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PREFACE

1. **Content of the preface.** The theory of Mean Field Games (MFG, in short) is a branch of the theory of Differential Games which aims at modeling and analyzing complex decision processes involving a large number of indistinguishable rational agents who have individually a very small influence on the overall system and are, on the other hand, influenced by the mass of the other agents. The name comes from particle physics where it is common to consider interactions among particles as an external mean field which influences the particles. In spite of the optimization made by rational agents, playing the role of particles in such models, appropriate mean field equations can be derived to replace the many particles interactions by a single problem with an appropriately chosen external mean field which takes into account the global behavior of the individuals.

The introduction of such a "social" component in this optimization criterion makes this theory very flexible to be applied to various fields and, for this reason, it is attracting an increasing interest from economists (micro and macro), biologist describing the animal behavior and possibly sociologists, engineers and urban planners.

From a strictly mathematical point of view, MFG comprise very general models which include as a particular case other important theories, such as the fluid-dynamical formulation of optimal transportation problem by Benamou-Brenier, systems arising in high frequency approximation of the wave equations or semi-classical limit of the Schrödinger equation, just to give a few relevant examples.

The mathematical formulation of Mean Field Games proposed and investigated by Lasry and Lions in a series of papers in 2006/07 (see also P.-L. Lions cours on the site of the Collège de France http://www.college-de-france.fr for more recent developments) leads to study a system of PDE's, where the classical Hamilton-Jacobi-Bellman equation is coupled with a Fokker-Planck equation for the density of the players, in a peculiar forward-backward way. Related ideas have been developed independently and at about the same time by Huang, Caines and Malhamé.

The aim of this special issue of Network and Heterogeneous Media is to provide a flavor of recent trends in this fast growing research area. It contains nine original papers dealing with various aspects of the MFG theory: some of them present new theoretical results whereas others deal with numerical algorithms and applications of the theory. All the papers have been presented during the workshop "Mean Field Games and Related Topics" held at INdAM, Rome, May 12-13, 2011 (http://www.mat.uniroma1.it/ricerca/convegni/2011/mfg/). The workshop has gathered more that 50 researcher in this area also attracting young researchers from neighbouring areas. We take this opportunity to gratefully acknowledge the sponsors of that successful event: GNAMPA-INDAM, MIUR project PRIN 2007 "Viscosity, metric and control theoretic methods in nonlinear PDE's", Progetto Ateneo SAPIENZA 2009 "Analisi, algoritmi e metodi di calcolo per una classe di

equazioni alle derivate parziali nonlineari" and ITN Marie Curie SADCO "Sensitivity Analysis for Deterministic Controller Design" (FP7-PEOPLE-2010-ITN", Grant Agreement number 264735).

Finally, special warm thanks are due to Prof. B. Piccoli, Editor-in-Chief of Network and Heterogeneous Media, for proposing us this editorial opportunity and to all the contributors and the referees which helped us in making the initiative feasible. We hope that this special issue will contribute to the development of new activities in the field.

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