Research article

Style investing and the ICAPM

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Abstract: Empirical multifactor models of excess returns are theoretically grounded in Merton's ICAPM. Merton modeled investors who maximize expected utility of their own consumption. Multiple priced factors arise as "hedge portfolios" most closely correlated with state variables that drive intertemporal changes in the investment opportunity set. It is puzzling that empirically useful factors have not been convincingly identified as those portfolios, despite massive effort. But the majority of asset demand now arises from *style investors*, i.e. institutionally managed funds that try to either meet (index funds) or beat (actively managed) returns from style-specific benchmark portfolios. So we modify Merton's derivation to incorporate the aggregate demands derived from style investors' different objective functions. In addition to resolving the aforementioned puzzle, we show that this style investing version of the ICAPM is more consistent with recent empirical evidence documenting comovement among assets held in a style benchmark. Finally, the model casts doubt on the widespread belief that individual investors will necessarily benefit from funds with positive multifactor alpha.

Keywords: ICAPM; style investing; factor models; continuous time
JEL codes: E21, G11, G12

1. Introduction

Robert Merton's (1990) Intertemporal Capital Asset Pricing Model (ICAPM) predicts that assets' expected returns will be a linear function of the instantaneous expected excess returns (i.e. a "beta" relationship) of the tangency portfolio, as in the static CAPM, as well as a number of additional "hedge" portfolios. Each additional portfolio is the one most highly correlated with a time-varying (diffusion-generated) state variable that affects investors' indirect utilities (i.e. their Bellman value functions), derived using a time-additive, expected utility of own-consumption objective function. Merton defined the state variables to be the subset of coefficients of the assets' return processes (i.e. their instantaneous drifts and volatilities) that are time-varying, thus causing intertemporal changes in the investment opportunity set. Hence those attempting to theoretically rationalize empirical

multifactor regression models of asset returns have tried to establish that the factor portfolios optimally (i.e.using the aforementioned objective function) react to (a.k.a.*hedge*) Merton's hypothesized intertemporal changes in the investment opportunity set.

There are two reasons to reconsider the assumption that the only state variables affecting agents' value functions are in the subset of time-varying coefficients of the assets' return processes. First, there is little *direct behavioral* evidence that *typical* individual investors' asset demands include the predicted additional portfolios that optimally hedge Merton's posited intertemporal changes in the investment opportunities, either through their direct portfolio choices or through purchases of mutual funds serving those purposes. Inherently weaker, largely correlational evidence that attempts to tie candidate state variables to the empirically useful factor portfolios has not been conclusive. Fama and French (1992) admitted their own inability to rationalize the two non-market factor portfolios (size (SMB) and book-to-market (HML)) in their own empirical multifactor model:

> "Finally, there is an important hole in our work. Our tests to date do not cleanly identify the two consumption-investment state variables of special hedging concern to investors that would provide a neat interpretation of the results in terms of Merton's (1973) ICAPM or Ross' (1976) APT."

This admission did not deter numerous others from trying and subsequently claiming to succeed. Much attention has focused on the possibility that "distress risk" is a state variable that might account for one of the Fama-French non-market factors. But in a recent survey of the contradictory opinions of earlier papers, and their own extension of the evidence, De Groot and Huij (2011) concluded that:

> "Finally, our results indicate that the empirical explanatory power of the Fama-French (1993) SMB and HML factors cannot be attributed to these factors being exposed to default risk."

And as noted by Daniel and Titman (1997):

> "There are now more than a dozen factor models that explain the returns on portfolios sorted on size and book-to-market. It can't really be the case that all these factor models are "correct", in the sense that they all explain the cross-section of stock returns...our explanation of this is that these tests are designed in such a way that the tests lack statistical power. Specifically, we argue that, under very weak conditions, *any* factor will appear to explain the average returns of size and book-to-market sorted portfolios. That is, these tests will make it appear that returns are consistent with the factor model even when they are not."

Other observers (e.g. see Cochrane, 2001) have lamented the failure of many econometric studies to incorporate additional theoretical restrictions implied by Merton's model, essentially "fishing" for state variables based on their in-sample premia, without prior restriction on both the number and names of the state variables. This approach runs afoul of Fama's (1998) insight that:

> "When we know the number of state variables, but not their names, confident conclusions about even the number of them that produce special risk premiums are probably impossible, unless the number is zero, so the ICAPM reduces to the CAPM."

Despite these sentiments, conventional empirical work continues to promote additional variables as candidates for "priced factors" in the linear multifactor model. In what should become a

paradigm-shifting paper, Harvey et al. (2016) surveyed that vast literature to catalog three hundred and sixteen (yes, that's 316) factors proposed and advocated within the union of 313 research articles, yet still believe that this "likely underrepresents the factor population". Promoters of these alternatives often reported that use of their favored candidates in factor models explained the cross-section of expected returns at least as well as the aforementioned Fama-French three-factor or Carhart (1997) four-factor choices. For example, Wagner and Winter (2013) use the STOXX Europe 600 to construct factors similar to the four chosen by Carhart, and add another two factors that respectively proxy for idiosyncratic risk and liquidity. They conclude that "No single risk factor is dominated and hence our six factor model may serve as a valid performance benchmark." Novy-Marx (2013) found that " gross profits-to-assets, has roughly the same power as book-to-market predicting the cross section of average returns." Gregory et al. (2013) report "We find that versions of the four-factor model using decomposed and value-weighted factor components are able to explain the cross-section of returns in large firms or in portfolios without extreme momentum exposures."

The claims for "explaining the cross-section of returns" are typically based on classical Fisherian hypothesis testing of model specifications and their implications. But Harvey et al. (2016) provide compelling evidence that when applied to evaluating factor models, such Fisherian hypothesis testing has sometimes but not always produced evidence for priced factors that has its orthodox textbook interpretation. They attribute this to data mining, publication bias, and other real-world imperfections that aren't properly accounted in standard textbook applications of the Fisherian framework.*

While neither we nor Harvey et al. (2016) can use this to criticize any *particular* paper among the hundreds in this vein, perhaps it is time for some fresh thinking about Merton's ICAPM model and the theoretical basis for what *should* be present in the linear multifactor model implied by it.

Second, index or exchange-traded funds and actively managed funds (mutual funds, pensions, endowments, trusts, private client, etc.) delegated to professionals, i.e. *style investing*, has rapidly grown to account for a very significant share of total investment. Hence the asset demands of institutional style investors cannot be ignored in any credible equilibrium asset pricing model. Institutionally invested funds larger than $100 million comprised only 26.8% of the US equity market in March 1980, but grew rapidly to account for 51.5% of the market as early as the end of 1996 (see Metrick and Gompers, 2001). Many possible *reasons* for this growth have been suggested, including the advent and growing popularity of tax-favored retirement accounts (for a discussion, see Rydqvist et al., 2014). Nagel (2005) showed that significant trading by mutual funds is done to maintain consistency with their respective pledged styles, and that "style investing is an important source of trading volume."

Few think that a style investor's objective function, determining its asset demands, is identical to (Merton's assumed) investors' intertemporal utility functions of their own respective consumption. For example, as noted by Roll (1992),

> "Today's professional money manager is often judged by total return performance relative to a prespecified benchmark..."

Roll accordingly modified the static mean-variance model underlying the static CAPM, incorporating the manager's "prespecified benchmark" as a state variable of the manager's objective function.

Merton's demand-supply equilibrium *requires* that the asset demands of managers using this (or

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*If President Trump were a financial econometrician, he might very well refer to the published "fake factor news".

any other) benchmark-dependent objective function *must* be added to the demands of individuals using Merton's hypothesized time-additive expected utility of own-consumption. We will see that doing so has (at least) two important testable implications: (i) investors with an objective function embodying the goal of beating a benchmark portfolio will invest a fraction of their wealth in it, and (ii) the most popular benchmark portfolios will be factors in the ICAPM multifactor expected return equilibrium relationship. We provides a summary of existing empirical evidence, and argue that it is neatly consistent with both of these implications – consistency that cannot be achieved by the *sole* adoption of Merton's state variables.

Finally, this more realistic version of the ICAPM calls into question the widespread normative interpretation of a positive multifactor model "α" as an indicator a fund's potential benefit to an individual investor. Subsequent to the work of Fama and French (1992) and its extension by Carhart (1997), a three or four-factor estimated "α" has been used in most modern studies to measure "abnormal" returns (for example, see Wermers, 2000). When the factors arise because they are correlated with the returns on benchmark portfolios that style investors are trying to either meet (as index or exchange-traded funds do) or beat (actively managed funds do), there is no basis for interpreting a positive multifactor α as a measure of abnormal performance that is relevant for individual investors. Hence one cannot validly argue that individual investors will benefit by investing in a strategy with a positive multifactor α, even if there if there is no error in estimating it.

2. Methods: Merton's ICAPM

Our development of the style investing extension of Merton's ICAPM is substantially facilitated by reviewing Merton (1990), who posited that the *i*th individual asset's price is generated by an Itô process:

$$\frac{dP_i}{P_i} = \mu_i dt + \sigma_i dz_i \tag{1}$$

where μ_i denotes the instantaneous *drift* and σ_i denotes the instantaneous *volatility* of the instantaneous rate of return of asset i. Merton used ρ_{ij} to denote the instantaneous correlation coefficient between the Wiener processes dz_i and dz_j. Merton (1990) assumed that Itô processes governed the interest rate r_f and possibly one or more of the rest of these parameters, i.e. they are allowed to be random coefficients. Individual investor k treats them as state variables when solving the following joint consumption/portfolio problem[†]

$$\max_{c^k, w^k} E_0 \left[\int U^k(c^k(t), t) dt \right] \; s.t. \; dW^k = \sum_i [w_i^k(\mu_i - r_f) + r] W^k dt + \sum_i w_i^k \sigma_i dz_i - c^k dt \tag{2}$$

Following Breeden (1989), the presence of *any* time-varying state variables affecting an agent's choice problem, *whether or not these state variables are the greek letters defined above*, can be modelled as a column vector $s \equiv s_1, \ldots, s_S$ entering the stochastic dynamic programming decomposition for the problem:

$$\max_{c^k, w^k} \{U^k(c^k(t), t) + E_t[J^k(W^k, s, t)]\} \tag{3}$$

[†]Merton admitted both finite and infinite horizons for this investment problem. In the former case, he added a bequest term B to the objective function to prevent the unrealistic exhaustion of wealth at a finite time T, while in the latter case he multiplied the concave utility function by an exponentially decaying discount factor to help ensure convergence of the integral. He also admitted the possibility of wage income, denoted y. But none of these features change the form of the derived ICAPM relationship.

where J^k denotes the *Bellman value* function of the problem. Next, Itô's Lemma is used to calculate dJ^k:

$$dJ^k = J_t dt + J_W dW + J_s' ds + \frac{1}{2}\left(J_{WW}(dW)^2 + 2J_{Ws}' dWds + ds' J_{ss} ds\right) \qquad (4)$$

where the notation for individual k is suppressed on the right hand side. J_{Ws} denotes the vector of cross partial derivatives of J_W with respect to the state variable vector s, and J_{ss} denotes the Hessian matrix of J with respect to s. Now substitute dW from the budget constraint in (2), ds from the diffusion processes for the hypothesized state variables, and simplify using the stochastic calculus multiplication rules $dtdt = 0$, $dz_i dt = 0$, $dz_i dz_i = dt$, $dz_i dz_j = \rho_{ij} dt$. Then set the term in dJ^k that multiplies dt equal to zero, which must be satisfied when the expression is evaluated along the time paths for the vector of risky asset weights w^k and consumption c^k that maximize the objective function in (2) (i.e. the Hamiliton-Jacobi-Bellman (HJB) equation). The first order condition for w^k, multiplied by wealth W^k, yields Breeden's expression (op.cit., eqn. 10) for the individual's instantaneous expenditures on the assets (i.e. asset demand) at time t :

$$w^k W^k = T^k [V_{aa}^{-1}(\mu - r_f 1)] + [V_{aa}^{-1} V_{as}] H_s^k. \qquad (5)$$

To interpret (5), use the partial derivatives in (4) to calculate the scalar $T^k = -\frac{J_W^k}{J_{WW}^k}$. V_{aa} denotes the (A x A) covariance matrix of the instantaneous asset returns, with typical element $\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}$; $\mu - r_f 1$ denotes the (A x 1) vector of instantaneous expected values of the assets' returns in excess of the riskfree rate r_f; V_{as} denotes the (A x S) cross covariance matrix of the instantaneous asset returns with the state variables, with typical element $\sigma_i \sigma_{s_j} \rho_{is_j}$; and the (S x 1) vector $H_s^k = -\frac{J_{Ws}^k}{J_{WW}^k}$.

The first bracketed vector on the right hand side of (5) has the same form as the weight vector for the familiar risky asset tangency portfolio of static mean-variance (a.k.a. *modern*) portfolio theory. But under Merton's assumption that at least some of the μ_i, σ_i, ρ_{ij}, and/or r_f are state variables governed by diffusion processes, *those weights will no longer be static*. They will vary continuously, in accord with the continuous changes in the (product of the) instantaneous covariance matrix and the instantaneous expected excess return vector.

Because there *are* state variables, the second term in the right hand side of (5) will be nonzero. The j th column of its bracketed expression is, in Breeden's (1989) words, the weight vector of

> "the portfolio of assets that is most highly correlated in return with movements in state variable j..." "Thus, those S portfolios are the best hedge portfolios available for individuals to use in hedging opportunity set changes...H_s^k gives individual k's holdings of those hedge portfolios (which may be positive or negative)."

The presence of these hedge portfolios (i.e. the second term in (5)) is the sole difference between the continuous time, dynamic portfolio choice rules and the static mean-variance portfolio choice rules popularized in college finance textbooks.

The equilibrium market portfolio's vector of asset expenditures, denoted $w^M M$, is found by aggregating the demand (5) from Merton's individual investors:

$$\sum_{k=1}^{K} w^k W^k \quad =$$

$$\left(\sum_k T^k\right)[V_{aa}^{-1}(\mu - r_f 1)] + [V_{aa}^{-1}V_{as}]\left(\sum_k H_s^k\right) \equiv$$

$$T^M[V_{aa}^{-1}(\mu - r_f 1)] + [V_{aa}^{-1}V_{as}]H_s^M = w^M M \qquad (6)$$

so that the market portfolio weight vector is

$$w^M = \frac{T^M}{M}[V_{aa}^{-1}(\mu - r_f 1)] + [V_{aa}^{-1}V_{as}]\frac{H_s^M}{M} \qquad (7)$$

Premultiply both sides of (7) by the matrix $V_{aa}\frac{M}{T^M}$, and solve for the instantaneous expected asset excess return vector

$$\mu - r_f 1 = V_{aM}\frac{M}{T^M} - V_{as}\frac{H_s^M}{T^M}. \qquad (8)$$

Equation (8) is an expected return vs. risk relationship, expressed as a weighted sum of the assets' instantaneous covariances with the market portfolio and the state variables. *This expression also makes clear that investor k's impact on the expected return vs. risk tradeoff depends on the size of the investor's T^k in the total T^M and H_s^k in the total H_s^M.*

This fact is obscured by the more familiar time-varying multiple β-form of this relationship, i.e. Merton's ICAPM. As summarized in Breeden (1993), the ICAPM is found by premultiplying both sides of (8) by the transpose of the market portfolio weight vector w^M to obtain an expression for the instantaneous expected excess return of the market portfolio. Similarly, one successively premultiplies both sides of (8) by the transpose of each column j of the second bracketed term in (5), i.e. the hedge portfolio weight vector denoted $w_{s_j^*}$ for the hedge portfolio s_j^*, producing expressions for their instantaneous expected excess returns. All these instantaneous expected excess returns will be expressed as a linear combination of the two unobserved quantities $\frac{M}{T^M}$ and $\frac{H_s^M}{T^M}$, which can be found by inversion of the linear equations and substituted back into (8) to obtain the ICAPM β-relationship for the individual assets:

$$\mu_i - r_f = \beta_{iM}(\mu_M - r_f) + \sum_{j=1}^{S}\beta_{is_j^*}(\mu_{s_j^*} - r_f) \qquad (9)$$

where the (A x S+1) matrix of betas is computed by multiplying the (A x S+1) matrix of instantaneous covariances of the assets' returns with both the market and the S hedge portfolios, by the (S+1 x S+1) inverse of the matrix of instantaneous covariances of the returns from the market portfolio and the S hedge portfolios (see Breeden, 1989).

In summary, the presence of any time-varying (governed by diffusions) state variables that enter some agents' choice problems result in demands for additional, non-tangency portfolios that are the most highly correlated with the state variables. Merton's (1990) choice of state variables were S of the μ_i, σ_i, ρ_{ij}, and r_f described by (1) and the paragraph following it, which jointly determine the instantaneous investment opportunities. Hence the S additionally demanded, non-tangency portfolios are those most highly correlated with intertemporal changes in the instantaneous investment opportunities, and each will be another priced factor in the linear multifactor expression (9) for an asset's expected returns.

3. Results: adding style investors to Merton's model

The above review of Merton's ICAPM clears or path for adding style investors to it. As noted in section 1 above, knowledgable analysts agree that style investing is widespread and that style investors' asset demands are influenced by signicantly different objectives than Merton's individual investors were assumed to have. Few knowledgable analysts think that a fund manager or advisor's optimization is identical to Merton's (2) when managing *other* people's money. For example, Roll's (1992) tracking error variance (TEV) model assumed (de-facto, when returns are normally distributed) that style investors maximize the following single-period expected utility:

$$E[-e^{-\theta(W-W_b)}] \qquad (10)$$

where W_b denotes the wealth that would have been earned had the funds been invested in the investor's benchmark portfolio, rather than the investor's own portfolio worth W. When returns are normally distributed in the single period model, problem (10) is equivalent to maximizing $E[W-W_b]-\frac{\theta}{2}Var[W-W_b]$. Written in terms of the rates of return for the manager's portfolio and the benchmark portfolio, a manager with a fixed positive value of θ will choose a portfolio with the minimum value of the *tracking error variance* among those having the same θ-dependent expected return in excess of the benchmark's expected return. Not surprisingly, this managerial objective function was also assumed in a repeated, single-period model by Becker et al. (1999) in order to test mutual funds' market timing abilities, and in the static asset pricing model of Brennan (1993). Additional implications for portfolio choice are developed in Jorion (2003), and generalized in Wagner (2002). An index or exchange-traded fund acts as-if it has a very large value of θ in (10), thus emphasizing the importance of minimizing its tracking error variance with respect to its benchmark index, i.e. it tries to meet the index more than it tries to beat the index.

This is not the only managerial objective function that captures the professional investment insights that motivated Roll. In the intertemporal context, it is perhaps more plausible to posit the following intertemporal style investing objective:

$$-\lim_{t\to\infty} \frac{1}{t} \log E_t \left[e^{-\theta(\log W_t - \log W_{bt})} \right] \qquad (11)$$

Collecting the log terms in (11) and simplifying shows that this is closely related to the asymptotic growth rate of an expected power utility of the relative wealth ratio W_t/W_{bt}, analogous to that assumed in Grossman and Zhou (1993), Bielecki, Pliska, and Sherris (2000), and Fleming and Sheu (1999). But as we show below, the parametric specification of the fund manager objective function does not affect the qualitative form of the ICAPM relationship (9), just as the parametric specification of the individual investors' possibly different functional forms for the utility functions (indexed by superscript k in (2)), as permitted in Merton's ICAPM, does not affect the qualitative form of (9).

To most simply illustrate the advantages of incorporating asset demands derived by style investing objectives like (10) or (11), temporarily suppose (for illustrative purposes) that the investment opportunity set is constant, i.e. the greek letters are constants. Then only the first term in (5) specifies the asset demands of individual investors $k = 1, \ldots, K$. If these were the only investors in the markets, the CAPM would hold (see Merton (1990), inconsistent with the empirical evidence for multiple factors. But Sensoy (2009) reports that "the vast majority of actively managed, diversified U.S. equity

funds use a S&P or Russell benchmark index that is defined on size or value/growth dimensions." So we now assume that there are S classes of style investors, with each class measuring wealth relative to a style-specific benchmark portfolio worth s_j, $j = 1, \ldots, S$. The members of class j try to either meet (i.e. index funds) or beat (i.e. actively managed funds) the same benchmark portfolio worth W_{b_j}.

Member n of style investor class j has a value function that can be denoted $J^n(W^n, s_j, t)$. It would be determined by the specific objective function of those members, e.g. (11) using s_j to denote its W_b, when optimized subject to the own-wealth constraint in (2) after omitting its consumption term. The derivation of both the HJB equation for this member, and its first order condition with respect to the member's portfolio weight vector w_j^n, proceeds identically as in the previous section, producing the following analog of (5):

$$w_j^n W_j^n = T^n[V_{aa}^{-1}(\mu - r_f 1)] + [V_{aa}^{-1} V_{as_j}] H_{s_j}^n \tag{12}$$

where the scalar $H_{s_j}^n = -\frac{J_{W s_j}^n}{J_{WW}^n}$.

Using Breeden's (1989) aforementioned interpretation, the second bracketed term in (12) is "the portfolio of assets that is most highly correlated in return with movements in state variable j." Unlike Merton's definition of state variables, there is no additional calculation needed to find the portfolio most highly correlated with s_j, because it *is* s_j itself, i.e. it is benchmark portfolio j. In Breeden's notation used above, $s_j^* \equiv s_j$.

Because (12) is derived from a style investing objective, e.g. (11) with $s_j \equiv W_b$, the derivatives $J_{s_j}^n < 0$ and $J_{W s_j}^n > 0$. So $H_{s_j}^n = -\frac{J_{W s_j}^n}{J_{WW}^n} > 0$, we have the following result:

Proposition 1: Style investors in class j will hold a positive amount $H_{s_j}^n$ of their benchmark.

The equilibrium market portfolio's vector of asset expenditures, denoted $w^M M$ in (6), is now found by adding the aggregate demand of Merton's individual investors, calculated by summing (5) without its second term [‡], to the aggregate demand of all members of each of the S classes of style investors, found by summing (12) over all members n of each class j (denoted $n \in s_j$). This yields:

$$\sum_{k=1}^{K} w^k W^k + \sum_{j=1}^{S} \sum_{n \in s_j} w_j^n W_j^n \quad =$$

$$(\sum_k T^k)[V_{aa}^{-1}(\mu - r_f 1)] + \sum_{j=1}^{S} \left(\sum_{n \in s_j} T^n[V_{aa}^{-1}(\mu - r_f 1)] + [V_{aa}^{-1} V_{as_j}] H_{s_j}^n \right) \quad =$$

$$(\sum_k T^k + \sum_{j=1}^{S} \sum_{n \in s_j} T^n)[V_{aa}^{-1}(\mu - r_f 1)] + \sum_{j=1}^{S} \sum_{n \in s_j} [V_{aa}^{-1} V_{as_j}] H_{s_j}^n \quad \equiv$$

$$T^M[V_{aa}^{-1}(\mu - r_f 1)] + [V_{aa}^{-1} V_{as}] H_s^M \quad = \quad w^M M. \tag{13}$$

where the jth component of the (S x 1) vector H_s^M is $\sum_{n \in s_j} H_{s_j}^n$.

Comparison of the equilibrium condition (13) that incorporates the aggregate demand of style investors, to the equilibrium condition (6) that does not, shows that the two differ only in their structural interpretations. So the rest of the derivation leading to the ICAPM β-relationship (9) can

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[‡]Recall that for illustrative purposes only, we assumed that the investment opportunity set was constant, i.e. none of the state variables *defined by Merton* are present. Hence there are no individual hedging demands, i.e. only its first term in (5) is nonzero.

still be used. So the ICAPM linear multifactor expression for expected asset returns still holds, but with a different interpretation. The second term in the traditional model's equation (6) arises solely because of Merton's sophisticated individual investors' adjustments to cope with Merton's posited intertemporal changes in the investment opportunity set. But the second term in the style investing model's last line of (13) arises *solely* from style investors' attempts to meet or beat specific benchmark portfolios $b_1, \ldots, b_S$, and hence *will exist even when the investment opportunity set is constant* (unlike in Merton), in which case we have the result:

> *Proposition 2:* Even with a constant investment opportunity set, the incorporation of style investors causes the benchmarks denoted b_j to be priced factors in the linear multifactor relation:

$$\mu_i - r_f = \beta_{iM}(\mu_M - r_f) + \sum_{j=1}^{S} \beta_{ib_j}(\mu_{b_j} - r_f) \tag{14}$$

4. Discussion

The above assumption of a constant investment opportunity set was made for illustrative purposes only. If state variables cause intertemporal changes in the asset price process parameters in (1), each one will result in an additional factor in (9), i.e. the portfolio of assets most closely correlated with it. For example, it is possible that the intertemporal asset demands of benchmark investors will *in and of themselves* cause the asset price process parameters to change, and hence become state variables in Merton's sense. But because the demands of the style investors would still represent a significant sum in the new expression for the demand in the equilibrium equation (13), the benchmark portfolios will still be priced factors.

One might be prompted to guess that differences in the objective functions of agents, i.e. Merton's investors' (2) and the style investors' (11), might lead to theoretical arbitrage opportunities. If actually present, such arbitrage opportunities would destroy the claim that (9) is an equilibrium relationship. But any such claim cannot be staked *solely* on the basis of heterogeneous objectives. Recall that Merton made no restriction on the heterogeneity of individual investors' utility functions (represented by the superscript "k" in (2)–(6)), nor did he make the (unlikely) assumption that financial markets were complete. If the heterogeneity of agents' objectives *per-se* resulted in theoretical arbitrage opportunities, such opportunities would also be present in Merton's original formulation. The subsequent literature has not established that. Hence heterogeneous objective functions are not theoretically problematic.

Moreover, there is nothing sacred in Merton's assumption that an individual investor behaves as if maximizing an intertemporal expected utility dependent solely on her own consumption. In attempts to explain the equity premium puzzle and related anomalies, many leading researchers replaced own-consumption with an argument of own-consumption measured relative to an "external habit", e.g. see the popular graduate finance texts of Cochrane (2001), and Campbell et al. (1997). The external habit plays a role in individual investors' objective functions that is mathematically isomorphic to the role played by the benchmark portfolio in our style investors' objective functions. If external habit formation is thought to be a sensible argument for an individual investor's objective function, isn't (10) or (11) a similarly sensible objective for an index or managed fund? If anything, there is far less *direct behavioral* evidence that individual's act as-if they have an external habit than that index or managed

funds act as-if they have a benchmark they try to meet or beat, respectively. And there is no accepted normative framework arguing that individual investors *should* incorporate an external habit into their objective functions. So how could it be less acceptable to incorporate style investors' objectives like (10) or (11) into Merton's model, as is done here? And if doing so raises the issue of possible arbitrage opportunities, why hasn't that issue been raised with respect to habit formation in comsumption-based asset pricing models?

A more behaviorally "deep" theory of equilibrium with institutional investors is provided by Kaniel and Cuoco (2011). This approach has both strengths and weaknesses relative to the approach employed herein. A strength of their approach is that instead of assuming a stylized fund manager objective function like (10) or (11), Kaniel and Cuoco (2011)) assume that a fund manager maximizes the expected utility of fees received from fund investors, which in turn are *assumed, with no "deep" behavioral rationalization* to be a (reality-motivated) function of a benchmark and parameters that would be set by a manager's firm. This enabled them to make predictions about the sensitivity of a fund manager's portfolio choice to changes in the fee function's parameters, and to work out some implications for equilibrium asset prices. But they were unable to derive the widely-used multifactor relationship (9) as an equilibrium consequence of their posited managerial behavior, and hence have not rationalized use of linear multifactor models as a legitimate focus of inquiry. Hence there is, as yet, no "deeper" explanation for the linear multifactor relationship (9) than that provided by either Merton's original framework, or the style investing version of it developed herein.

4.1. The alpha has no clothes

In the static CAPM, i.e. Merton's ICAPM equilibrium when the investment opportunity set is constant, the only risky asset portfolio purchased by investors is the mean-variance efficient (MVE) tangency portfolio, which is the portfolio with maximum Sharpe Ratio. Hence all agents agree that the Sharpe Ratio is the correct risk-adjusted performance measure for risky asset portfolios. It can be shown that the agents could further increase the Sharpe Ratio by also incorporating a different portfolio, if the *unknown population* affine relationship between its expected excess return and MVE tangency portfolio's excess return has a positive intercept, i.e. a positive Jensen's alpha parameter. While the existence of a positive alpha portfolio is inconsistent with the equilibrium, this is the foundation for using the *estimated* alpha from a single (i.e. market) factor statistical model as a portfolio performance measure. Moreover, Buchner and Wagner (2016) argue that proper treatment of a levered firm's equity as a call option on the value of the firm's assets implies that *estimated* alpha will be downward biased. As a result, shorting a portfolio that has a negative *estimated* alpha (thereby producing a positive estimated alpha) will not necessarily lead to a performance improvement.

But it has become commonplace to use the estimated alpha from empirical multifactor models for this purpose, e.g. the aforementioned Fama-French three factor or Carhart four factor model (e.g., see Wermers (2000)). In Merton's ICAPM equilibrium, neither portfolios held by individual investors solving (2) nor the market portfolio are MVE. Instead, they are what Fama has dubbed "multifactor-efficient". Fama (1996) proved that "the typical multifactor-efficient portfolio of the ICAPM combines an MVE portfolio with hedging portfolios that mimic uncertainty about consumption-investment state variables."

But style investors have different objectives (e.g. the relative wealth objectives (10) or (11)) than the intertemporal utility of individual investors' own-consumption in (2). If the additional factors

arise from demands derived from style investors, Fama's multifactor-efficiency need not characterize the ICAPM relationship. As a result, there is no basis for interpreting the multifactor model α as a performance measure adjusted for the effects of *individual* investor risk [§]. This is problematic for mutual fund performance analyses whose conclusions are altered by using a multifactor model's α as a performance measure instead of Jensen's α.

4.2. Consistency with statistical findings

Proposition 1 in section 3 is that style investors trying to either meet or beat a benchmark portfolio will invest a fraction of their wealth in the benchmark portfolio. In a dynamic world, this effect increases the propensity for the portfolio's assets to be traded together, and hence for their prices to move together. Hence stocks added/deleted from a popular benchmark index portfolio should start to covary more/less with other stocks in that index. Sensoy (2009) found that the S&P 500 is the most widely adopted benchmark, used by funds accounting for 61.3% of assets under management in actively managed mutual funds. So in light of Proposition 1, it isn't surprising that a recent study by Boyer (2011) cited studies by Vijh (1994) and Barberis et al. (2005) that corroborated this prediction, showing that stocks' respective S&P 500 index βs go up/down when added/deleted, even after controlling for the change in non-S&P 500 stocks' βs.

Moreover, Boyer (2011) performed a number of statistical tests using the (S&P/BARRA) growth and value components of the S&P 500, which are defined by sorting the stocks according to book-to-market, placing those above (below) a cutoff point into the value (growth) component. Sensoy (2009) found that large growth and large value benchmarks were the third and fourth most popular benchmark portfolios. So again it isn't surprising that Boyer concluded that:

> "Stocks in the S&P 500 are among the most liquid and closely watched by analysts. This paper shows that arbitrary, economically meaningless labels cause the prices of these stocks to diverge from fundamental value through the trading activity of style investors who use these labels for capital allocation decisions."

In summary, the addition of style investing to the ICAPM provides a theoretical model consistent with the empirical evidence for comovement of assets contained in benchmark portfolios adopted by style investors. Such consistency is strained (or perhaps completely lacking) when trying to interpret this as a consequence of "hedging" against changes in the investment opportunity set, as required by Merton's choice of state variables.

Now if stocks' covariances (i.e. the 2nd moments of joint returns) can be affected by style investing, it is reasonable to assume that expected returns (i.e. the 1st moments of joint returns) might also be affected. To do otherwise is tantamount to making an unwarranted, ad-hoc assumption that only 2nd order moments of the joint returns distribution can be affected by style investing. Proposition 2 of section 3 shows how 1st order moments should be affected by style investing – they should arise as factors in standard linear multifactor regressions. So again it is not surprising that the most recent statistical findings do support the superiority of benchmark indices as factors in that model specification. Exhaustive comparisons by Cremers et al. (2013) established the following findings. First, popular passive benchmark portfolios (e.g. the S&P 500) are mispriced by the

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[§]The benchmark portfolios would have to be those most closely correlated with state variables of *individual* investors' value functions in order for the α to have this interpretation.

Fama-French and Carhart multifactor models, e.g. they report that "the academic factor models assign large nonzero alphas for extended periods of time to passive benchmark indices." This empirical finding is consistent with our style investing modification of the ICAPM: positive multifactor alpha should not be used as a performance measure when the factors arise from both actively managed and index funds' use of them as benchmarks to either meet (index funds) or beat (actively managed funds). Second, Cremers et al. (2013) showed that *benchmark indices serve as better factors than the Fama-French and Carhart factors over the recent period they studied*, to wit:

> "As alternatives to the well-known three and four-factor models, we test models with modified versions of the Fama-French factors as well as models based on the common benchmark indices. We analyze tracking error volatility across a broad cross-section of mutual funds to see which models best explain the common variation in returns and thus most closely track the time series of fund returns. The index-based models produce the lowest out-of-sample tracking error, thus outperforming the traditional Fama-French and Carhart models.
>
> When applied to the cross-section of average mutual fund returns, the index-based models explain average returns well, producing alphas close to zero for all fund groups."

This provides the most direct confirmation possible for Proposition 2 of section 3. Those findings were corroborated in Hunter et al. (2014), who found that an equally weighted index of all funds in a particular style group typically worked very well as an additional factor in a linear multifactor model for separate funds' returns in that style group.

5. Conclusions

Robert Merton's Intertemporal Capital Asset Pricing Model (ICAPM) models the asset demands from investors who maximize the intertemporal expected utility of their own consumption. He showed that that assets' expected returns will be a linear function of the instantaneous expected excess returns of the market portfolio, as in the static CAPM, as well as a number of additional "hedge" portfolios. Each additional portfolio is the one most highly correlated with a time-varying (diffusion-generated) state variable that affects investors' indirect utilities (i.e. their Bellman value functions) via its effects on the investment opportunity set. In accord with this, those attempting to theoretically justify empirical multifactor statistical models of asset returns have tried to establish that the factor portfolios optimally (in Merton's hypothesized consumption/investment problem) hedge Merton's hypothesized intertemporal changes in the investment opportunity set. Forty years of subsequent empirical studies have established the explanatory value of several factors, but have not established that those factors could be the hedge portfolios that Merton's model requires them to be.

Also during the subsequent 40 years, it has become widely acknowledged that index and professionally, actively managed funds (e.g. mutual, pensions, endowments, private clients, etc.) now determine a hefty majority of total investment, and typically are run with the objective of either meeting (index) or beating (actively managed) designated style-specific benchmark portfolios. Hence we modified Merton's ICAPM to incorporate such style investors, and highlighted two implications of that model that differ from Merton's. The first is that style investors will hold a positive amount of the benchmark portfolio that they are either trying to meet (index funds) or beat (actively managed

funds). The second is that the most popular style-specific benchmark portfolio(s) should appear in successful empirical specifications of the popular linear, multifactor model of excess returns.

We argued that the conclusive evidence of comovement among assets present in popular style-specific benchmark portfolios is consistent with the aforementioned first implication of our style investor modification of Merton's model. We also summarized recent conclusive evidence that style-specific benchmark indices perform better in linear, multifactor regressions than do the usual Fama-French and Carhart factors, which provides direct corroboration of the second implication described above. Neither form of empirical evidence is easy to square with Merton's model, which would require the phenomena to arise as a result of sophisticated intertemporal hedging of changes in the investment opportunity set. In fact, changes in the investment opportunity set are not even required to imply a linear multifactor model when style investors' demands are added to Merton's individual investors' demands, again unlike Merton's model.

Finally, the style investing version undermines the foundation for the nearly universal interpretation of a positive multifactor alpha as evidence of superior fund performance. In addition to derived demands of individual investors, aggregate asset demands in the style investing version reflect the demands of style investors, who have objective functions that do not depend on individual investor consumption or wealth. As such, there is no foundation for designating positive alpha funds as *necessarily* beneficial for individual investors.

Conflict of interest

The authors declare no conflict of interest.

References

Barberis N, Shleifer A, Wurger J (2005) Comovement. *J Fin Econ* 75: 283–317.

Becker C, Ferson W, Myers D, et al. (1999) Conditional market timing with benchmark investors. *J Fin Econ* 52: 119–148.

Bielecki T, Pliska S, Sherris M (2000) Risk Sensitive Asset Allocation. *J Econ Dyn Cont* 24: 1145–1177.

Boyer B (2011) Style related comovement: Fundamentals or labels? *J Fin* 66: 307–332.

Breeden D (1989) Intertemporal portfolio theory and asset pricing, In: Eatwell, J. Milgate, M. Newman, P. Authors, *The New Palgrave: Finance*. W.W. Norton.

Brennan M (1993) Agency and asset pricing. Finance Dept. working paper 6-93, Anderson Graduate School of Management, University of California, Los Angeles.

Buchner A, Wagner N (2016) The betting against beta anomaly: Fact or fiction? *Fin Res Lett* 16: 283–289.

Campbell J, Lo A, MacKinlay C (1997) *The Econometrics of Financial Markets*, Princeton, NJ: Princeton University Press.

Carhart M (1997) On persistence in mutual fund performance. *J Fin* 52: 57–82.

Cochrane J (2001) *Asset Pricing*, Princeton, NJ: Princeton University Press.

Cremers M, Petajisto A, Zitzewitz E (2013) Should benchmark indices have alpha? *Crit Fin Rev* 2: 1–48.

Daniel K, Titman S (1997) Evidence on the characteristics of cross-sectional variation in stock returns. *J Fin* 52: 1–33.

De Groot W, Huij J (2011) Is the value premium really a compensation for distress risk? Finance Dept., Rotterdam School of Management.

Fama E (1996) Multifactor portfolio efficiency and multifactor asset pricing. *J Fin Quant Anal* 31: 441–465.

Fama E (1998) Determining the number of state variables in the ICAPM. *J Fin Quant Anal* 33: 217–231.

Fama E, French K (1992) The cross-section of expected stock returns. *J Fin* 47: 427–465.

Fleming W, Sheu SJ (1999) Optimal long term growth rate of expected utility of wealth. *J App Prob* 9: 871-903.

Gregory A, Tharyan R, Christidis A (2013) Constructing and testing alternative versions of the Fama-French and Carhart models in the U.K. *J Bus Fin Acct* 40: 172–214.

Grossman S, Zhou Z (1993) Optimal investment strategies for controlling drawdowns. *Math Fin* 3: 241–276.

Harvey C, Liu Y, Zhu H (2016) ...And the cross section of expected returns. *Rev Fin Stud* 29: 5–68.

Hunter D, Kandel E, Kandel S, et al. (2014) Mutual fund performance evaluation with active peer benchmarks. *J Fin Econ* 112: 1–29.

Jorion P (2003) Porfolio optimization with constraints on tracking error. *Fin Anal J* 58: 70–82.

Kaniel R, Cuoco D (2011) Equilibrium prices in the presence of delegated portfolio management. *J Fin Econ* 101: 264–296.

Merton R (1990) *Continuous Time Finance*, Oxford: Basil Blackwell.

Metrick A, Gompers P (2001) Institutional investors and equity pricing. *Q J Econ* 116: 229–259.

Nagel S (2005) Trading styles and trading volume. Finance Dept. working paper, Graduate School of Business, Stanford University, Stanford.

Novy-Marx R (2013) The other side of value: the gross profitability premium. *J Fin Econ* 108: 1–28.

Roll R (1992) A mean/variance analysis of tracking error. *J Port Mgmt* 18: 13–22.

Rydqvist K, Spizman J, Strebulaev I (2014) Government policy and ownership of equity securities. *J Fin Econ* 111: 70–85.

Sensoy B (2009) Performance evaluation and self-designated benchmark indexes in the mutual fund industry. *J Fin Econ* 92: 25–39.

Vijh A (1994) S&P 500 trading strategies and stock betas. *Rev Fin Stud* 7: 215–251.

Wagner N (2002) On a model of portfolio selection with benchmark. *J Ass Mgmt* 3: 55–65.

Wagner N, Winter E (2013) A new family of equity style indices and mutual fund performance: Do liquidity and idiosyncratic risk matter? *J Emp Fin* 21: 69–85.

Wermers R (2000) Mutual fund performance: An empirical decomposition into stock-picking talent, style, transactions costs, and expenses. *J Fin* 55: 1655–1703.