification.
The conditional asymmetry parameter, which controls the shape of the distribution of returns, and it is used as the conditional skewness variable in the main specification is:

$$\lambda_t = 1 - \frac{2}{1 + e^{h_{ts}}} \tag{A7}$$

where

$$h_t = \gamma_0 + \gamma_N u_{t-1}^- + \gamma_P u_{t-1}^+ + \gamma_h h_{t-1} \tag{A8}$$

The measures $u_t^- = |u_t|$ for $u_t < 0$ and zero otherwise, and $u_t^+ = |u_t|$ for $u_t > 0$ and zero otherwise are proxies for downside and upside shocks, respectively (Feunou et al., 2012). The intercept γ_0, is a measure of unconditional asymmetry, while the coefficient γ_N measures the marginal impact of downside price shocks on the asymmetry index h_t and the asymmetry parameter λ_t. In contrast, the coefficient γ_P measures the marginal impact of past price shocks on h_t and λ_t. The coefficient γ_h measures the persistence of past upside and downside shocks on the conditional values of h_t and λ_t (see Delis et al., 2020 for further details).

Finally, the conditional shape parameter (used as the conditional kurtosis variable in the main specification) is defined by using (see also Mazur and Pipień, 2018):

$$k_t = k_U - \frac{k_U - k_L}{1 + e^{g_t}} \tag{A9}$$

where

$$g_t = d_0 + d_N u_t^- + d_P u_t^+ + d_h g_{t-1} \tag{A10}$$

u_t^- and u_t^+ are as defined previously, and k_L and k_U are the predetermined lower and upper limits for the time varying shape parameter k_t. For the estimation, k_L and k_U are set to 0.3 and 1.8, respectively. The parameters d_N and d_P control the shape of the distribution while d_h proxies for the persistence effect.