

## SPECIAL ISSUE ON CONTEMPORARY TOPICS IN CONSERVATION LAWS

During last 20 years the theory of Conservation Laws underwent a dramatic development. *Networks and Heterogeneous Media* is dedicating two consecutive Special Issues to this topic. Researchers belonging to some of the major schools in this subject contribute to these two issues, offering a view on the current state of the art, as well pointing to new research themes within areas already exposed to more traditional methodologies.

Since the seminal works of Euler and Riemann, the development in Conservation Laws has always been based on a strong interplay between analytic theory and the consideration of specific examples. In particular, the Euler equations of compressible inviscid fluids have been the driving paradigm for generations of mathematicians working on Conservation Laws. Nowadays additional applications provide motivation, problems, and examples for the development of the general theory. At the same time, the subtleties of abstract measure theory allowed the construction of surprising examples. Indeed, various theoretical results were obtained, so that *Conservation Laws* can now be considered a fully autonomous discipline within the very broad area of Partial Differential Equations.

Notwithstanding that these two issues are not covering the full breadth of the research frontier, new results within traditional research themes are presented, along with contributions and reviews of recent results in new research directions. As is well known, whenever realistic applications are considered, the available theoretical tools do not provide detailed information on the qualitative agreement between models and reality. Numerical methods play a key role in testing the coherence of analytic models against realistic features of the phenomenon at hand. Accordingly, the importance of having reliable numerical algorithms can hardly be overstated. These two issues deal with numerical methods developed for specific applications.

Although largely arbitrary, the diversity between the articles chosen for the two Special Issues is partly motivated by the different weighting that each contribution gives towards “*specific models*” and “*general theory*”. In the first issue we include the contributions that give (slightly) more priority to specific applications.

Andreianov and Gazibo present a new strong formulation for the Cauchy-Dirichlet problem for a hyperbolic-degenerate parabolic conservation law. They provide a new interpretation of the boundary condition that utilizes maximal monotone graphs, and prove a well-posedness result.

The paper by Babadjian, Mifsud, and Seguin investigates the well-posedness of multi-dimensional Friedrichs systems under constraints, and proves the stability of solutions with respect to perturbation terms in the PDE (parabolic regularization and relaxation of the constraint). Friedrichs systems are linear PDEs, but here the solution has to stay in a given convex set, making the problem nonlinear.

The paper by Biler, Karch, and Zienkiewicz develops techniques for proving blow-up of solutions for Keller-Segel type models involving classical or fractional diffusion operators. Blow-up are shown to occur for initial data that are appropriately large in

a Morrey norm, depending on the ratio of the spatial dimension and the (fractional) order of the diffusion operator.

The paper by Brassart provides an overview of new theoretical results for scalar conservation laws perturbed by fractional diffusion operators. The equations are defined on a bounded domain with homogeneous Dirichlet boundary condition, and involve fractional powers of the Dirichlet Laplacian. Depending on the fractional order, well-posedness results for smooth or entropy solutions are presented.

Caravenna and Spinolo provide the interaction estimates for a front tracking algorithm for a suitable system of conservation laws, having in mind the construction of a counterexample which shows that Schaeffer's regularity theorem for scalar conservation laws can not be extended to systems.

Coclite and Di Ruvo consider the fifth order nonlinear Kawahara-Korteweg-De Vries equation and study the singular limit as the dispersion parameter goes to zero, proving the convergence of solutions to discontinuous weak solutions of the Burgers' equation.

M. Colombo, Crippa, and Spirito investigate multi-dimensional continuity equations with unbounded damping coefficient. Proofs based on logarithmic estimates are supplied for results concerning existence and uniqueness of renormalized solutions. Moreover, the involved techniques are shown to furnish alternative proofs for a series of recent results with the divergence of the velocity field belonging merely to  $BMO$ .

The paper by R. M. Colombo and Guerra considers two 1D immiscible fluids and demonstrates the convergence for the non-linear compressible to incompressible limit of the coupled dynamics as the Mach number tends to zero, i.e., in the limit one of the two fluids becomes incompressible.

Ghoshal's paper delivers new regularity results for scalar conservation laws with spatially discontinuous flux. The main result is a time dependent bound on the total variation of the solution. The analysis uses a generalization of the Lax-Oleinik formula for the Hamilton-Jacobi formulation of the problem.

Finally, Panov proposes new sufficient non-degeneracy conditions ensuring strong precompactness of bounded sequences of functions satisfying nonlinear first-order differential constraints. Moreover, he makes use of this precompactness property to establish the decay of periodic entropy solutions to scalar conservation laws.

Most of the contributions were presented at the *CoToCoLa* meeting held in Besançon between February 9th and February 12th, 2015. The meeting was funded by the CoToCoLa project and the Franche-Comté region, through the project "Les Trimestres du laboratoire de mathématiques de Besançon - Printemps 2015 : Modélisation et équations aux dérivées partielles". The editors thank the organizers of this meeting — Boris Andreianov, Matthieu Brassart, Carlotta Donadello, and Ulrich Razafison — for initiating and assisting in preparing these Special Issues.

Guest Editors:

Rinaldo M. Colombo (Brescia)

Kenneth H. Karlsen (Oslo)

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Andrea Marson (Padova)