NETWORKS AND HETEROGENEOUS MEDIA
©American Institute of Mathematical Sciences
Volume **11**, Number **1**, March **2016**
doi:10.3934/nhm.2016.11.1i
pp. **i–ii**

SPECIAL ISSUE ON CONTEMPORARY TOPICS IN CONSERVATION LAWS

During last 20 years the theory of Conservation Laws underwent a dramatic development. *Networks and Heterogeneous Media* is dedicating two consecutive Special Issues to this topic. Researchers belonging to some of the major schools in this subject contribute to these two issues, offering a view on the current state of the art, as well pointing to new research themes within areas already exposed to more traditional methodologies.

Since the seminal works of Euler and Riemann, the development in Conservation Laws has always been based on a strong interplay between analytic theory and the consideration of specific examples. In particular, the Euler equations of compressible inviscid fluids have been the driving paradigm for generations of mathematicians working on Conservation Laws. Nowadays additional applications provide motivation, problems, and examples for the development of the general theory. At the same time, the subtleties of abstract measure theory allowed the construction of surprising examples. Indeed, various theoretical results were obtained, so that *Conservation Laws* can now be considered a fully autonomous discipline within the very broad area of Partial Differential Equations.

Notwithstanding that these two issues are not covering the full breadth of the research frontier, new results within traditional research themes are presented, along with contributions and reviews of recent results in new research directions. As is well known, whenever realistic applications are considered, the available theoretical tools do not provide detailed information on the qualitative agreement between models and reality. Numerical methods play a key role in testing the coherence of analytic models against realistic features of the phenomenon at hand. Accordingly, the importance of having reliable numerical algorithms can hardly be overstated. These two issues deal with numerical methods developed for specific applications.

Although largely arbitrary, the diversity between the articles chosen for the two Special Issues is partly motivated by the different weighting that each contribution gives towards *"specific models"* and *"general theory"*. In the first issue we include the contributions that give (slightly) more priority to specific applications.

In the present issue, three papers are related to the shallow water equations, a classical field of applications for balance laws. First, Aïssiouene, Bristeau, Godlewski, and Sainte-Marie introduce a new finite volume-finite element method, in the spirit of Chorin and Temam, to solve a shallow-water model with non-hydrostatic effects that is a Green-Nagdhi type system modeling the flow of an incompressible free surface fluid. Next, the paper by Delestre, Ghigo, Fullana, and Lagree considers a shallow water type system of conservation laws for blood flow in arteries with a varying section and elasticity. The main contribution is a well-balanced numerical method that avoids oscillations and preserves relevant steady states. Finally, for the shallow water equations with bottom topography, Hiltebrand and Mishra put forward a discontinuous Galerkin method with both shock capturing and streamline diffusion terms. They prove that the numerical method, which utilizes unstructured grids, is energy stable and preserves relevant stationary solutions.

In the last few years the macroscopic modeling of vehicular traffic dynamics developed a variety of new models, mostly based on conservation or balance laws. The paper by Andreianov, Donadello, Razafison, Rolland, and Rosini studies an extension of the Aw-Rascle-Zhang (ARZ) traffic flow model, incorporating point constraints to limit the flux, mimicking a traffic light, toll booth or construction site. They provide a well-posedness framework as well as a Riemann solver that is utilized in a finite volume method. Garavello proves the existence of weak solutions to an initial-boundary value problem for a system of 1D conservation laws, which is a phase transition model of traffic flow on unidirectional roads. A rather new research direction deals with non-local modeling of traffic dynamics. To this area belongs the contribution by Goatin and Scialanga, which establishes the well-posedness and proposes a finite volume method for the Lighthill-Whitham-Richards (LWR) model with non-local velocity.

Related to traffic flow is the modeling of large sets of individuals, including aggregation, flocking, and swarming. James and Vauchelet give a survey of recent theoretical and numerical results for a first order PDE model for aggregation in one spatial dimension. The model consists of a conservative transport equation for a probability measure, with a velocity field that is a non-local function of the unknown. To this area belongs also the paper by Karlsen and Ulusoy, which analyzes a hyperbolic Keller-Segel system with degenerate, nonlinear fractional diffusion. Colombo, Marcellini, and Rossi review recent mathematical and numerical results for a predator-prey PDE system and a model for the laser cutting of metal. Both models involve nonlocal conservation laws exhibiting nontrivial pattern formation.

Finally, the paper by Gwiazda, Kropielnicka, and Marciniak-Czochra considers a problem that has been less studied in the analytic literature. It proves convergence of the escalator boxcar train method for a system of age-structured population equations of the McKendrick-von Foerster type, a commonly used numerical algorithm in computational biology.

Most of the contributions were presented at the *CoToCoLa* meeting held in Besançon between February 9th and February 12th, 2015. The meeting was funded by the CoToCoLa project and the Franche-Comté region, through the project "Les Trimestres du laboratoire de mathématiques de Besançon - Printemps 2015 : Modélisation et équations aux dérivées partielles". The editors thank the organizers of this meeting — Boris Andreianov, Matthieu Brassart, Carlotta Donadello, and Ulrich Razafison — for initiating and assisting in preparing these Special Issues.

Guest Editors:
Rinaldo M. Colombo (Brescia), Kenneth H. Karlsen (Oslo)
Frédéric Lagoutière (Orsay) and Andrea Marson (Padova)