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Article

Examination of modelling in K-12 STEM teacher education: Connecting theory with practice

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Abstract: The goal of this paper is to examine the place of modelling in STEM education and teacher education. First, we introduce modelling as a cyclical process of generating, testing, and applying knowledge while highlighting the epistemological commonalities and differences between the STEM disciplines. Second, we build on the four well-known frameworks, to propose an Educational Framework for Modelling in STEM, which describes both teacher and student roles in the modelling cycle. Third, we use this framework to analyze how modelling is presented in the new mathematics and science school curricula in two Canadian provinces (Ontario and British Columbia), and how it could be implemented in teacher education. Fourth, we emphasize the epistemological aspects of the Educational Framework for Modelling in STEM, as disciplinary epistemological foundations may seem too abstract to both teacher educators and teachers of STEM school subjects. Yet, epistemologies are the driving forces within each discipline and must be considered while teaching STEM as a unified field. To nurture critical thinkers and innovators, it is critical to pay attention to what knowledge is and how it is created and tested. The Educational Framework for Modelling in STEM may be helpful in introducing students and future teachers to the process of modelling, regardless of if they teach it in a single- or a multi-discipline course, such as STEM. This paper will be of interest to teacher educators, teachers, researchers, and policy makers working within and between the STEM fields and interested in promoting STEM education and its epistemological foundations.

Keywords: educational framework for modelling in STEM, educational technology, modelling, STEM teacher education, STEM epistemology

1. Introduction

This research has grown out of the joint desire of two colleagues, long-term research collaborators and teacher educators – a mathematician (DM) and a physicist (MMB) – to examine epistemological, curricular and pedagogical connections and tensions between the subjects comprising science, technology, engineering and mathematics (STEM) education, and their implications for teacher education. During this examination, the concept of *modelling* kept coming up repeatedly both in literature and in our own reflections on teaching. We soon realized that modelling can serve as a pedagogical glue connecting otherwise distinct subjects into a unified STEM concept and supporting the goals of recent STEM education initiatives [1-3]. While modelling initially found place in post-secondary classrooms, currently, it features at all levels of schooling [4-11]. As a process, modelling involves aspects that are pertinent to all four STEM fields (e.g., inquiry, quantitative and design thinking, and use of technological tools, multiple representations, algorithms, and models). Yet, in mathematics education, modelling may be easily confused with problem-based learning and problem-solving, which might limit its utility [12]. In science education, also related to modelling, an inquiry-based learning is often inadequately taught [13]. Both in schools and in teacher education programs, inquiry-based learning may be largely driven by one's curiosity combined with superficial and surface understanding of the content, while lacking the "epistemic framing relevant to the discipline" (p. 941) [13]. In our decades-long work with future teachers of mathematics and science, we have noticed that their beliefs about the nature of STEM fields are often taken for granted, overlooking how teachers' personal epistemological stances affect their teaching [14]. Since modelling is usually associated with solving ill-structured problems, there is a concern that university graduates are not well equipped or motivated to rise to this challenge. Hofer suggests that "education that focuses on the progression of epistemological thinking has the potential for addressing this critical need" (p. 369) [14].

Sharing the stated concerns, in this paper we highlight the distinguishing features of modelling and situate it in STEM teacher education, building on the Kolb's Experiential Learning Cycle [15], Gardiner's Framework for Epistemic Control [16], Windschitl et al.'s [13] model-based inquiry learning, and Carlson et al.'s [17] framework for teaching modelling. With a goal of enabling students to handle complex real-life phenomena, which is the crux of modelling, and making a case that modelling can become a powerful glue holding the STEM teacher education together, we strive to contribute to envisioning authentic and meaningful K-12 STEM education and consequently STEM teacher education [18].

The research questions guiding this theoretical work are:

RQ1: How can modelling help solidify epistemological foundations of STEM learning environments?

RQ2: How is modelling featured in the new mathematics (Ontario) and science (British Columbia) school curricula?

RQ3: How might modelling be implemented in STEM teacher education?

We start by defining the key terms and frameworks, and then present our answers to these research questions.

1.1. In search for authentic STEM learning environments

The term STEM has become ubiquitous in K-12 curricula, policy documents [19], and teacher education. However, the omnipresence of the term does not necessarily mean that its nature, epistemological underpinnings [20], and pedagogical implications [21-23] are clear. Subsequently, it is yet to be determined what STEM teacher education in the 21st century is or could be, and how to educate teachers who will be able and willing to make STEM learning meaningful for their students. With the limited understanding of STEM education and the driving forces behind it, it is unclear what glues separate S.T.E.M. fields into a unified STEM education [24].

Our search for the common language and complementary pedagogical approaches amid the dissatisfaction with the limited attention STEM teacher education has received in the recent decades, was not accidental. Together, we have been educating future mathematics and science teachers for almost half a century. We have been actively involved in the international STEM education movement [25, 26], and have been collaborating with the STEM fields' experts, and instructors of undergraduate and graduate courses. Yet, time and again, we noticed that mathematics and science educators and teacher educators rarely collaborate to create authentic STEM learning environments for their own students, including future teachers. The same applies to policy makers, as the mismatch between mathematics and science curricula in our provinces [27-29] shows the lack of communication between the curriculum writers and the researchers working in STEM fields.

To complicate things even further, the popularity and ubiquity of the STEM acronym has its challenges. There is ample research evidence showing that labeling the 'same old learning environments' as STEM, does not make STEM any more appealing to students who are traditionally underrepresented in these fields [30-32]. Moreover, secondary school teacher education is often rather compartmentalized, focusing on the preparation of mathematics and science teachers but not STEM teachers [27, 29]. Besides, contemporary STEM movement is slowly morphing into STEAM, where A can stand for Arts, Architecture and sometimes even for All subjects [33-35]. But, what does it mean in practice? At the elementary school level, at least in the North American context, teachers are generalists which potentially allows for a more integrated approach, but many teachers lack the solid mathematics and science background while often holding negative attitudes towards STEM subjects [36-40]. Consequently, understanding what brings STEM fields together and what pulls them apart becomes even more important for teacher educators as it prompts them to challenge the often taken-for-granted assumptions in the search for authentic learning environments [35].

1.2. The need for epistemological foundations of STEM teacher education

As each field hidden behind the STEM acronym has its own idiosyncratic epistemology [22], it is reasonable to assume that there might exist some epistemological commonalities and tensions between these disciplines relevant to the K-12 teacher education. In our previous work [41], we have examined epistemologies of STEM disciplines in view of different models for designing STEM teacher education programs. We agreed with Michael Marder [42], who suggested that educators should "approach STEM more as an opportunity than a threat […; to] identify a common core of scientific practices that integrate science, mathematics, engineering, and technology, and make this core a goal for every educated citizen" (p. 150) [42]. Since the STEM construct encompasses "widely different modes of thought and action, all of them important" (p. 149) [42], we recommended that this goal could be reached through collaborations between the stakeholders—STEM disciplines' instructors/teachers, teacher educators, and experts (i.e., those involved in bona fide STEM fields' research and practice). This could be done by co-teaching STEM courses and creating communities of practice. However, for fruitful conversations across the interdisciplinary groups to happen, we needed a framework. To that end, we found Gardiner's [16] metacognitive framework for epistemic control promising in helping educators "to go beyond often fixed adversarial critical thinking approaches and to develop an epistemic position based on inclusive collaboration and emergent creativity" (p. 1) [16]. While Gardiner's framework was primarily developed for collaborations between students, we found it useful for any group working in the domain where epistemological sensitivity is advisable.

Gardiner's framework asks for metacognitive introspection from all collaborators. Generally, metacognition is defined as "the monitoring and control of one's own thought" (p. 361) [43], the purpose of which is to become a more efficient learner and problem solver. However, the metacognition that Gardiner [16] writes about refers to one's thinking as a disciplinary knowledge holder in the interdisciplinary context; it is related to the epistemology of a discipline one is educated in. When learners face a complex problem or work in an interdisciplinary team, they need to move through the states of epistemic (a) awareness, (b) humility, and (c) empathy, to reach (d) epistemic control. By understanding what and how members of their discipline know and what the limitations of that knowledge are, one develops epistemic awareness and humility. Seeing that others' ways of knowing are valuable and worth contributing individually, is epistemic empathy. After reaching epistemic empathy, one becomes open to multiple approaches and perspectives, leading to adaptability in practicing multiple ways of knowing, which is epistemic control.

For STEM teachers, engaging in this process would be invaluable. Regardless of if they consider themselves members of one discipline or are the so-called generalists, learning about and valuing different forms of knowledge would allow them to overcome rigidity of any singular approach to dealing with a phenomenon—a skill they would be able to pass onto their students. While this reflective process may produce some discomfort in those involved, its facilitators need to ensure "the discomfort is intellectual rather than personal and/or social" (p. 7) [16]. Gardiner suggests that "epistemic control can be conceptualized through the phrase 'think like a' [physicist, mathematician, artist…]" (p. 8) [16], an exercise through which the collaborators go collectively.

As teacher educators, we suggest that emphasizing and exercising the interdisciplinary epistemic control would be useful during activities that involve modelling—teachers creating learning situations in which students purposefully implement epistemologies used by the practitioners of science, technology, engineering and mathematics fields.

1.3 What is modelling?

For mathematicians, scientists and engineers, modelling is a methodology, along with design and experimentation [44]. It may be seen as "the most relevant characteristic of the scientific mode of knowledge production" (p. 870) [44]. Education literature prizes modelling as a novel pedagogical approach in teaching STEM subjects.

While it is a common approach for science, so far, K-12 mathematics curricula have paid limited attention to helping students develop modelling skills [7, 12]. Even when modelling is included in the curriculum, teachers often do not have the necessary background to incorporate it into their teaching [45] and rarely see it as relevant to mathematics they are teaching [46]. Yet, modelling could help struggling students to learn how to communicate mathematics, use multiple representations, and connect abstract concepts to real-life [12, 47]. Modelling can support the growth of students' mathematical content knowledge as well as the process skills required to be successful in mathematics [48].

Modelling can be seen as the process of creating models for various purposes, such as deepening understanding of a particular relationship or a mechanism underlying a specific phenomenon, solving a practical problem, or building a theory or an artifact. According to Blum [12], by using the term 'applications and modelling', mathematics education researchers emphasize "both the products and the processes in the interplay between the real world and mathematics" (p. 73) [12]. A science education researcher, Hestenes [49], writes that in both mathematics and science education, "A *model* is a *representation of structure* in a given *system*. A system is a set of related *objects*, which may be real or imaginary, physical or mental, simple or composite. The *structure* of a system is a set of relations among its objects. The system itself is called the *referent* of the model." (p. 17, emphasis in the original) [49]. A model can be represented visually (using graphs, diagrams, sketches), verbally or symbolically.

English and Mousoulides [50] describe engineering-based modelling in elementary schools as a method for solving open-ended real-life problems by going through an iterative process of understanding the context, and developing, testing and revising a working model. In addition, students have to document their work and argue regarding their model's utility (explore its strengths and limitations, and answer the "what-if" questions). For Gil and Gibbs [51], models are conceptual systems that describe the (more) natural world and are central in developing statistical reasoning. By answering the questions "why" and "how," models "explain phenomena observed in data, capturing relations and structures in graphical, textual or mathematical form" (p. 164). After briefly describing the process of modelling, and situating it in the STEM education context, we describe theories that inspire the emerging framework.

2. Developing a conceptual framework: Making connections across the frameworks

To develop our conceptual framework, dubbed Educational Framework for Modelling in STEM (see Table 1), we first turn to Kolb's Experiential Learning Cycle [15, 52]. In order to enhance students' learning, educators support them in developing and utilizing the abilities to engage in, reflect on and abstract from a concrete experience. Through, the so-called active experimentation, the students then implement and test new knowledge in a different situation. Kolb and Kolb [53], specify that "the learning cycle [consisting of experiencing, reflecting, thinking, and acting stages] is a recursive circle or spiral" (p. 8) along which the learning advances with an "increasing depth of understanding and skill" (p. 9) . While the student can enter the learning cycle at any of the four stages, the learning still happens in a linear form (i.e., in an exact sequential order) and must encompass all four stages to be effective.

Abdulwahed and Nagy [54] criticize traditional lab sessions in engineering education, which often turn "into an algorithmic following of the lab manual instead of actively constructing meaningful knowledge out of it" (p. 285). As a countermeasure, they suggest organizing a virtual lab session in the first stage of the Kolb's cycle and a hands-on lab session in the fourth stage. Their post-secondary students better comprehended the material that way; the virtual lab sparked the students' thinking, while the hands-on lab consolidated the active experimentation stage. Abdulwahed and Nagy posit that while doing the laboratory experiments or simulations, students go through different stages of the Kolb's learning cycle, and that a careful interlay of the experimentations, reflections, conversations, investigations, and assessment, lead to better learning outcomes.

Despite its popularity, the Kolb's [15] theory, which is the base of the Experiential Learning Cycle, was also criticized. For example, Seaman [55] calls it ideology, rather than theory. Based on a recent systematic review of the literature on experiential learning, Morris [56] revised Kolb's [15] model, especially focusing on the aspect of concrete experience. This revised model highlights that "learner responsibility was the underpinning theme of the [experiential learning] concept" (p. 1067) [56] and describes the learning stages as consisting of "*contextually rich* concrete experience, *critical* reflective observation, *contextual-specific* abstract conceptualization, and *pragmatic* active experimentation" (p. 1064, emphasis in the original).

With these modifications and clarifications [i.e., 53, 56, 57, 58], we find the revised Kolb's cycle useful for organizing students' STEM learning experiences. By putting emphasis on student responsibility and social aspects of learning, Morris's description [56] fits our ideas of learning in the context of epistemological complexity (e.g., STEM).

The original Kolb's [15] learning cycle described the process of experiential learning in general. As teacher educators, we sought to better understand teacher's role in staging the learning cycle for their students; we needed a framework that would include starting and even intermittent endpoints (i.e., goals), more applicable to in-school STEM learning. We found such a framework in Windschitl, Thompson, and Braaten's [13] five-step Model-Based Inquiry (MBI) in science. The MBI stresses the importance of the initial step in inquiry, such as when a teacher chooses a phenomenon, introduces it to the students, and helps them connect it to their experiences. MBI's goal is to "develop defensible explanations of the way the natural world works" (p. 15) [13], keeping in mind that any new knowledge produced in the process is just a stepping stone in seeking deeper understanding. In this way, the MBI is faithful to the epistemological commonalities of scientific fields.

While the MBI framework, in the words of its creators, "offers a more epistemically congruent representation of how contemporary science is done" (p. 964) [13], for STEM teachers it would be also beneficial to utilize Gardiner's [16] Framework for Epistemic Control. This framework goes beyond the five epistemic features of scientific knowledge (i.e., such that is testable, revisable, explanatory, conjectural, and generative; [13]) and offers extensions applicable to other STEM fields. For example, mathematicians create knowledge that is not revisable, while engineers incorporate art-based design. Contrary to science, these two fields are not necessarily concerned with "how the natural world works," as mathematics deals with purely abstract phenomena and engineering with artificial, human-made phenomena. This motivated us to add Gardiner's [16] framework to our conceptualization of modelling in K-12 STEM teacher education. The students who work with real-life phenomena must recognize the limitations of their own knowledge and ways of knowing, critically reflect on the phenomena, assess their approach, and when needed start all over or revise it. The students should continue following the epistemological learning cycle until they, their peers, and the external evaluator(s) are satisfied with the results [16].

Finally, the fourth framework for teaching modelling in elementary grades came from Carlson, Wickstrom, Burroughs, and Fulton [17]. It consists of three phases—posing questions, building solutions, and validating conclusions. Before the first phase, teacher develops the activity and anticipates potential problems. During the lesson, the teacher organizes student groups, monitors them and regroups them when necessary. After the students complete the modelling activity, the teacher may revisit it to consolidate knowledge or to make relevant curricular connections. The stages from the four frameworks are aligned in Table 1: the experiential learning cycle [56], the modelling cycle [13], the framework for epistemic control [16], and the framework for teaching modelling [17].

Table 1. Aligning the existing frameworks into the educational framework for modelling in STEM

<table>
<tr><th>Stage</th><th>Revised Kolb's Learning Cycle [56]</th><th>MBI learning cycle [13]</th><th>Epistemological foundations [16]: Stages are cumulative</th><th>Teaching modelling [17]</th></tr>
<tr><td>0</td><td></td><td>Setting the parameters: Select a phenomenon that is within students' reach and interest.</td><td>Preparing students for epistemic introspection: Discuss, how is the epistemology reflected in modelling.</td><td>Getting ready: Develop activities, anticipate student difficulties, questions, potential challenges, etc.</td></tr>
<tr><td>1</td><td>Immersing in contextually rich concrete experiences: Students engage through both mind and body, while working in groups in authentic contexts.</td><td rowspan="2">Organizing what students know and what they want to know: Students are given resources; initial questions emerge.</td><td>Gaining epistemic awareness: Understand what and how members of one's discipline come to know and how each member of the group can contribute.</td><td>Enacting: Organize students, guide and scaffold modelling activities, keep them relevant and focused, thus opening opportunities for deep learning to occur.</td></tr>
<tr><td>2</td><td>Conducting critical reflective observations: In an investigator-like manner, students weigh what they know and what knowledge the situation requires.</td><td>Developing epistemic humility: Students recognize the limitations of their knowledge and assess how the present situation challenges and extends what they know.</td><td rowspan="2">Enacting: Monitor students' work; provide adaptive interventions when needed (Blum, 2015) to help students formulate their own questions and seek answers.</td></tr>
<tr><td>3</td><td>Conducting contextual-specific abstract conceptualization: Students propose work-ing hypotheses; under-stand that all knowledge is provisional & needs testing in context.</td><td>Generating testable, revisable, explanatory, conjectural, & generative hypotheses: Students propose patterns, models, theories that might explain the relationships between observed phenomena.</td><td>Acquiring and practising epistemic empathy: Use different perspectives of the group members to interpret and understand the phenomenon. What new insights does this process bring?</td></tr>
<tr><td>4</td><td>Pragmatic active experimentation: Testing if and how abstract conceptualizations agree with new concrete experiences.</td><td>Seeking evidence to test suggested hypotheses: Collect new evidence; use proposed models to generate new data. Constructing an argument: Explain the phenomenon, allow for alternative explanations.</td><td>Exercising epistemic control: "Think like a …" The group critically examines their model and tests it in view of the ill-structured context-based conditions. If it fits, consider the work done or start a new cycle of inquiry.</td><td>Enacting: Teacher monitors students' work, asks questions, and regroups students when required.</td></tr>
<tr><td>1*</td><td>Returning to stage 1 with enhanced understanding of the phenomenon.</td><td>Returning to stage 1 with a set of new questions as a motivation for a new cycle.</td><td>Returning to stage 1: Start a new cycle of inquiry at a deeper epistemological level.</td><td>Reflecting, modifying, revising: Teacher consolidates or revisits activity, with modifications/follow-up.</td></tr>
</table>

Examining and juxtaposing the existing seemingly unrelated frameworks, helped us see how they could complement each other in the context of STEM education. This motivated us to propose a new Educational Framework for Modelling in STEM that will be further clarified in Table 3. This novel, six-stage unified framework will outline how to organize (as a teacher) or engage (as a student) in STEM modelling in epistemologically authentic ways. By combining the four distinct frameworks into one, we encourage teachers to consider how they might address some of the issues mentioned earlier through modelling activities in STEM education. If pre-service and in-service teachers learn about this framework, recognize its origins, and practice applying it, they will be ready to engage their students in meaningful STEM learning through modelling. In the next section, we focus on addressing our research questions.

3. Answering research questions

To answer our research questions, we reflected on a role of modelling in teacher education, how modelling features in the related K-12 curricula in our jurisdictions, and ways in which the Educational Framework for Modelling in STEM could be implemented in educating new teachers.

3.1. How can modelling help solidify epistemological foundations of STEM learning environments?

Modelling is becoming more prominent in both mathematics and science school curricula [4, 27, 29] and consequently in teacher education [45]. There is mounting evidence that the emphasis on real-life connections in modelling leads to remarkable learning gains, especially in underserved student populations and students at risk, or as Lesh, Young, and Fennewald [59] write, "modeling is virtually unparalleled in the successes that it has produced" (p. 283) [59].

To find how modelling can help solidify epistemological foundations of STEM learning environments, we unpacked the relationship between the three pillars supporting the growth of the next generation of STEM educators: (a) formal STEM teacher education; (b) STEM K-12 curricula; and (c) STEM fields' education, as shown in Figure 1.

***Formal STEM Teacher Education*:** In British Columbia and in Ontario, teacher education programs consist of four distinct segments that provide the necessary knowledge and practice to the emerging professionals: (a) General courses, that focus on children development, course management, educational technology, etc.; (b) methods courses, that examine content-specific pedagogical approaches for select subjects; (c) school-based practicum, where future teachers gain some teaching experience while interacting with school students, teachers and communities; and (d) Bachelor's degree courses, that could have been taken before enrolling into a Bachelor of Education (B.Ed.) program or concurrently with it. Of the latter three, Bachelor's degree courses reflect knowledge and epistemology of the related fields; methods courses integrate the knowledge of the content, pedagogy, epistemology, and curriculum of the school subjects; while during the school practicum, future teachers implement general and content-specific pedagogies.

STEM K-12 Curricula: Since the K-12 curriculum is rather comprehensive, it is impossible to "cover" it all due to time constraints. Therefore, there are always parts of it that are not addressed adequately during teacher education. Similarly, the teacher education courses include content that goes beyond the curriculum, such as history of STEM fields, Educational Psychology, use of technology, context of discoveries, and STEM – society connections. That is why we envision the two circles (Curriculum and Teacher Education) only partially overlapping (see Figure 1). K-12 STEM curricula likely focus on the subject-specific big ideas and skills (i.e., content and competencies), core competencies that are common for all subjects (e.g., communication, thinking; personal and social), disciplinary connections to society, history, ethical considerations, etc. Yet, K-12 STEM curricula rarely go in depth with content and epistemology of the fields of study.

STEM Fields' Education: Undergraduate or graduate education in one of the STEM fields (e.g., mathematics, physics, chemistry, engineering) constitutes STEM Fields' Education. Both Teacher Education and K-12 Curriculum intersect with the corresponding STEM Fields' Education (given that only some of the STEM fields are school subjects). Curriculum only tangentially addresses the epistemological underpinnings of the related fields, such as the nature of knowledge, truth, and evidence: How do we know what we know? How do we explain the world? What explanations are accepted and why? How is new knowledge produced and validated? How is progress made in a specific field? What are the current limits of knowledge?

The intersection of the three pillars: In our vision of the relations between these three pillars, there is a region that belongs to all three, where the STEM K-12 Curricula, STEM Teacher Education, and STEM Fields' Education overlap. We believe that this overlap holds significance for teacher education. Importantly, since it incorporates content, pedagogy and epistemology of the STEM fields, modelling is situated in the overlap of these three pillars (Figure 1). Modelling clearly belongs to all STEM fields, and is increasingly featured in the school curricula, which makes a case for modelling to become more prominent in teacher education.

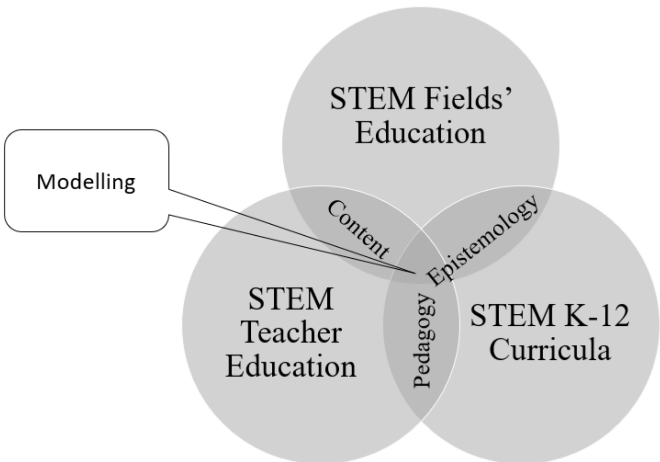

Figure 1. Connections between the STEM Fields' Education, K-12 Curricula, and Teacher Education

3.2. How is modelling featured in the new mathematics and science K-12 curricula?

Next, we describe approaches to modelling found in the new curricula from the Canadian provinces of Ontario and British Columbia, and analyze them in view of our proposed Educational Framework for Modelling in STEM (see Table 1).

3.2.1. Modelling in the new mathematics 1-8 curriculum in Ontario

In the new mathematics 1-8 curriculum [29], modelling is described as a process in which students work on rich, real-life, problems. Situated in Algebra strand, modelling can be applied to various contexts, allowing students to bring in learning from other strands (i.e., Social-Emotional Learning; Number; Data; Spatial Sense; and Financial Literacy). The process involves "four key components that are interconnected and applied in an iterative way, where students may move between and across, as well as return to, each of the four components as they change conditions to observe new outcomes until the model is ready to be shared and acted upon" (p. 86). These four components are:

1. Understanding the problem
2. Analyzing the situation
3. Creating a mathematical model, and
4. Analyzing and assessing the model.

In the diagram (Figure 2), components 1, 2, and 4 belong to the "real-life situation" circle, while 3 is separate and it is unclear where it belongs. There is also a text "Share and Act upon Model(s)," written above component 4. This raises the following questions: If this is an additional component, why is it not labeled as such? When should the students share and act upon models? Is saving space the only reason for not using 'mathematical' in conjunction with the word 'model'?

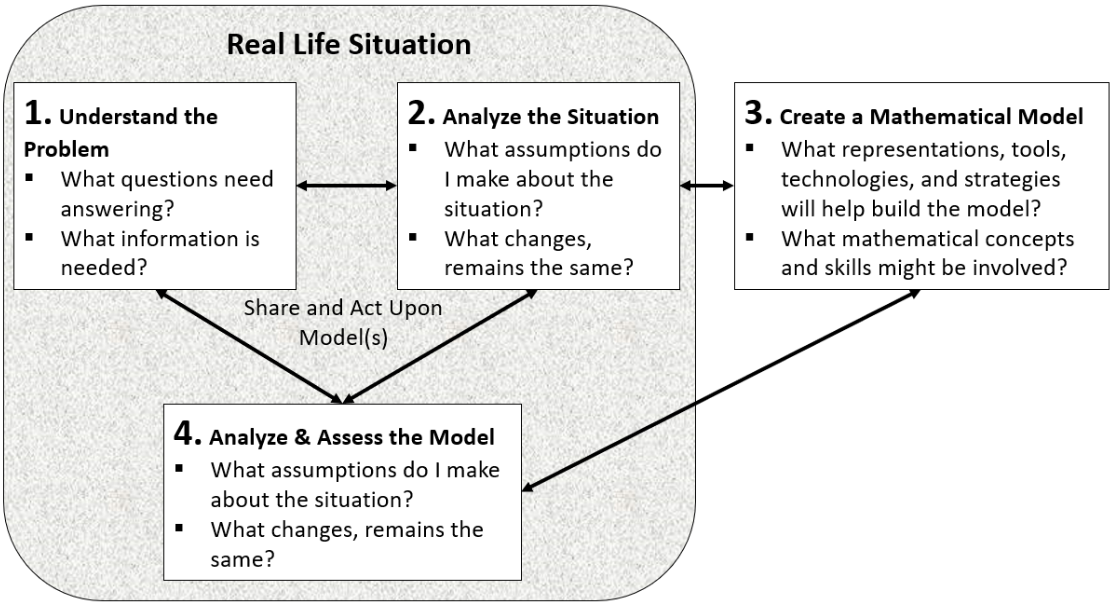

Figure 2. Visualization of the process of mathematical modelling in the new Ontario 1-8 mathematics curriculum [29]

Although all the components in Figure 2 are connected with bi-directional arrows, the numeration suggests that the modelling activity should be organized in a specific order and that it should preferably contain all the components. However, our analysis points that the process of modelling could be presented to teachers more clearly. By noticing the three cycles, teachers could realize the similarities and differences to some pedagogies already familiar to them:

(a) 1-2-3-4 cycle: This is a full process that consists of four steps and takes the learner(s) from the real-life situation (1, 2) to a mathematical abstraction/representation (3) and back to real-life situation (4). If the students (or evaluators) are not satisfied with the model, they may be directed back to (1) or (2) or (3). The process continues in a spiral-like fashion.

(b) 2-3-4 cycle: This process is a simplified version and more akin to problem-solving. The student seemingly has the content knowledge required to complete the task. However, if the analysis (4) shows that students need to re-assess the situation or re-do the model, they could be reverted to (2) or (3).

(c) 1-2-4 cycle: This process is completely conducted in a real-life environment and may be organized in situations when students are given models to choose from and assess.

The sections in the diagram (see Figure 2) illustrate that there are multiple approaches to modelling that teachers should consider appropriate for different learning situations. The bidirectional arrows in the diagram emphasize that despite the numbering of the four steps, skipping a step or reverting to a previous one is possible. This opens opportunities for teachers to design flexible activities thus addressing different learning goals. At the same time, the diagram focusses on what the students should be doing during the modelling activity, but it completely ignores the teacher. As a result, teachers may not see it as relevant or misinterpret it [45, 46].

The curriculum is primarily guide for teachers, and should include reasons for engaging students in modelling, teacher's actions before, during, and after a modelling activity, as well as the desired educational outcomes. Epistemological stance is a big part of STEM modelling, important to consider even when modelling in a single subject, such as mathematics, biology, or General Science. In preparation for modelling (see Table 1) and adjusted to grade level, teachers should discuss with the students: How is the epistemology reflected in modelling? What does it mean to model? How to evaluate the fitness of a model? How to use the model to make predictions? What evidence may support or refute the model? This step should be followed by showing the students examples from different fields, highlighting the similarities and differences between the models, and how they were created, tested, and applied.

3.2.2. Modelling in the new science and mathematics K-12 curricula in British Columbia

The new British Columbia curriculum claims to be concept-based and competency-driven [60]. It is based on a premise that:

> **…** essential for 21st-century learning [are]: a concept-based approach to learning and a focus on the development of competencies, to foster deeper, more transferable learning. These approaches complement each other because of their common focus on active engagement of students…, learning… through "doing" rather than through passive listening or reading… [and engaging] in authentic tasks that connect learning to the real world. (Curriculum model section, para.10)

Yet, the new British Columbia Curriculum is doing a lip service to modelling by missing many opportunities for achieving curricular goals. In the new mathematics and science curricula, the word "modelling" is mentioned only a few times (e.g., in Foundations of Mathematics 12, where it discusses the modelling from data and understanding of the variety of functions, and in Computer Science 11, where it discusses modelling with mathematics). Table 2 shows how the Computer Science Curriculum elaborates the modelling process:

Table 2. Elaboration of modelling in the new British Columbia Computer Science Curriculum [27, https://curriculum.gov.bc.ca/curriculum/search Keyword "model"] (emphasis and capitalizations in the original)

Model with mathematics in **situational contexts**	Computer Science 11	Reasoning and modelling	**Keyword**: Model	**Elaboration:** Use Mathematical Concepts And Tools To Solve Problems And Make Decisions (E.G., In Real-Life And/Or Abstract Scenarios) Take A Complex, Essentially Non-Mathematical Scenario And Figure Out What Mathematical Concepts And Tools Are Needed To Make Sense Of It
			Keyword: Situational Contexts	**Elaboration:** Including Real-Life Scenarios And Open-Ended Challenges That Connect Mathematics With Everyday Life
Ways to model **mathematical problems**	Computer Science 11	No CCG	**Keyword:** Mathematical Problems	**Elaboration:** Estimate Theoretical Probability Through Simulation represent Finite Sequences And Series solve A System Of Linear Equations, Exponential Growth/Decay solve a Polynomial Equation calculate Statistical Values Such As Frequency, Central Tendencies, Standard Deviation Of Large Data Set compute Greatest Common Factor/Least Common Multiples

Curricular elaborations in this document are too vague for defining measurable educational outcomes. This curriculum confuses modelling and problem solving [61], models and modelling, and provides little guidance on how the students [or teachers] may make sense of mathematical concepts and mathematics models. In many other occurrences in the new British Columbia curriculum, the focus is not on the *process* of modelling but on its outcomes and on the evaluation of the already existing models. Moreover, the new curriculum uses the same terminology across subjects as if modelling is invariable across the fields. Finally, mixing modelling and problem solving is problematic. While the two are related, they are distinct and should not be confused. We are convinced that while modelling has a number of general features, it is not universal. As the ways of modelling are grounded in the field's epistemology, every STEM field has its own way to model phenomena, to verify and test models, as well as to present the outcomes of the modelling process. The new British Columbia curriculum presents modelling in such general terms for it to fit all fields. Being so imprecise, makes modelling meaningless for both teachers and students, becoming inefficient and counterproductive for building epistemological bridges. Thus, the challenge that the new British Columbia curriculum failed to address is having a fine balance between being narrow and prescriptive, as opposed to being broad and open. By mixing the results of modelling (i.e., models) and modelling as a process, at the same time "sprinkling" the term everywhere and failing to define it, modelling loses its potential as pedagogy both in K-12 classrooms and in STEM teacher education [49, 50].

4. How might modelling be implemented in STEM teacher education?

With a vague curriculum, the onus of equipping teachers with adequate knowledge and skills starts with teacher education programs. In this section we discuss the implementation of modelling in STEM teacher education. Building on the connections elaborated in Table 1, we suggest the following pedagogical approaches for STEM teacher education:

- Familiarize pre-service STEM teachers with epistemological foundations of their core disciplines. In addition, they should learn about and practice using the Gardiner's [16] Framework for Epistemic Control. In that way, each stage of the Kolb's learning cycle [15] becomes an opportunity for "epistemic discourse and dialogue" with their peers.
- Emphasize reasons for incorporating modelling at all grade levels. Clarify the difference between modelling and problem solving, as well as between the traditional cycle of inquiry and MBI as defined by Windschitl et al. [13].
- Clarify the role technology has in all stages of modelling, as well as in learning and teaching all STEM disciplines. The MBI framework [13] highlights the importance of students' bodily sensations during the learning. Immersing one in the context seems crucial for having a contextually rich concrete experience [56]. However, there is plenty of evidence that technology (e.g., virtual reality, simulations) can provide such an environment [54]. STEM teachers should consider laboratory activities, which are prevalent in science, technology, and engineering education, as a fertile ground for students to go through the cycle of learning.

In Figure 3, we expand the STEM Teacher Education circle from Figure 1. We emphasize that STEM content, pedagogies and epistemologies are the foundation of STEM teacher education. At the same time, school practicum and the STEM K-12 curricula allow pre-service teachers to apply these theoretical foundations into practice. At the core of both knowledge and practice is modelling, as it is built upon the knowledge and epistemology of STEM disciplines; as a pedagogy, it belongs to both curriculum and school practice.

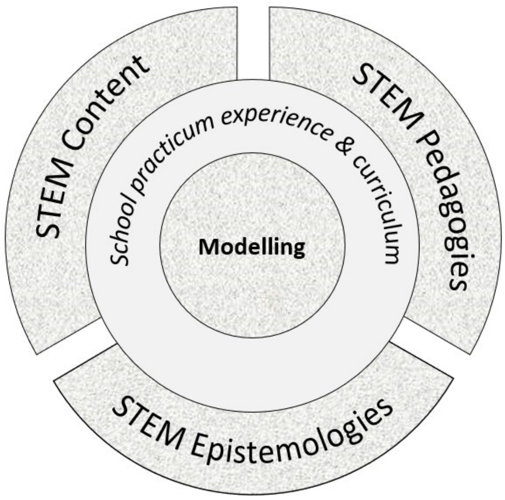

Figure 3. Modelling is at a centre of STEM teacher education

The diagram presents pre-service teachers' knowledge of curriculum, pedagogy, and content and epistemology as often disconnected. For STEM teachers, these different knowledge systems merge and support each other during teaching practice, and particularly during modelling.

5. Summary and Conclusions

This paper aims at addressing three research questions:

RQ1: How can modelling help solidify epistemological foundations of STEM learning environments?
RQ2: How is modelling featured in the new mathematics (Ontario) and science (British Columbia) school curricula?
RQ3: How might modelling be implemented in STEM teacher education?

Since modelling belongs to all four STEM disciplines, we made a case that it can help solidify epistemological foundations of STEM learning environments. As an example of implementation, we briefly described how is modelling featured in the mathematics and science curricula in Ontario and British Columbia. Our analyses present these curriculum changes as promising but vague, requiring a lot of explanation and professional learning.

Despite good intentions, if modelling is emphasized in a curriculum [e.g., 27, 29], but not well explained and delineated from its other meanings used in everyday language, misinterpretations may arise. For example, as a noun, model is a representation or example; as an adjective, model means exemplary; as a noun, model(ing) may mean to plan according to a model, to work as a model, or to create a model. While it is encouraging that modelling is finding its way into the curriculum, it remains more researched than implemented in schools.

For teaching STEM, curricular implementation of modelling could become a litmus test for identifying the epistemological tensions and gaps behind the intended STEM curricula. We recall how for Driver, Newton, and Osborne [62] knowing science means,

> that one knows not only *what* a phenomenon is, but also *how* it relates to other events, *why* it is important, and *how* this …view of the world came to be. Knowing any of these aspects in isolation misses the point. (Emphases in the original, p. 297)

We believe that such a statement could be said about any field of study. This presents one of the biggest challenges for education systems that have generalist teachers beyond the primary school, because it is virtually impossible to have teachers who know (in Driver et al.'s way) all the fields that ground the subjects they teach. Similar challenge is in assuming that any teacher who specializes teaching one of the STEM fields is suitable to teaching STEM. There is a need to organize teacher education programs for STEM teachers.

In answer to our third question regarding ways in which modelling might be implemented in STEM teacher education, we created an Educational Framework for Modelling in STEM. It combines four frameworks that we find important for teaching STEM: the experiential side of learning that allows for conceptualization of real-life phenomena [15, 56], epistemological pliability and fidelity when dealing with multi-disciplinary problems [16], and the process of modelling in science [13] and how to teach it [17]. All four frameworks are cyclical in nature and Gardiner's is also cumulative (i.e., each new level subsumes the previous ones). In Table 1, we aligned student and teacher roles in modelling with the stages in which experiencing a phenomenon and understanding the origins of knowledge happens.

In Table 3, we present a refined version of our framework, suitable for sharing during the pre-service classes and in-service professional learning activities. The sidebar illustrates how the teacher's control diminishes though the modelling process. This is consistent with Blum [12], Morris [56], Kolb [15], and Windschitl, Thompson, and Braaten [13] who emphasize that in and through modelling student responsibility for learning increases, while teacher intervenes more in the beginning and then to make sure that all stages are covered and meaningful.

Table 3. The refined version of the Educational Framework for Modelling in STEM. The Stages I-VI correspond to Stages 0-1* in Table 1

Stage	Teacher's and students' roles during modelling*	Release of control
I	**Teacher** prepares students**:** Discusses how the epistemology is reflected in modelling. **Teacher** prepares a lesson: Selects a phenomenon, develops activities, anticipates student difficulties, questions, potential challenges, etc.	Teacher releases control of student learning as students advance from Stage I to VI ↓
II	**Students** get immersed in contextually rich concrete experiences; discuss how each member of the group can contribute. **Teacher** provides students with resources; records initial questions; organizes what students know and what they want to know; organize them into groups; discusses what and how members of one's discipline come to know; guides and scaffolds modelling activities.	
III	**Students** contribute what they know and what knowledge the situation requires; assess how the present situation challenges and extends what they know. **Teacher** organizes the activity, provides resources, organizes students' initial questions, discusses limitations of each individual knowledge, monitors their work.	
IV	**Students** propose working hypotheses; propose patterns, models, theories, etc. that might explain the relationships between observed phenomena. They discuss and use different perspectives of the group members to interpret and understand the phenomenon. **Teacher** monitors students' work.	
V	**Students** test if and how the results of Stage IV agree with new concrete experiences; collect new evidence; use proposed models to generate new data, explain the phenomenon, allow for alternative explanations; test the models. Decide if the work is done and could be reported or start a new cycle of inquiry. **Teacher** monitors students' work, asks questions, and regroups students when required.	Students gain control of their learning as they advance from Stage I to VI
VI	**Students** return to Stage II with enhanced understanding of the phenomenon, with a set of new questions as a motivation for a new cycle. **Teacher** consolidates or revisits the modelling activity, suggests modifications for the follow-up.	

The process of teacher's release of control during a modelling activity is emphasized by the arrow. As teachers relinquish control of the learning process, the students assume it by taking more responsibility for decision making and consequently for their learning. We agree with Blum [12] that, "For quality teaching of applications and modelling …teacher education is crucial" (pp. 88-89) [12]; teaching experience and degree in the field are not enough to provide teachers with necessary teaching competences. Examining STEM fields' epistemologies is often considered to be too theoretical or advanced to be included in teacher education. Moreover, STEM teacher educators rarely have an authentic experience of practicing the field they teach. Yet, the nature of knowledge and of knowledge acquisition have profound effects on the progress in any field. Therefore, parallel to what we suggest for schoolteachers, STEM teacher educators would benefit from collaborations with STEM practitioners and a wider education community.

In conclusion, we would like to mention some challenges in implementing STEM modelling in teacher education programs that have to be addressed to implement this pedagogy successfully with future teachers. These challenges include:

- Leaving the comfort of subject silos and considering how different STEM subjects are interrelated
- Collaborating with colleagues from other STEM fields
- Creating authentic STEM curriculum, while incorporating various aspects from the curricula of separate STEM fields
- Being aware of different epistemologies of various STEM fields and being able to incorporate them into teaching
- Promoting divergent thinking.

We hope that our analysis and the framework will help teacher educators as well as curriculum developers to introduce modelling activities in their courses.

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