



Research article

Wavelet-based systematic risk estimation: application on GCC stock markets: the Saudi Arabia case

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Supplementary

In this appendix we provide the empirical results due to the 3-scale law. The new approach in estimating the systematic risk of an asset in the CAPM is based on the application of a wavelet multi-scaling by decomposing a given times series on a scale by scale basis. At each scale, we estimated the wavelet variance of the market return and the wavelet covariance between the market return and a return stock to estimate next the stock's beta. We have applied two different scales, a first one consisting of the 2-scale law which is always used in wavelet analysis even in other fields and a second one consisting of a 3-scale law to reinforce the results obtained with the 2-scale and thus confirm the role of the time factor in the analysis of markets. Recall that all the studies applying wavelets especially in financial and economic fields restrict to the 2-scaling wavelets. However, splitting the whole period for studying markets into sub-periods of the form 2^j may not be always adequate. Recall that the size of the time series whenever being different from the 2-scale form yields generally real problems at the boundaries. For this reason, we thought to test a different scale. To avoid the divisibility by 2 again, we thought that prime scales will be adequate. In fact, we prepare in the present study the arguments to develop as future direction a random time scale to express more adequately the uncertainty in markets movement. We also provide the estimation of the determination coefficient R^2 as for the previous cases in Table b.

Table a. Estimations of the return excess of actions on the market for 3-scale law.

STOCKS	Beta for scales 1 to 6					
	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
AADC	4.2924	3.6979	4.0744	-0.6988	0.6194	-0.0752
AAIC	-2.1138	2.1907	-0.4069	2.4240	0.5999	-0.1915
AKSC	0.6964	1.4596	-0.8958	1.7118	-0.3511	0.0106
AAOMC	-1.6754	1.4722	0.7010	-0.8129	0.6893	0.0104
ABALAwal	1.5193	0.6199	0.2278	0.3631	0.0201	0.1376
ABAINma	-0.0739	0.7812	-0.6264	1.7059	-0.4877	0.0696
ABIDC	0	0	0	0	0	0
ABPTC	4.4997	0.1302	-0.6417	0.2017	0.6494	0.7416
AACI	-5.1350	1.4548	-2.3335	2.7024	0.5124	0.2476
ACAbdullatif	1.3791	0.7426	-0.2186	-1.0152	0.4179	0.3914
ACAImaraI	-1.2346	0.4067	0.4287	1.1875	-0.4446	-0.0159
ACAlujain	-5.0553	-0.3420	-0.9928	-1.3141	0.8793	-0.1457
ACCAC	0.4442	1.5939	-0.9842	1.1130	-0.4699	0.0851
ACI	-2.5332	4.0168	-3.6271	2.0066	0.7739	-0.8392
ACIC	2.6265	0.8749	-2.1012	0.1153	0.4612	0.5757
ACIG	3.6239	-0.0932	-3.1657	5.5101	-1.1403	-0.0549
ADC	-1.1153	0.2146	-0.9818	0.7942	-0.0207	0.0643
AICC	-1.8933	1.6367	-2.1876	0.7715	2.8441	-0.1979
AIG	-1.0871	-0.9683	-0.0344	-0.2061	-1.0655	0.1742
AIHG	6.5438	1.7925	-0.2757	3.1007	0.9920	-0.1478
AJADC	-0.4901	1.1101	0.4374	-0.7206	0.5606	-0.0898
AJCC	-1.2902	0.8597	-0.8760	0.8970	-0.2107	0.0181
AKGIS	-0.9641	-1.0325	-0.5946	0.1401	0.4152	0.1468
ANB	0.8399	0.6677	0.7857	-0.7738	-0.2217	-0.0115
APC1	0.0700	1.0097	1.1770	-0.6824	-0.1131	0.1291
APC2	-0.2525	1.5737	-0.4140	1.3011	-1.2576	-0.0952
APTS	0.3711	0.9815	0.6247	0.5860	-0.4446	0.1449
ARB	0.0626	0.4847	-0.6628	0.0766	0.0980	0.0323
ARCI	0.6699	1.9911	-1.6486	1.7647	-0.3865	-0.0071
ASAXASA	-1.9057	0.4453	-0.9331	-0.0929	0.2926	-0.1352
ASCIC	-2.1583	1.8581	-0.8889	-3.6034	-0.1380	-0.2701
ASCIC2	-3.5710	3.5923	-0.3326	-0.3789	-1.2178	-0.1125
ASDC	0.0897	2.9155	-0.8600	1.4112	-0.8456	0.1170
ASTIG	0.4289	2.1020	1.4421	-0.0684	-0.3170	-0.3071
ASVON	-4.3642	-2.0703	-3.0547	1.6010	0.1579	-0.0453
ATC	-1.2858	0.6636	-0.3312	0.4921	-0.2160	-0.0327
ATEC	0.4200	0.9231	0.3658	1.2121	-0.5331	-0.0217
ATTM	-0.2684	0.8497	-0.2324	1.7885	-0.7882	-0.0147
BA	-0.4786	-0.7160	-1.7815	1.2378	-0.6094	-0.0446

STOCKS	Beta for scales 1 to 6					
	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
BA2	0.2943	0.4516	-0.5153	1.5031	0.5661	0.0664
BACI	-1.2344	3.8738	-1.9305	-1.1344	-0.2589	-0.6491
BCIC	0.4468	0.3446	-0.0457	-0.8279	-0.4967	-0.0825
BCIC2	0.7415	2.7417	1.3318	4.8612	-0.6509	-0.0786
BSF2	0.9505	0.4715	-0.9422	0.1761	-0.4261	0.2103
DARED	1.7736	1.4033	-3.0122	2.5296	-0.9359	0.1303
DH	-0.4552	0.3606	-0.4105	0.6109	-0.3078	-0.0840
EAT	3.0644	-1.3772	4.1812	1.0425	-0.0278	-0.1699
EEC	0.4412	0.0670	-1.3632	0.2482	0.2985	0.1865
EPCC	-0.8664	-0.8131	-0.3768	0.0736	0.2639	-0.1365
ETEC	-0.2397	1.6825	-2.7424	1.8923	-0.8176	0.1719
FAAC	2.1075	0.4908	2.0932	0.5864	0.4061	0.1370
FFS	-2.0767	-1.1048	0.5325	-0.0565	-0.4522	-0.1235
FPC	-0.3528	2.1198	-0.9292	0.9266	-1.2378	-0.3381
FPMC	0.1225	1.6415	0.1049	0.8749	-1.7205	-0.3093
GGCI	-1.4147	2.8228	0.2042	4.6504	2.4996	-0.3598
GUCI	4.2604	3.3091	2.5820	0.2833	0.2289	-0.0134
HB	-0.7187	2.9968	2.3670	-0.6530	1.1773	-0.0931
HCC	-0.9853	1.8563	-2.2351	0.6604	-0.6040	-0.0302
HFSC	-1.7657	-0.2215	-2.6238	-1.4931	0.3344	0.0513
JDC	0.0466	2.2127	-2.0561	0.5896	-1.4171	-0.0769
JMC	-0.0812	0.6700	-0.2289	0.3678	0.0474	0.0096
JODC	1.2236	1.5196	-1.9431	0.5912	-0.4794	0.0342
KEC	1.3241	3.2024	1.3406	0.7103	-0.5503	-0.1057
KHC	-1.4892	0.1534	-2.1175	0.7722	-0.7576	-0.0548
MCC	-0.6512	1.0481	-1.7724	-0.0581	-0.3898	0.0625
MCDC	0.1928	0.5161	-0.6142	1.3808	-0.1747	-0.0029
MCIC	1.0590	2.1076	0.9083	-1.6697	0.2412	0.1438
MESCC	-4.4835	-0.7082	-3.0450	1.6281	-0.7435	-0.1016
MMSC	1.0374	0.1244	0.1565	-0.9513	0.8307	-0.5516
MTC	-0.7934	-0.8841	1.2844	0.7335	-0.2976	0.3618
NADC	-0.8041	0.7923	0.4664	0.6841	0.3484	0.0612
NAMC	-0.7954	1.9237	-1.0392	3.4280	-0.6462	0.0259
NCC	-1.9029	1.3075	-1.3462	0.1007	-0.1653	-0.1346
NCC2	9.7917	-1.9440	-9.0473	-2.2796	-0.3999	0.0647
NGC	0.8138	1.0541	-0.3292	1.7894	-1.6098	0.0083
NGIC	3.7250	1.5839	-1.1698	0.4601	-0.2052	-0.2195
NIC	0.2138	-0.1305	0.0890	-0.0660	-0.4861	0.0936
NMMC	2.3026	1.7187	-0.3980	0.0533	-0.7632	-0.1724
NPC	0.4096	1.4641	-1.9242	0.9042	-0.2682	-0.0859
QAC	-0.6998	2.1853	0.5195	0.0672	0.4542	0.0635
RB	-0.2324	0.4215	-0.0396	0.9110	-0.1578	0.4056

STOCKS	Beta for scales 1 to 6					
	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
RRPC	-0.0755	0.4500	0.4770	-0.2716	0.1597	-0.0103
RSHSC	-1.0297	1.6269	-0.2027	1.0055	-0.7055	0.2543
RUCI	-0.7599	1.2756	-0.0157	1.0032	0.5653	-0.1931
SAAC	1.7293	0.9250	0.2846	0.4116	-0.4203	-0.0184
SACI	-0.4992	1.7648	0.6824	-3.0713	-1.5034	0.2844
SAFC	0.9927	0.4277	0.3577	1.3158	-0.3900	-0.1082
SAHARA	-0.9938	0.9293	-0.6001	-0.0299	-0.4424	0.0508
SAIC	-0.1398	-0.5782	-1.7506	1.1359	-0.7491	0.0774
SAMC	1.3545	0.7586	-0.3732	1.9642	0.3296	0.0305
SARC	1.1086	1.5145	-1.7019	1.9290	-2.1307	-0.1266
SASC	0.4795	1.2989	-0.4032	0.7388	-1.7529	-0.1510
SBIC	0.6191	0.1843	-0.0687	-0.3008	-0.5071	0.0134
SCC	0.4951	0.8062	0.1014	0.6135	-0.0138	-0.1208
SCC2	-13.1221	-1.1067	-6.1694	1.2210	-0.0370	-0.2744
SCC3	0.3564	0.9548	-0.5633	1.1985	0.2466	0.0240
SCC4	-0.3089	0.6655	0.7219	1.4424	-0.1629	-0.1607
SCIC	-0.7058	2.1323	-1.5656	2.0632	0.6772	-0.1886
SDFC	0.0312	1.0516	0.0763	-0.0362	-1.3091	0.2765
SECI	-0.5003	-0.8781	-3.5378	-0.1502	0.0117	-0.1789
SEC	-0.0970	0.3918	-0.4570	0.2374	-0.0142	0.1522
SFC	-1.3497	1.6665	-0.3834	1.5223	-0.5635	-0.0298
SFG	1.6867	0.7072	-1.7037	0.0415	0.1419	0.3576
SG	-0.0253	0.0353	0.4188	-0.1922	0.8736	0.1222
SICI	3.2996	-0.5895	1.5018	0.9886	0.4170	0.1036
SIDC	-0.6633	1.5571	-2.2043	1.7334	-1.1781	-0.0108
SIEC	1.5941	3.5012	4.6512	0.4867	1.2396	0.2141
SIIG	1.2006	1.5233	-0.7745	0.0649	-0.0112	0.0745
SIPC	-0.2977	0.9888	-0.3495	-0.1883	-0.1651	0.0176
SISC	-0.6413	1.9129	-1.1857	1.4031	-0.6859	0.0499
SKPC	0.0812	0.7725	-1.1090	0.2012	-0.1782	-0.1034
SPA	0.2387	0.6698	-0.2733	1.5702	-0.1266	-0.0122
SPCC	1.9991	-0.2112	0.5019	-0.7145	-0.2001	0.0537
SPMC	-0.8409	-0.0294	0.3039	1.8621	-0.0524	0.0499
SPPC	3.7600	1.0794	0.2336	0.4256	-1.3013	0.0214
SPTC	0.6383	1.9522	-0.6595	0.7845	0.4288	-0.0322
SRCR	1.6304	3.6902	2.7053	-0.9771	0.6928	-0.1438
SREC	-1.4476	0.5246	-1.0289	1.2213	-0.3665	0.0496
SRMG	4.5818	0.5364	2.5228	1.0270	-0.2112	0.0788
SSPC	1.4786	0.8836	0.2231	0.9175	0.2966	-0.1686
SSTC	-0.5705	1.5994	-1.9148	-0.5088	1.7745	0.9039
ST	0.7008	0.0413	0.7269	-0.6057	-0.8033	0.0901
STIC	0.2391	0.9560	4.0787	0.7055	-1.2061	-0.0679

STOCKS	Beta for scales 1 to 6					
	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
ST2	-0.2128	1.8956	-2.6301	0.7706	-1.4999	-0.1857
SUCI	-1.4362	2.1685	1.4232	6.3373	-1.2602	-0.3278
SVCPC	0.0653	-1.2164	0.6120	0.2654	0.3125	-0.1523
TADC	0.5046	1.8929	-0.9728	-0.5650	-0.1891	-0.1008
TAI	0.6489	1.9062	0.6219	1.0651	-1.0217	-0.0939
TAPR	1.6816	0.1541	-1.0529	-1.2287	0.0678	0.0660
TCC	0.6937	-0.0057	-0.4397	-0.0830	-0.0562	-0.0362
TCCI	0.4380	1.0374	0.6075	2.5994	-0.3833	-0.0635
TEC	-0.6439	1.5582	0.0326	2.6171	-0.5705	-0.5852
THC	-0.7950	1.3151	-1.4128	0.5303	0.1691	0.0548
TMGIC	-2.0924	-0.7485	-0.6337	0.2781	0.0586	-0.0499
TNCGI	-0.2443	1.4623	-0.5289	0.5995	-0.2888	0.0479
TNSC	2.7294	1.0353	0.4952	-0.5993	-0.7396	-0.0171
TQCC	3.3108	0.3626	-0.0539	-0.0263	-0.4272	0.0260
TSBB	1.2229	0.7830	-0.9420	0.9410	-0.1318	0.2430
TSIB	-0.4294	0.6599	0.1897	0.2005	0.2470	-0.0206
UCAC	2.3890	-0.9296	-0.6391	-0.9856	-0.4224	-0.0933
UEC	-1.6611	0.6225	0.3290	0.9929	0.7430	0.1900
UITC	-0.2579	0.6391	0.2311	0.0058	0.3589	-0.0425
UWFC	-1.5828	0.8919	0.3592	1.4907	-0.4044	0.1473
WIC	-0.9842	1.7331	-1.6097	2.6897	-0.6946	-0.0773
YCC	-0.1291	0.2268	2.1286	0.6855	-0.4673	0.0325
YNPC	0.6353	0.4762	0.5719	0.9181	-0.3168	-0.0935
YSCC	-0.0861	0.7109	0.8683	0.4520	0.0900	0.0238
ZIIK	0.4672	0.8111	-0.8627	-0.4459	-0.5825	-0.2724
Mean	0.0972	0.9537	-0.4026	0.6516	-0.1810	-0.0106

Table b. The determination coefficient R^2 relative to the 3-scale law.

STOCKS	R^2 for each scale					
	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
AADC	0.6422	0.5182	0.2790	0.0136	0.0174	0.0011
AAIC	0.3038	0.1540	0.0019	0.0343	0.0030	0.0005
AKSC	0.0522	0.1221	0.0466	0.1480	0.0046	0.0000
AAOMC	0.2398	0.1583	0.0184	0.0487	0.0194	0.0001
ABALAwal	0.4663	0.2589	0.0028	0.0097	0.0001	0.0041
ABAINma	-0.0739	0.7812	-0.6264	1.7059	-0.4877	0.0696
ABIDC	<i>NaN</i>	<i>NaN</i>	<i>NaN</i>	<i>NaN</i>	<i>NaN</i>	<i>NaN</i>
ABPTC	0.7169	0.0017	0.0649	0.0029	0.0216	0.0443
AACI	0.9781	0.0449	0.0560	0.0477	0.0020	0.0011
ACAbdullatif	0.3491	0.0980	0.0144	0.0427	0.0044	0.0531
ACAImaraI	0.4539	0.0223	0.0274	0.2037	0.0087	0.0002
ACAlujain	0.4692	0.0357	0.0273	0.0171	0.0187	0.0049
ACCAC	0.0033	0.0844	0.1773	0.0221	0.0354	0.0043
ACI	0.7089	0.6406	0.0983	0.0366	0.0041	0.0299
ACIC	0.4963	0.0216	0.0134	0.0000	0.0006	0.0047
ACIG	0.6006	0.0002	0.0893	0.1557	0.0047	0.0002
ADC	0.1322	0.0022	0.0207	0.0276	0.0001	0.0013
AICC	0.2734	0.1078	0.0663	0.0056	0.0983	0.0075
AIG	0.3000	0.0849	0.0013	0.0044	0.0607	0.0221
AIHG	0.9394	0.3853	0.0012	0.1488	0.0220	0.0027
AJADC	0.1791	0.1746	0.0127	0.0348	0.0393	0.0090
AJCC	0.5621	0.0520	0.0353	0.1019	0.0082	0.0005
AKGIS	0.0986	0.1123	0.0209	0.0019	0.0399	0.0127
ANB	0.4369	0.1890	0.0828	0.0202	0.0085	0.0001
APC1	0.0009	0.0533	0.0403	0.0346	0.0016	0.0094
APC2	0.0120	0.1108	0.0053	0.0654	0.0388	0.0020
APTS	0.0069	0.0987	0.0652	0.0445	0.0175	0.0123
ARB	0.0009	0.0697	0.0388	0.0004	0.0036	0.0009
ARCI	0.4563	0.1740	0.0276	0.0488	0.0047	0.0000
ASAXASA	0.4797	0.0124	0.0199	0.0001	0.0010	0.0005
ASCIC	0.3806	0.1120	0.0102	0.1264	0.0004	0.0138
ASCIC2	0.5653	0.7235	0.0013	0.0008	0.0136	0.0007
ASDC	0.0003	0.2595	0.0115	0.0429	0.0233	0.0020
ASTIG	0.0354	0.4370	0.2610	0.0003	0.0028	0.0181
ASVON	0.2935	0.2835	0.0345	0.0182	0.0001	0.0003
ATC	0.1592	0.0137	0.0026	0.0036	0.0013	0.0002
ATEC	0.0464	0.0535	0.0063	0.0426	0.0151	0.0002
ATTM	0.0084	0.0275	0.0030	0.1001	0.0578	0.0001
BA	0.0874	0.0751	0.2280	0.0460	0.0400	0.0005
BA2	0.0040	0.0131	0.0137	0.0364	0.0167	0.0012

STOCKS	R ² for each scale					
	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
BACI	0.3377	0.2047	0.0495	0.0184	0.0041	0.0911
BCIC	0.0567	0.0042	0.0001	0.0066	0.0164	0.0022
BCIC2	0.2622	0.1184	0.0132	0.1047	0.0073	0.0010
BSF2	0.2200	0.2203	0.1573	0.0024	0.0837	0.0344
DARED	0.3715	0.0680	0.1549	0.0443	0.0520	0.0057
DH	0.0490	0.1020	0.0158	0.1213	0.0077	0.0046
EAT	0.1381	0.0896	0.1285	0.0141	0.0000	0.0033
EEC	0.8163	0.0023	0.0162	0.0057	0.0150	0.0206
EPCC	0.0670	0.0873	0.0089	0.0007	0.0267	0.0196
ETEC	0.0023	0.0899	0.1654	0.0404	0.0464	0.0082
FAAC	0.2871	0.0259	0.4712	0.0132	0.0148	0.0003
FFS	0.6931	0.4638	0.0358	0.0004	0.0199	0.0051
FPC	0.0675	0.1773	0.0140	0.0150	0.0610	0.0195
FPMC	0.0017	0.0926	0.0001	0.0158	0.1518	0.0158
GGCI	0.1038	0.2028	0.0017	0.1620	0.0460	0.0129
GUCI	0.8515	0.2650	0.1633	0.0005	0.0006	0.0000
HB	0.0872	0.4668	0.1851	0.0056	0.0807	0.0036
HCC	0.3457	0.2297	0.1680	0.0226	0.0261	0.0005
HFSC	0.5050	0.0080	0.3572	0.1269	0.0056	0.0016
JDC	0.0002	0.1859	0.0743	0.0070	0.0898	0.0019
JMC	0.0016	0.1939	0.0277	0.0258	0.0007	0.0002
JODC	0.3039	0.0933	0.2040	0.0115	0.0179	0.0005
KEC	0.3825	0.4486	0.0429	0.0512	0.0148	0.0021
KHC	0.0782	0.0051	0.4455	0.0141	0.0159	0.0012
MCC	0.0404	0.0323	0.1270	0.0001	0.0100	0.0023
MCDC	0.0760	0.2005	0.2281	0.0872	0.0134	0.0000
MCIC	0.2064	0.1667	0.0086	0.0184	0.0026	0.0063
MESCC	0.8964	0.1370	0.1028	0.0536	0.0172	0.0028
MMSC	0.3152	0.0120	0.0022	0.0739	0.0468	0.0915
MTC	0.0357	0.0146	0.0261	0.0036	0.0040	0.0027
NADC	0.3029	0.1103	0.0382	0.0137	0.0196	0.0004
NAMC	0.0322	0.1545	0.0304	0.1454	0.0205	0.0001
NCC	0.2382	0.2026	0.0947	0.0004	0.0063	0.0058
NCC2	0.3402	0.0650	0.3687	0.0407	0.0042	0.0012
NGC	0.0553	0.1398	0.0113	0.1527	0.0919	0.0000
NGIC	0.7900	0.1385	0.2301	0.0186	0.0071	0.0285
NIC	0.0197	0.0011	0.0026	0.0004	0.0291	0.0039
NMMC	0.4820	0.1066	0.0023	0.0001	0.0294	0.0088
NPC	0.0733	0.5306	0.1033	0.0354	0.0040	0.0021
QAC	0.0453	0.1737	0.0015	0.0001	0.0056	0.0009
RB	0.0247	0.1986	0.0021	0.0442	0.0353	0.0029
RRPC	0.0033	0.0512	0.0365	0.0082	0.0014	0.0000

STOCKS	R ² for each scale					
	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
RSHSC	0.2635	0.1356	0.0019	0.0376	0.0325	0.0040
RUCI	0.5468	0.0535	0.0000	0.0144	0.0159	0.0092
SAAC	0.7130	0.4856	0.0342	0.0190	0.0181	0.0003
SACI	0.0735	0.1214	0.0043	0.0682	0.0161	0.0116
SAFC	0.5141	0.0896	0.0527	0.0669	0.0228	0.0014
SAHARA	0.1780	0.0817	0.0485	0.0001	0.0210	0.0019
SAIC	0.0008	0.0078	0.1048	0.0314	0.0156	0.0014
SAMC	0.1818	0.1094	0.0196	0.1110	0.0137	0.0005
SARC	0.5752	0.1034	0.0434	0.0825	0.0818	0.0026
SASC	0.0338	0.0658	0.0044	0.0155	0.1288	0.0110
SBIC	0.1760	0.0062	0.0005	0.0049	0.0303	0.0003
SCC	0.1500	0.1279	0.0017	0.0308	0.0001	0.0094
SCC2	0.8119	0.0675	0.4031	0.0550	0.0001	0.0192
SCC3	0.0174	0.2213	0.1274	0.1542	0.0199	0.0008
SCC4	0.0331	0.0651	0.0655	0.2674	0.0021	0.0206
SCIC	0.0549	0.1673	0.0308	0.0250	0.0068	0.0079
SDFC	0.0001	0.2032	0.0005	0.0001	0.1013	0.0266
SECI	0.0034	0.0110	0.0573	0.0000	0.0000	0.0079
SEC	0.0049	0.0931	0.1304	0.0680	0.0002	0.0465
SFC	0.2341	0.1018	0.0040	0.0673	0.0100	0.0002
SFG	0.2609	0.0711	0.3556	0.0001	0.0045	0.0016
SG	0.0011	0.0002	0.0220	0.0020	0.0635	0.0111
SICI	0.9707	0.0137	0.0211	0.0099	0.0021	0.0007
SIDC	0.0233	0.0758	0.0715	0.0351	0.0460	0.0000
SIEC	0.1602	0.4687	0.2089	0.0016	0.0221	0.0028
SIIG	0.2083	0.2189	0.0607	0.0005	0.0000	0.0033
SIPC	0.0685	0.6548	0.0201	0.0274	0.0060	0.0002
SISC	0.0479	0.1386	0.0298	0.0585	0.0323	0.0016
SKPC	0.0078	0.0415	0.1432	0.0012	0.0041	0.0063
SPA	0.7638	0.1432	0.0085	0.0684	0.0019	0.0001
SPCC	0.4320	0.0044	0.0218	0.0511	0.0093	0.0031
SPMC	0.2790	0.0034	0.0163	0.0247	0.0003	0.0005
SPPC	0.6790	0.1979	0.0030	0.0130	0.0327	0.0000
SPTC	0.1365	0.2632	0.0103	0.0205	0.0137	0.0002
SRCR	0.6821	0.4179	0.1746	0.0218	0.0253	0.0063
SREC	0.4666	0.0384	0.0740	0.0670	0.0156	0.0004
SRMG	0.7713	0.0115	0.0658	0.0255	0.0031	0.0019
SSPC	0.8347	0.0879	0.0118	0.0558	0.0043	0.0106
SSTC	0.1712	0.0906	0.1048	0.0020	0.0639	0.0820
ST	0.6086	0.0010	0.1104	0.0116	0.0978	0.0038
STIC	0.1195	0.0933	0.0956	0.0103	0.0175	0.0007

STOCKS	R ² for each scale					
	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
ST2	0.0014	0.0928	0.1120	0.0045	0.0324	0.0081
SUCI	0.3679	0.1453	0.0115	0.1596	0.0155	0.0123
SVCPC	0.0014	0.2538	0.0722	0.0076	0.0096	0.0058
TADC	0.0672	0.1317	0.0263	0.0071	0.0010	0.0021
TAI	0.0809	0.1316	0.0058	0.0489	0.0277	0.0019
TAPR	0.1655	0.0063	0.2403	0.0177	0.0001	0.0006
TCC	0.1262	0.0000	0.0256	0.0005	0.0006	0.0015
TCCI	0.1066	0.2441	0.0325	0.1179	0.0255	0.0017
TEC	0.0402	0.1633	0.0000	0.1070	0.0101	0.0489
THC	0.0567	0.2558	0.2066	0.0137	0.0034	0.0014
TMGIC	0.3461	0.3202	0.0209	0.0074	0.0001	0.0005
TNCGI	0.0072	0.0804	0.0433	0.0376	0.0066	0.0021
TNSC	0.6759	0.0814	0.0140	0.0055	0.0573	0.0001
TQCC	0.6463	0.0500	0.0009	0.0002	0.0348	0.0010
TSBB	0.1280	0.2328	0.0920	0.0583	0.0034	0.0027
TSIB	0.0271	0.1545	0.0217	0.0037	0.0182	0.0001
UCAC	0.5288	0.2634	0.0039	0.0101	0.0073	0.0002
UEC	0.6727	0.0706	0.0081	0.0665	0.0630	0.0258
UITC	0.0146	0.1957	0.0063	0.0000	0.0109	0.0004
UWFC	0.4298	0.0793	0.0055	0.1271	0.0187	0.0019
WIC	0.0937	0.0709	0.0295	0.0602	0.0082	0.0004
YCC	0.0083	0.0052	0.4901	0.0426	0.0430	0.0008
YNPC	0.0860	0.0164	0.0203	0.0524	0.0095	0.0047
YSCC	0.0014	0.1082	0.0919	0.0255	0.0029	0.0006
ZIIK	0.0626	0.0810	0.2098	0.0318	0.0340	0.0565

In Figure A we test as for the 2-scale law the linearity between the market return and the actions. It shows here also some linearity between the individual stock and the market portfolio where on the horizontal axis the excess of the market return is assigned versus the excess of the stock return assigned to the vertical axis. We notice that the linear relationship is particularly strong at the high scales especially scale 5 and 6. Medium and low scales show some perturbation in the movement of the market.

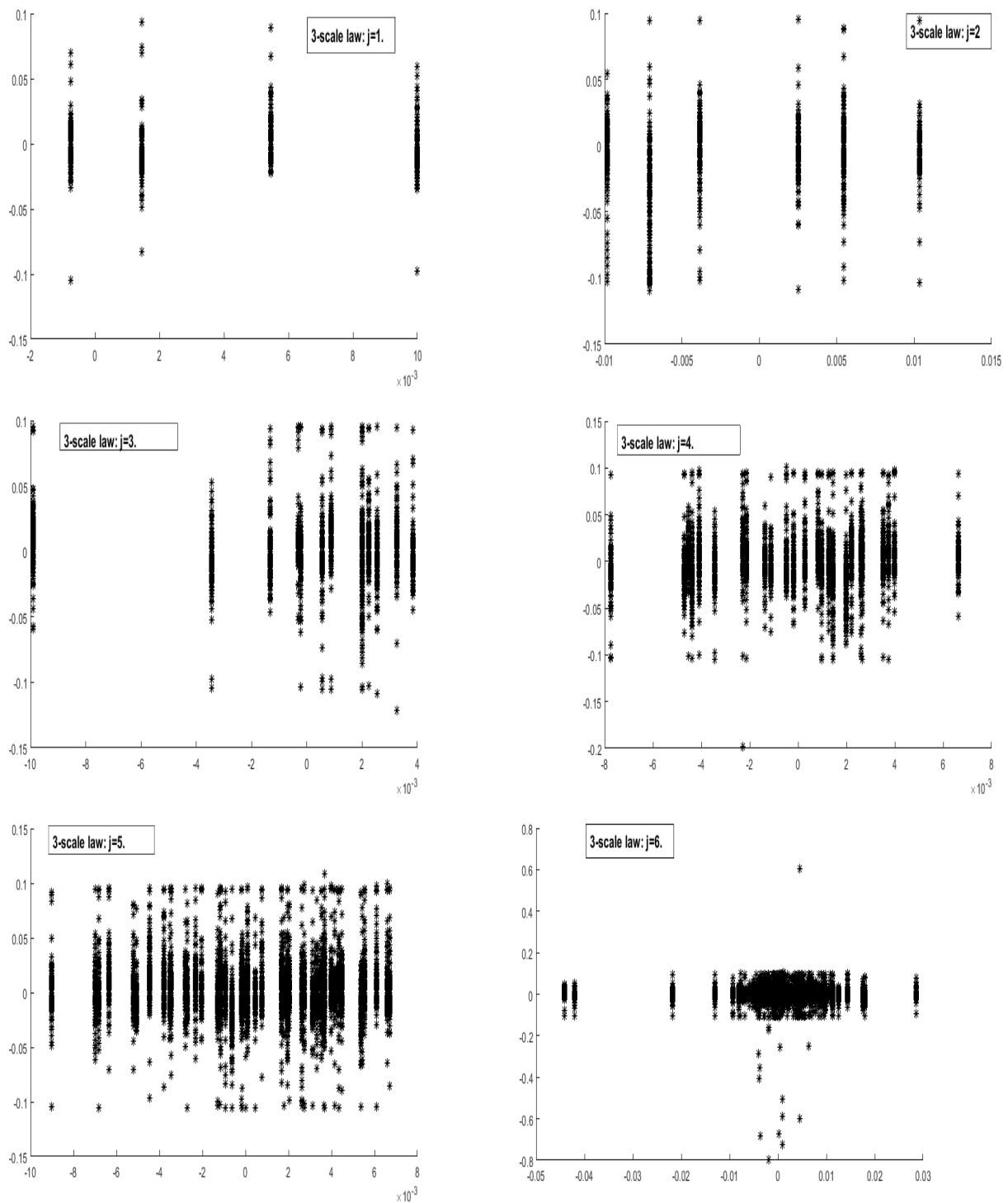


Figure A. Excess market return (horizontal axis) versus excess return of the action (vertical axis) for different time scales with the 3-scale law.



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