

## PREFACE

Professor Hiroshi Matano was born in Kyoto, Japan, on July 28th, 1952. He studied at Kyoto University, where he prepared his doctoral thesis under the supervision of Professor Masaya Yamaguti. He obtained his first academic position as a research associate at the University of Tokyo. He then moved to Hiroshima University in 1982 and came back to Tokyo in 1988. He has been a Professor at the Graduate School of Mathematical Sciences at the University of Tokyo since 1991.

He has also been a visiting Professor at universities all over the world, including the University of Arizona, the University of Wisconsin, the University of Minnesota, the University of Tours, the Universities of Paris, Orsay and Cergy-Pontoise, the University of Rome, the University of Heidelberg, Johns Hopkins University, the University of Chicago, as well as a number of various other institutions.

His main field of research is the qualitative theory of nonlinear diffusion equations and nonlinear elliptic equations. He is particularly interested in how the geometry of the spatial domain influences the properties of solutions such as stability. He is also interested in studying the global dynamical structure of various nonlinear diffusion equations by using the theory of infinite-dimensional dynamical systems.

Among other things, he studied nonlinear diffusion equations in several space dimensions and discovered that nonconstant stable stationary states can exist even in a spatially uniform diffusive media provided that the domain is dumbbell-shaped (1979).

His other work includes: the introduction of the lap-number argument (or zero-number argument) in the qualitative study of one-dimensional parabolic equations; the development of the theory of order-preserving dynamical systems and its applications to various partial differential equation; the study of blow-up phenomena and other singularities in nonlinear heat equations and elliptic equations; geometrical and dynamical properties of the Stefan problem and other free boundary problems. More recently he has also been interested in some singular perturbation problems and in travelling waves in heterogeneous media.

In addition to his fundamental contributions in mathematics, Hiroshi Matano's profound work has had and continues to have considerable impact in applications reaching beyond the field of partial differential equations. His lectures are extremely clear and elegant and he brings an enormous inspiration to his colleagues and students; he is and has been an invited lecturer at most prestigious institutions and conferences. In particular he was an invited speaker at the International Congress of Mathematics ICM 94 in Zürich and a laureate of the Spring Prize of the Mathematical Society of Japan in 1990. He is on the editorial board of a number of international journals including the Proceedings of the Royal Society of Edinburgh, the journal of Dynamics and Differential Equations, Discrete and Continuous Dynamical Systems, and others.

Hiroshi Matano promotes collaboration between Japanese, European and American scientists, and organizes many international conferences, among which the

SNP Workshops, Singularities Arising in Nonlinear Problems, where he invites scientists from all over the world. Let us add that he is a coordinator of ReaDiLab (Reaction-Diffusion Laboratory), which is an international laboratory (LIA) of the French CNRS. Today Hiroshi is 60, and his lectures are as beautiful and inspiring as ever. We wish him good health and hope that he will continue to produce deep mathematical articles for very many years.

The aim of this special issue of the Journal Network and Heterogeneous Media is to present a collection of papers which represent recent progress in the field of nonlinear partial differential equations, as well in the theoretical aspects as in their applications to complex systems. The idea of this issue grew while organizing the International Conference entitled : "Nonlinear Partial Differential Equations: Theory and Applications to Complex Systems, An International Conference in honor of Hiroshi Matano", which was held at IHES, Bures-sur-Yvette, France in June 2012. This conference, whose themes were centered around reaction-diffusion systems, was organized by the five editors of the present special issue.

This volume covers a large area in the field of elliptic and parabolic equations, ranging from mathematical modeling in biology to asymptotic analysis and large time behavior, partly based on self-similarity and traveling wave solutions.

Around the modeling articles, let us mention the study of a cross-diffusion model for the self-aggregation of German cockroaches; the authors discuss the approximation by a related reaction-diffusion system as well as its derivation from a microscopic model. A time evolution problem for the transport of solutes in nephrons is presented, both from the modeling and the analysis view points. It turns out that the corresponding semi-group enjoys a contraction in  $L^1$  property and that it stabilizes at an exponential rate. Another article presents a number of partial differential equation problems arising in biology. Striking is the fact they often take the form of free boundary problems for coupled systems of elliptic, hyperbolic and parabolic equations. Also included is a new integro-differential model describing the time evolution of a population subject to mutations and selection due to competition between individuals; concentration phenomena appear in the limit of small mutations.

Another essential field covered in this volume includes recent results in the theory of nonlinear elliptic equations, in particular the existence of periodic solutions for semi-linear elliptic systems on  $R^n$  with a periodic forcing term, as well as the existence of  $2k$ -end solutions for the  $n$ -dimensional stationary Allen-Cahn equation in the whole plane; multiple end solutions asymptotically converge along half-lines to the solution of the corresponding ordinary differential equation. Liouville theorems, in other words non existence results, are presented for a class of nonlinear Schrödinger systems. We should also mention an application to control theory where the cost functional explicitly depends on the location of the free boundary.

Other articles deal with qualitative properties of time evolution parabolic partial differential equations. An essential theme is the large time behavior of solutions; it includes the study of a semi-linear parabolic equation defined on a circle and a characterization of the Sturm global attractors. The Cauchy problem for a semi-linear heat equation with Sobolev nonlinearity is also studied, in particular threshold solutions which blow up in infinite time or slowly decay at various rates. Other results involve self-similarity, for instance for a diffusion equation with nonlinear absorption and a boundary source term, which possesses self-similar profiles in the large time limit as well as in the limit of infinitely strong source at the boundary.

Another example concerns a two-point free boundary problem in a sector for a quasi-linear parabolic equation together with special inhomogeneous boundary conditions. It turns out that there exist an expanding solution and a shrinking solution which are self-similar at discrete times. Other essential notions for the study of large time behavior are those of traveling waves and their generalizations. An article involves the existence and uniqueness of periodically growing solutions of a semi-linear parabolic equation modeling crystal growth in the framework of strongly monotone semi-flows. Such notions also permit to characterize the spreading speed for a Stefan free boundary problem for the logistic equation.

It is of essential interest to master symmetry properties of solutions. In this context, a paper concerns the symmetry and monotonicity properties of the limiting profiles of solutions of fully nonlinear parabolic equations on bounded reflectionally symmetric space domains. Symmetry properties also imply complicated bifurcation phenomena. A normal form on the center manifold is used to study oscillatory dynamics in a reaction-diffusion system with triply degenerate bifurcation points.

An important approach in the theory of nonlinear partial differential equations concerns singular perturbation methods. In this volume, a forward-backward parabolic equation is studied by means of a pseudo-parabolic regularization. This approach permits to deduce admissibility conditions between the phases from entropy inequalities in the case of a three phase solution. The method of matched asymptotic expansions is applied to prove the convergence of solutions of the Navier-Stokes equations to solutions of the Euler equations on a curved domain with a non characteristic boundary as the viscosity tends to zero. Another article deals with the asymptotic limit of a FitzHugh-Nagumo system when the activator satisfies an Allen-Cahn equation with a large reaction coefficient while the inhibitor satisfies a parabolic equation with a small parameter multiplying its time derivative.

We would like to thank all the authors for their efforts to make this special issue a reality. Special thanks are due to Benedetto Piccoli and Corrado Lattanzio for giving us this editorial opportunity and to the many referees of this issue. It is our great pleasure to be able to join the authors in dedicating this volume to Professor Hiroshi Matano.

**Guest Editors:**

Henri Berestycki, Danielle Hilhorst, Frank Merle,  
Masayasu Mimura and Khashayar Pakdaman