

ERRATUM AND ADDENDUM TO “FEEDBACK  
STABILIZATION OF A COUPLED STRING-BEAM SYSTEM” BY  
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In this note we give a correct stability result and proof (where we just replace  $\theta = \frac{1}{2}$  by  $\theta = 1$  in the Lemma 6.3 and in the proof of the second assertion of Theorem 1.2) of the second assertion of Theorem 1.2 and we add some stability results (better than the result given in the second assertion of Theorem 1.2):

On the one hand, if we take  $\theta = 1/2$ , then we only have  $|\beta_n|^{1/4} |v_{1,n}(1)| \rightarrow 0$ , and not  $|\beta_n|^{1/2} |v_{1,n}(1)| \rightarrow 0$ , which is used in the proof (equation before [1, (46)]). On the other hand, if we take  $\theta = 1$ , then we have  $|\beta_n|^{1/2} |v_{1,n}(1)| \rightarrow 0$ , and we can thus continue the proof.

So we should make same changes in the proof of the second assertion of [1, Theorem 1.2]:

1. In the Theorem 1.2, assertion (ii), we change the estimate (6) to the following estimate:

$$E_2(t) \leq C \frac{\ln^4(t)}{t^2} \|(\vartheta_1^0, \vartheta_2^0, \vartheta_1^1, \vartheta_2^1)\|_{\mathcal{D}(\mathcal{A}_2)}^2, \forall t > 0.$$

2. In the Lemma 6.3 we change  $\theta = \frac{1}{2}$  to  $\theta = 1$ .
3. In the proof of the second assertion of Theorem 1.2 we change (41) to

$$\limsup_{|\beta| \rightarrow \infty} \frac{1}{\beta} \|(i\beta - \mathcal{A}_2)^{-1}\| < \infty,$$

change (42) to

$$\|\beta_n (i\beta_n I - \mathcal{A}_2) Z_n\|_{\mathcal{H}} \rightarrow 0 \quad \text{as } n \rightarrow \infty,$$

change (43), (44) and (45) respectively to

$$\beta_n (i\beta_n y_{1,n} - v_{1,n}, i\beta_n y_{2,n} - v_{2,n}) \equiv (f_n, h_n) \rightarrow 0 \quad \text{in } V,$$

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$$\beta_n \left( i\beta_n v_{1,n} - \frac{d^2 y_{1,n}}{dx^2} \right) \equiv g_n \rightarrow 0 \quad \text{in } L^2(0, 1),$$

$$\beta_n \left( i\beta_n v_{2,n} + \frac{d^4 y_{2,n}}{dx^4} \right) \equiv k_n \rightarrow 0 \quad \text{in } L^2(1, 1 + \ell).$$

Finally we change the first identity in the “First step” of the proof of the second assertion of Theorem 1.2 by

$$\|\beta_n (i\beta_n I - \mathcal{A}_2) Z_n\|_{\mathcal{H}} \geq |\Re(\langle \beta_n (i\beta_n I - \mathcal{A}_2) Z_n, Z_n \rangle_{\mathcal{H}})| =$$

$$|\beta_n| \left( |v_{1,n}(1)|^2 + \left| \frac{dv_{2,n}}{dx}(1) \right|^2 \right).$$

The proof now continues as in the original paper [1].

**Addendum to the second assertion of Theorem 1.2.**

By Lemma 6.3 and by applying [2, Theorem 2.4] with  $\alpha = 1$  we obtain a better stability result than (6) in Theorem 1.2; precisely we obtain the following stability result:

$$E_2(t) \leq \frac{C}{t^2} \|(\vartheta_1^0, \vartheta_2^0, \vartheta_1^1, \vartheta_2^1)\|_{\mathcal{D}(\mathcal{A}_2)}^2, \forall t > 0.$$

**REFERENCES**

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