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ON THE MODELING OF CROWD DYNAMICS: LOOKING AT THE BEAUTIFUL SHAPES OF SWARMS

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Abstract. This paper presents a critical overview on the modeling of crowds and swarms and focuses on a modeling strategy based on the attempt to retain the complexity characteristics of systems under consideration viewed as an assembly of living entities characterized by the ability of expressing heterogeneously distributed strategies.

1. **Introduction.** This paper presents some perspective ideas on crowd modeling motivated by the aim of taking into account the characteristics of pedestrian crowd viewed as a living, hence complex, system.

The study of complex systems, namely systems of many individuals interacting in a non-linear manner has received in recent years a remarkable increase of interest among applied mathematicians and physicists due also to the conceptual difficulties to treat complexity [10]. These systems are characterized by the difficulty to understand and model them based on the sole description of the dynamics and interactions of a few individual entities localized in space and time.

The above reasonings amount to state that the traditional modeling of individual dynamics does not lead in a straightforward way to a mathematical description of collective emerging behaviors. The reader interested in the modeling of complex systems can refer to the book [6], to papers [9, 10, 12], and therein cited bibliography, to recover an appropriate information on the literature in the field. The recently edited book [56] offers a broad range of interesting applications in different fields of life sciences.

The dynamics of crowds and swarms definitely exhibits characteristics that are typical of complex system. However rarely the existing literature takes into account this relevant specificity. In fact, methods of classical fluid mechanics, either at the scale of particles or that of continuum mechanics, are used despite that self-propelled particles have the ability to develop specific strategies that remarkably distinguish them from classical particles.

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This paper develops a critical analysis followed by a strategy to overcome the afore-said gap. The analysis starts from the existing literature documented in the review papers [15, 42], which mainly refer to the dynamics of vehicular traffic, which has some analogy with that of crowds. However, several important differences distinguish the dynamics of vehicles from that of crowds and swarms.

The approach developed in this paper is based on the kinetic theory of active particles [6, 12], which has been applied in several fields of applied and life sciences as documented, among others, in the modeling of social competition [2, 19], spread of epidemics [34, 35], theory of evolution [13], and in various other fields of life sciences. This method is further developed in this paper to take into account the specific features of the system under consideration, for instance, nonlinear interactions and learning dynamics. It is worth mentioning that such a theory includes the heterogeneous behavior of individuals as originally motivated in the field of behavioral economy [51], while interactions are not classical, but modeled by stochastic games [60].

After the above introduction, a description of the contents can be given. Section 2 presents an overview of the specific characteristics of crowds viewed as a living, and hence complex, system. Special emphasis is given to focus some specific features that are typical of living systems such as the ability to develop a strategy based on that expressed by the surrounding pedestrians. Subsequently, a critical overview on the different observation and representation scales, that can be used to the modeling approach, in view of selecting the most appropriate strategy towards modeling. Section 3 focuses on the mathematical approach of the kinetic theory of active particle and indicates some perspective ideas to be properly developed towards the modeling of the class of systems under consideration. The implications of the transition from normal to panic flow conditions are also treated. Section 4 moves from the modeling of crowds to that of swarms and provides some perspective ideas towards some conceivable approach to this challenging problem that keeps capturing the attention of applied mathematicians and physicists.

2. Crowd viewed as a living (complex) system.

The modeling approach proposed in this paper looks at the dynamics of crowds as the output of the interactions involving individuals, who have the ability to express specific strategies related to that of the other individuals in their action domain. This section presents the main characteristics which, according to the authors' bias, should be retained by the modeling approach. The selection of the most appropriate strategy towards modeling is a consequence. The contents are developed through four subsections and is referred to the existing literature in the field.

2.1. Five key features of crowd dynamics.

Let us consider a crowd in a bounded two-dimensional domain of pedestrians, who interact and have the objective of reaching an exit zone. The crowd can also be localized in an unbounded domain and have the objective of reaching a meeting point. Five features are extracted, according to the authors' bias, to be considered the key ones in the modeling approach.

1. Ability to express a strategy: Pedestrians are capable to develop a specific *organization ability* that depends on the state of the surrounding environment including the state and localization of the other pedestrians in their interaction domain. This strategy can be expressed without the application of any external organizing principle;

2. Heterogeneity: The ability to express a strategy is *heterogeneously distributed*. In some cases, the heterogeneous behavior includes a hierarchy.

3. Interactions: Interactions involve immediate neighbors, but in some cases also distant particles. In some cases, the topological distribution of a fixed number of neighbors can play a prominent role in the development of the strategy and interactions. Generally, the action on a pedestrian from those in the interaction domain is nonlinearly additive, namely it is not the sum of the individual actions.

4. Stochastic games: Interactions modify the state of pedestrians according to the strategy they develop. Living entities *play a game at each interaction*, so that the output is not due to deterministic causality principles. This dynamics is also related to the fact that living systems receive a feedback from their environments, which enables them to learn from their experiences;

5. Large deviations related to passage from normal to panic conditions: The expression of the strategic ability and the characteristics of interactions among pedestrians can be largely modified when panic conditions occur. Consequently emerging behaviors, very different from those observed in normal flow conditions, can appear.

2.2. **Scaling and representation.** The first step of the modeling approach consists in selecting the scale, which appears to be the most appropriate to describe, by mathematical equations, the class of systems under consideration. Classically, the following types of descriptions can be considered:

Microscopic description, which refers to individually identified entities. The overall state of the system is delivered by individual position and velocity of pedestrians. Mathematical models are generally stated in terms of systems of ordinary differential equations.

The ***macroscopic description*** is used when the state of the system is described by gross quantities, namely density, linear momentum, and kinetic energy, regarded as dependent variables of time and space. These quantities are obtained by local average of the microscopic state. Mathematical models are stated by systems of partial differential equations.

The ***kinetic theory description*** is used when the microscopic state of pedestrians is still identified by the individual position and velocity, however their representation is delivered by a suitable probability distribution over such microscopic state. Mathematical models describe the evolution of the afore-said distribution function by means of nonlinear integro-differential equations.

Let us consider a crowd of pedestrians in a domain Ω, with boundary $\partial\,\Omega$, which may also contain inner obstacles as shown in Figure 1. Such domain generally has an inlet and an outlet flow. It is useful, for each scale, using dimensionless variables referred to the following quantities:

n_M is the maximum density of pedestrians corresponding to maximal full packing density;

V_M is the maximum admissible mean velocity, which can be reached by pedestrians in free flow conditions.

$V_\ell = (1+\mu)V_M$, with $\mu > 0$, is a limit velocity which can be reached by a speedy isolated pedestrian;

x, y are the dimensionless space variable obtained dividing the real space by the length ℓ that corresponds to the largest dimension of Ω;

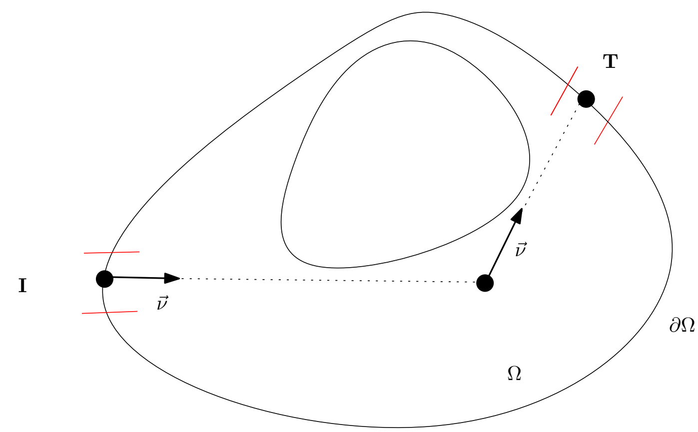

FIGURE 1. – Geometry of the domain Ω occupied by the crowd.

t is the dimensionless time variable obtained referring the real time to a suitable critical time T_c identified by the ratio between ℓ and V_M.

After the above preliminary definitions, it is possible assessing the mathematical structures underlying different models at the various scales.

• The ***microscopic representation*** is defined, for $i = 1, \dots, N$, by the position $\mathbf{x}_i(t) = \{x(t), y(t)\}_i$ in Ω of each i-th individual of a crowd of N pedestrians; and by the dimensionless velocity of each i-th individual $\mathbf{v}_i(t) = \{v_x(t), v_y(t)\}_i$ related to V_ℓ. Mathematical models are generally stated as a system of N ordinary differential equations, where $\mathbf{v}_i$ and $\mathbf{x}_i$ are the dependent variables. The structure underlying models at the microscopic scale is, with obvious meaning of notations, as follows:

$$\begin{cases} \dfrac{d\mathbf{x}_i}{dt} = \mathbf{v}_i \,, \\[2ex] \dfrac{d\mathbf{v}_i}{dt} = \mathbf{F}_i(\mathbf{x}_i, \dots, \mathbf{x}_N, \mathbf{v}_i, \dots, \mathbf{v}_N) \,. \end{cases} \tag{1}$$

The solution of Eq. (1), with given initial conditions, provides the time evolution of position and velocity of pedestrians. Macroscopic quantities are obtained by suitable averaging performed either at fixed time over a suitable space domain, or at fixed space over a suitable time interval. In both cases fluctuations cannot be avoided. Furthermore, the assessment of the parameters of the model needs a remarkable amount of empirical data that are very difficult and expensive to obtain also due to the dependence of the dynamics to environmental conditions.

• The ***macroscopic representation*** can be selected for high density, large scale systems in which the local behavior of groups is sufficient. In details, the macroscopic description is defined by the variables defined as follows:

$\rho = \rho(t, \mathbf{x})$ is the dimensionless density referred to the maximum density n_M of pedestrians,

$\mathbf{V} = \mathbf{V}(t, \mathbf{x}) = V_x(t, \mathbf{x})\,\mathbf{i} + V_y(t, \mathbf{x})\,\mathbf{j}$ is the dimensionless mean velocity, referred to V_M, where $\mathbf{x} = \{x, y\}$, while $\mathbf{i}$ and $\mathbf{j}$ denote the unit vectors of the coordinate axes.

The relationship between the flow rate, the mean velocity and the pedestrian density is given, in a dimensionless form, as follows: $\mathbf{q} = \rho\,\mathbf{V}$.

The mathematical structure, which underlies models at the macroscopic scale, is given by the equations of mass conservation and momentum equilibrium:

$$\begin{cases} \partial_t \rho + \partial_{\mathbf{x}} \cdot (\rho \mathbf{V}) = 0\,, \\ \\ \partial_t \mathbf{V} + \mathbf{V} \cdot \partial_{\mathbf{x}} \mathbf{V} = \mathcal{A}[\rho, \mathbf{V}; \boldsymbol{\nu}]\,, \end{cases} \tag{2}$$

where the dot-product denotes inner-product of vectors, while $\mathcal{A}$ is a psycho-mechanical acceleration acting on pedestrians in the elementary macroscopic volume of the physical space, which depends on the local density conditions and on the optimal trajectory in vacuum conditions that is identified by the local vector $\boldsymbol{\nu}$.

Some authors [14] suggest to split the afore said acceleration into the sum of a frictional-type acceleration $\mathcal{A}_F$, which is proportional to the difference between the actual velocity $\mathbf{V}$ and the mean equilibrium velocity $\mathbf{V}_e(\rho, \boldsymbol{\nu})$ corresponding to the local density, and an acceleration $\mathcal{A}_P$ determined only by the gradient of pedestrian density based on the idea that pedestrians chose an optimal path looking for minimal gradients.

• The ***kinetic (statistical) representation*** is defined by the statistical distribution of their position, and velocity:

$$f = f(t, \mathbf{x}, \mathbf{v}), \quad \mathbf{x} \in \Omega, \quad \mathbf{v} \in D_{\mathbf{v}}, \tag{3}$$

where $D_{\mathbf{v}}$ is the domain of the velocity $\mathbf{v}$ and f is normalized with respect to n_M. If f is locally integrable, $f(t, \mathbf{x}, \mathbf{v})\, d\mathbf{x} d\mathbf{v}$ denotes the number of individuals, which, at the time t, are in the elementary domain of the microscopic states $[\mathbf{x}, \mathbf{x} + d\mathbf{x}] \times [\mathbf{v}, \mathbf{v} + d\mathbf{v}]$ in the phase space.

Macroscopic observable quantities can be obtained, under suitable integrability assumptions, by moments of the distribution. In particular, the dimensionless local density is given by

$$\rho(t, \mathbf{x}) = \int_{D_{\mathbf{v}}} f(t, \mathbf{x}, \mathbf{v})\, d\mathbf{v}\,, \tag{4}$$

while the total number of individuals in Ω is given by

$$N(t) = \int_{\Omega} \rho(t, \mathbf{x})\, d\mathbf{x}\,, \tag{5}$$

which depends on time in the presence of inlet and/or outlet of pedestrians.

Analogously, the mean velocity can be computed as follows:

$$\mathbf{V}(t, \mathbf{x}) = E[\mathbf{v}](t, \mathbf{x}) = \frac{1}{\rho(t, \mathbf{x})} \int_{D_{\mathbf{v}}} \mathbf{v}\, f(t, \mathbf{x}, \mathbf{v})\, d\mathbf{v}\,. \tag{6}$$

Moreover, the velocity variance is similarly computed to provide a measure of the stochastic behavior of the system with respect to the deterministic macroscopic description.

The mathematical structure which underlies models delivered by the afore-said representation is given by a balance of number of particles in the elementary volume of the phase space, which can be formally written as follows:

$$\big(\partial_t + \mathbf{v} \cdot \partial_{\mathbf{x}}\big)\, f(t, \mathbf{x}, \mathbf{v}) = G[f; \boldsymbol{\nu}](t, \mathbf{x}, \mathbf{v}) - L[f; \boldsymbol{\nu}](t, \mathbf{x}, \mathbf{v})\,, \tag{7}$$

where G and L denote, respectively, the *gain* and *loss* of particles in the elementary volume of the phase space. The detailed expression of these terms correspond to different types of modeling interactions at the microscopic scale.

2.3. **Critical analysis on the selection of mathematical structures.** The various representation schemes given in the preceding subsection enable us to develop a critical analysis of mathematical models known in the literature. Out of such analysis, it appears that new ideas need to be developed with the aim of capturing the complexity features of the system under consideration.

The review paper [15] offers a broad panorama of the various models existing in the literature. Crowd modeling has initiated at the macroscopic scale. More precisely, Henderson's pioneering paper [48] proposed a model related to an homogeneous gas constituted of statistically independent particles in equilibrium in a two-dimensional space. Subsequently, Hughes [49] extended Henderson's fluid dynamics approach to allow for factors of human decision and interaction. This approach has been further refined in [14], while the analysis of the hyperbolic properties of the model are given in [39]. A detailed qualitative analysis of the initial value problem is given in [28] and [38]. The main problem in the modeling approach at the large scale consists in proposing a model for the acceleration term by an approach that should take into account density gradient fields [14, 53].

It is worth mentioning that the modeling can be developed also by first order models by using the mass conservation equation simply closed by a relation linking the mean velocity to the local flow conditions, formally $\mathbf{V} = \mathbf{V}[\rho, \boldsymbol{\nu}]$. For instance, Coscia and Canavesio [29] propose a model of this type which has been used to model the crowd flow on the Jamarat bridge and applied to study the dynamics of lively footbridges [64, 65]. A new approach has been invented by Piccoli and Tosin [58, 59], where the closure is obtained by probability measures, whose modeling is related to the dynamics at the lower scale, namely to the strategy developed by pedestrians, see also [30]. A technical application is proposed in [23].

The modeling approach at the microscopic scale has been mainly developed by Helbing and co-workers by the so-called *social force models*, introduced in [46]. This model is based, after a detailed analysis of individual behaviors [41], on the assumption that interactions among pedestrians are implemented by using the concept of a *social force* or *social field*. This approach can be technically adapted to take into account various mechanical behaviors. For instance, pedestrians keep a certain distance from other pedestrians. Such distance depends on the pedestrian density and walking speed. Suitable repulsive, short-range, potentials can be introduced to describe these phenomena. The interaction potential can be attractive, for long-range interactions, to model the aggregation phenomena of pedestrians, who often show a trend to walk in groups. Once separated (for instance if a pedestrian has to avoid an obstacle), the individual pedestrians try to reform the group. Moreover, pedestrians move with an individual speed, taking into account the situation, sex, age, handicaps, surroundings, and so on.

One of the crucial problems of the modeling at the microscopic scale consists in dealing with a large number of equations and in transferring the microscopic information to the macroscopic level, namely to physical quantities which can be possibly observed and measured. *Cellular automata models* can overcome this difficulty [21]. These models simulate pedestrians as entities (automata) in cells, however only very simple aspects of the dynamics are taken into account. On the other hand, more recent studies introduce a modeling of the self-organizing ability of pedestrians, which are modified at individual level, see [45, 47, 55]. A useful collection of empirical data is offered in the Technical Report [24].

Our reasonings show immediately that none of the afore-said representation scales is fully satisfactory. In fact, it is plain that the flow is not continuous, hence models derived at the macroscopic scale are not consistent with the classical paradigms of continuum mechanics. Moreover, the number of individual entities involved in the dynamics is not large enough to justify the use of continuous distribution functions within the framework of the mathematical kinetic theory. Finally, individual entities should be modelled as active particles due to their ability to modify their dynamics according to specific strategies. Therefore, new ideas different from the ones we have just seen above, should be possibly looked for, being related to the five sources of complexity reviewed in Section 2.

3. **Perspective ideas on the modeling of crowds.** This section aims at transferring the critical analysis proposed in the preceding section into perspective ideas to model pedestrian crowds. This aim is pursued in three steps. The first one focuses on the design of the mathematical structure to be assumed as a reference framework to derive specific models. The second step consists in modeling interactions at the microscopic scale. Finally the modeling of panic conditions is treated. This is an interesting topic [44], which is definitely worth future research activity to be properly related to well-defined models. The last subsection presents a critical analysis focusing on perspectives towards improving the modeling approach. This section takes advantage of [7], where some preliminary hints have been given. Here some further developments are proposed. In general, the modeling approach should pursue the following, at least, objectives:

1. Mathematical models should include a limited number of parameters related to well defined physical phenomena, which should be technically identified by experiments;
2. Empirical data should not be artificially plugged into mathematical models, which should reproduce them after a suitable choice of the parameters;
3. Models are required to reproduce, at least at a qualitative level, emerging phenomena which are observed in real flow conditions. In particular, the self-organizing ability is substantially modified by environmental conditions;
4. Modeling has to take into account also the fact that the human interpretation of danger is not, at least in some cases, correct. For instance, escaping a danger can be identified by the localization of overcrowded areas, which constitute additional danger, and a subsequent additional panic.

3.1. **A mathematical structure towards modeling.** The mathematical structure proposed in this subsection refers specifically to the complexity characteristics presented in Section 2. According to [6] an *activity variable* u is introduced in the microscopic state of pedestrians to model the heterogeneous distribution of their ability to express a strategy. Therefore, the microscopic state is defined by the variables: position $x \in [0, 1]$, speed $v \in [0, 1 + \mu]$, velocity direction $\theta \in [0, 2\pi)$, and activity $u \in [0, 1]$, where $u = 0$ and $u = 1$ correspond, respectively, to the worse and best ability of pedestrians. This variable includes the environmental conditions, namely $u = 1$ corresponds also to the best pedestrian in optimal conditions of the ambient where the crowds moves, while worse conditions contribute to reduce the value of u.

The space of microscopic states is subdivided into discrete cells of the space of the microscopic states to take into account the lack of continuity of the distribution function. More precisely the following sets identify the discrete values of the afore-said variables: $I_\theta = \{\theta_0 = 0\,, \ldots\,, \theta_i\,, \ldots\,, \theta_n = 2\,\pi\}$, $I_v = \{v_0 = 0\,, \ldots\,, v_j\,, \ldots\,, v_m = 1+\mu\}$, and $I_u = \{u_0 = 0\,, \ldots\,, u_w\,, \ldots\,, u_z = 1\}$.

The corresponding discrete representation is as follows:

$$f(t,\mathbf{x},\mathbf{v},u) = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{w=1}^{z} f_{ijw}(t,\mathbf{x})\,\delta(\theta-\theta_i)\;\delta(v-v_j)\,\delta(u-u_w)\,, \tag{8}$$

where $f_{ijw}(t,\mathbf{x}) = f(t,\mathbf{x},\theta_i,v_j,u_w)$, while it has been assumed that all pedestrians walk, namely $v > 0$, do not have a null activity $u > 0$, and that they cannot have a velocity larger than V_ℓ.

The macroscopic quantities are obtained by weighted sums. In particular, the number density and flow of pedestrians are, respectively, given by:

$$\rho(t,\mathbf{x}) = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{w=1}^{z} f_{ijw}(t,\mathbf{x})\,, \tag{9}$$

and

$$\mathbf{q}(t,\mathbf{x}) = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{w=1}^{z} (v_j\,\cos\theta_i\,, v_j\,\sin\theta_i\,)\,f_{ijw}(t,\mathbf{x}). \tag{10}$$

Higher order moments, corresponding to the energy and speed variance, can be computed by similar calculations.

It is worth stressing that the representation (8) is generally used in kinetic theory to reduce the computational complexity of the Boltzmann equation or related models. On the other hand, a discrete space of microscopic states was introduced in [36] in vehicular traffic to face Daganzo's criticism [32]. Subsequently, this idea has been followed by various authors as reported in the review paper [15]. More precisely, it is remarked in [32] that the number of vehicles is not large enough to justify the assumption of a continuous distribution function. The same reasonings are valid also in the case of pedestrians in a crowd. Hence the modeling approach is based on the idea of considering particles with state in a cell of finite dimension.

Let us consider first the relatively simpler case when the activity variable is heterogeneously distributed, but it is not modified by interactions. In this case the components $f_{ijw}(t,\mathbf{x})$ of the distribution function can be factorized as follows: $f_{ijw}(t,\mathbf{x}) = f_{ij}(t,\mathbf{x})g_w$, where $g_w = g(u = u_w)$ does not depend on time and space. The transition probability density involves only the velocity variables: $\mathcal{A}^{ij}_{hk,pq}$, and the time and space evolution of the distribution function f_{ij} can be obtained by equating the increase of time of f in the elementary volume of the space of the microscopic states to the net flow into such volume due to interactions. Accordingly, the following structure is proposed:

$$\begin{aligned}\big(\partial_t \;&+\; \mathbf{v}_{ij}\cdot\partial_{\mathbf{x}}\big) f_{ij}(t,\mathbf{x}) = \mathcal{J}[\mathbf{f}](t,\mathbf{x})\\ &=\; \sum_{h,p=1}^{n}\sum_{k,q=1}^{m}\int_\Lambda \eta[\rho(t,\mathbf{x}^*)]\sigma(\mathbf{x},\mathbf{x}^*)\mathcal{A}^{ij}_{hk,pq}[\rho(t,\mathbf{x}^*)] f_{hk}(t,\mathbf{x})\,f_{pq}(t,\mathbf{x}^*)\,d\mathbf{x}^*\\ &-\; f_{ij}(t,\mathbf{x})\sum_{p=1}^{n}\sum_{q=1}^{m}\int_\Lambda \eta[\rho(t,\mathbf{x}^*)]\,\sigma(\mathbf{x},\mathbf{x}^*) f_{pq}(t,\mathbf{x}^*)\,d\mathbf{x}^*,\end{aligned} \tag{11}$$

where $\mathbf{f} = \{f_{ij}\}$, and $\mathbf{v}_{ij} = (v_j\,\cos\theta_i\,,\,v_j\,\sin\theta_i)$, the visibility zone Λ may depend on space due to the geometry of Ω, while the modeling of the term $\mathcal{A}^{ij}_{hk,pq}$ should be consistent with the probability density properties:

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\mathcal{A}_{hk,pq}^{ij}=1,\quad \forall\, hp\in\{1,\ldots,n\},\quad \forall\, kq\in\{1,\ldots,m\}\,,$$

and for all conditioning local density. Indeed, the table of games reported in the following satisfies such property.

Equation (11), which underlies the derivation of specific models, corresponds to the following assumptions:

i) Pedestrians have a visibility zone $\Lambda = \Lambda(\mathbf{x})$, see Figure 2, which does not coincide with the whole domain Ω due to the limited visibility angle of each individual.

ii) Interactions involve three types of active particles: the ***test*** pedestrian with state (ij), which is representative of the whole system; the ***field*** pedestrians with state (pq) in the position $\mathbf{x}^*$ of the visibility zone Λ; and the ***candidate*** pedestrian with state (hk), at time t, in the position $\mathbf{x}$. The candidate pedestrian modifies, in probability, its state into that of the test pedestrian, due to interactions with the field pedestrians, while the test pedestrian looses its state due to interaction with the field pedestrians. The activity of the candidate pedestrian is u_w, which is not modified by the interaction.

All particles cannot be distinguished individually. Therefore, their state identifies them. In particular, candidate particles are field particles whose state, after the identification, reaches that of the test particles.

iii) The output of interactions is modeled by the discrete probability density function $\mathcal{A}_{hk,pq}^{ij}$, which denotes the probability density that a candidate (hk)-pedestrian modifies its state into that of the test (ij)-pedestrian due to interaction with the field (pq)-pedestrians. This transition density depends on the local number density.

iv) The frequency of the interactions is modeled by the term $\eta[\rho(t,\mathbf{x}^*)]$ depending on the distribution function of the field particles in the visibility zone. The intensity of the action on the candidate pedestrian due to the interactions with the field pedestrians is modeled by a weight function $\sigma(\mathbf{x},\mathbf{x}^*)$. The weight function is normalized with respect to integration over $\mathbf{x}^*$ over Λ.

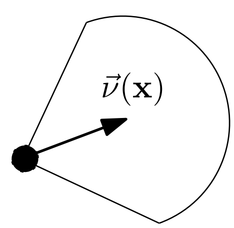

FIGURE 2. – Visibility zone

The derivation of specific models can take advantage of the above structures and is obtained by modeling the various terms that appear in Eq. (11) as shown in the next subsection. The more general case of interactions that modify the activity variable can be come relevant in the case of panic conditions and is outlined in subsection 3.3.

3.2. **Modeling interactions at the microscopic scale.** Specific models can be obtained by modeling the terms that appear in Eq. (11) to describe interactions at the microscopic scale, namely η, σ, and $\mathcal{A}$. This subsection is devoted to give appropriate hints towards this objective. The modeling necessarily needs heuristic assumptions towards the interpretation of the complex phenomenology of the system under consideration. Validation of models can be pursued by comparisons between their prediction and the output of empirical data. Papers [16, 36] have shown that an appropriate modeling of interactions at the microscopic scale leads to an accurate description of both the fundamental diagrams and emerging behaviors.

The approach of this present paper specifically refers to [16] by generalizing the model for traffic flow to crowd dynamics and, introducing new ideas related to specificity of pedestrian movements. The modeling refers to normal flow conditions, while various modifications needs to be applied in the case of panic conditions.

• ***Flow regimes:*** Three flow regimes are identified by two critical densities ρ_f and ρ_s. Namely, the *free flow regime* that occurs for $\rho \in [0, \rho_f[$ when the dynamics is influenced only by the geometry of the system, but not by the presence of other pedestrians; the *congested flow regime* that occurs for $\rho \in [\rho_f, \rho_s[$ when the dynamics is influenced by the afore-mentioned causes; and the regime that occurs for $\rho \in [\rho_s, 1]$ when pedestrians stop to avoid contact with other ones. The modeling proposed in the following is developed for $\rho_f \cong 0$.

• ***Interaction rate:*** The modeling of the term η, can be developed similarly to the case of vehicular traffic [16], namely by increasing the interaction rate with increasing local density in the free and congested regime. For higher densities, when pedestrians are obliged to stop, one may assume that η keeps a constant value, or may decay for lack of interest. The following model can be proposed among various conceivable ones:

$$\eta(\rho(t, \mathbf{x})) = (1 + \rho(t, \mathbf{x})) \exp\left(- \frac{\rho(t, \mathbf{x})}{1 + \rho_s} \right). \tag{12}$$

• ***Weight function:*** The function σ models how the pedestrian takes into account the flow nconditions depending on the distance. A general rule cannot be given as it depends on the size of the domain. For instance if the domain is small one can assume that the distance has not a practical influence, while in large domains σ may decay with the distance.

• ***Transition probability density:*** The modeling of the transition probability density $\mathcal{A}_{hk,pq}^{ij}$ refers to the interactions involving candidate with field particles. The approach proposed here is based on the assumption that particles are subject to three different influences, namely the ***trend to the exit point***, the ***influence of the stream*** induced by the other pedestrians, and the selection of the path with minimal density gradient. A simplified interpretation of the phenomenological behavior is obtained by assuming the factorization of the two modifications of the velocity direction and modulus:

$$\mathcal{A}_{hk,pq}^{ij} = \mathcal{B}_{hp}^{i}\, \mathcal{C}_{kq}^{j} = \mathcal{B}\big(\theta_h \to \theta_i | \theta_h, \theta_p, u_w, \rho^*\big)\, \mathcal{C}\big(v_k \to v_j | v_k, v_q, u_w, \rho^*\big), \tag{13}$$

where $\rho^* = \rho(t, \mathbf{x}^*)$.

Equation (13) is, of course, based on an heuristic assumption that cannot be justified by theoretical issues. However, it simplifies the modeling approach by separating the various causes that modify the dynamics. This assumption does not imply factorization of the probability density due to the mixing action of the variable ρ. Let us now consider separately, the modeling of the two transition densities.

– ***Modeling*** $\mathcal{B}^i_{hp}$: Let us consider the candidate particle in the position P moving in the direction θ_h and interacting with a field particle with direction θ_p. Moreover, let θ_ν be the angle from P to T or to the direction of the shortest path, see Fig. 1. Let us introduce three parameters $\varepsilon_1, \varepsilon_2$ and ε_3, which model, respectively the sensitivity of pedestrians to reach the target T, the sensitivity of pedestrians to chose the path with minimal gradients, and sensitivity of the pedestrian to follow the stream.

Moreover, it is assumed that the transition of velocity corresponding to the phenomena related to afore-said parameters ε_1 and ε_2, namely sensitivity to the target and to minimal gradients, depends on the ratio of vacuum $1-\rho$, while the transition related to the sensitivity to the stream, that corresponds to ε_3, depends on the density ρ. The selection of the direction, among the three $h-1$, h, and $h+1$, is identified by the terms $U_\rho^{h+1} = 1$ or $U_\rho^{h-1} = 1$, if the density gradient is the smallest on the directions $h+1$ or $h-1$, respectively; otherwise $U_\rho^{h+1} = 0$ or $U_\rho^{h-1} = 0$. If the smallest gradient is in the direction h no turning action is present.

Finally it is assumed that the dynamics is more active for higher values of the activity variable u_w. It is natural that an active individual shows a higher ability to modify the direction of the motion. Four cases can be considered for $\rho \in [0, \rho_s[$:

– *Interaction with a upper stream and target directions, namely* $\theta_p > \theta_h; \quad \theta_\nu > \theta_h$*:* It is assumed that, in addition to the selection of the path with minimal gradients, both actions contribute to an anticlockwise rotation:

$$
\begin{aligned}
\mathcal{B}^i_{hp} &= \varepsilon_1\, u_w(1-\rho) + \varepsilon_2\, u_w(1-\rho)\, U_\rho^{h+1} + \varepsilon_3\, u_w\, \rho \quad \text{if} \quad i = h+1\,,\\
\mathcal{B}^i_{hp} &= 1 - \varepsilon_1\, u_w(1-\rho) - \varepsilon_2\, u_w(1-\rho)[U_\rho^{h+1} + U_\rho^{h-1}] - \varepsilon_3\, u_w\, \rho \quad \text{if} \quad i = h\,,\\
\mathcal{B}^i_{hp} &= \varepsilon_2\, u_w\,(1-\rho)\, U_\rho^{h-1} \quad \text{if} \quad i = h-1\,.
\end{aligned}
$$

Analogous calculations can be done for the other cases. The result is as follows:

– *Interaction with a upper stream and low target direction* $\theta_p > \theta_h; \quad \theta_\nu < \theta_h$*:*

$$
\begin{aligned}
\mathcal{B}^i_{hp} &= \varepsilon_3\, u_w\, \rho + \varepsilon_2\, u_w(1-\rho) U_\rho^{h+1} \quad \text{if} \quad i = h+1\,,\\
\mathcal{B}^i_{hp} &= 1 - \varepsilon_1\, u_w(1-\rho) - \varepsilon_2\, u_w\,(1-\rho)[U_\rho^{h+1} + U_\rho^{h-1}] - \varepsilon_3\, u_w\, \rho \quad \text{if} \quad i = h\,,\\
\mathcal{B}^i_{hp} &= \varepsilon_1\, u_w\,(1-\rho) + \varepsilon_2\, u_w\,(1-\rho)\, U_\rho^{h-1} \quad \text{if} \quad i = h-1\,.
\end{aligned}
$$

– *Interaction with a lower stream and upper target direction* $\theta_p < \theta_h; \quad \theta_\nu > \theta_h$*:*

$$
\begin{aligned}
\mathcal{B}^i_{hp} &= \varepsilon_1\, u_w(1-\rho) + \varepsilon_2\, u_w(1-\rho)\, U_\rho^{h+1} \quad \text{if} \quad i = h+1\,,\\
\mathcal{B}^i_{hp} &= 1 - \varepsilon_1\, u_w(1-\rho) - \varepsilon_2\, u_w\,(1-\rho)[U_\rho^{h+1} + U_\rho^{h-1}] - \varepsilon_3\, u_w\, \rho \quad \text{if} \quad i = h\,,\\
\mathcal{B}^i_{hp} &= \varepsilon_3\, u_w\, \rho + \varepsilon_2\, u_w(1-\rho)\, U_\rho^{h-1} \quad \text{if} \quad i = h-1\,.
\end{aligned}
$$

– *Interaction with a lower stream and target directions* $\theta_p < \theta_h; \quad \theta_\nu > \theta_h$*:*

$$
\begin{aligned}
\mathcal{B}^i_{hp} &= \varepsilon_2\, u_w\,(1-\rho)\, U_\rho^{h+1} \quad \text{if} \quad i = h+1\,,\\
\mathcal{B}^i_{hp} &= 1 - \varepsilon_1\, u_w(1-\rho) - \varepsilon_2\, u_w\,(1-\rho)[U_\rho^{h+1} + U_\rho^{h-1}] - \varepsilon_3\, u_w\, \rho \quad \text{if} \quad i = h\,,\\
\mathcal{B}^i_{hp} &= \varepsilon_1\, u_w\,(1-\rho) + \varepsilon_2\, u_w(1-\rho)\, U_\rho^{h-1} + \varepsilon_3\, u_w\, \rho \quad \text{if} \quad i = h-1\,.
\end{aligned}
$$

– ***Modeling*** $\mathcal{C}_{kq}^j$: The modeling of the modifications of the velocity modulus due to interactions can be developed similarly to the approach of traffic flow. Therefore, detailed calculations are not repeated here. We refer to [36, 16], where the first paper uses a fixed grid and a parameter modeling the quality of the outer system, road and environment, while the second one introduces the concept of critical densities ρ_c and ρ_ℓ that, as already mentioned, separate the free from the, congested flow and the density that obliges pedestrians to stop. These densities are used as a parameter related to a parameter modeling the quality of the environment. These reasonings can be applied also to the case of pedestrian flows. Therefore the approach of [36, 16] can be straightforwardly used to model the transition probability density $\mathcal{C}_{kq}^j$. Empirical data to validate models are offered in [24]. The contents [16] show that the modeling reproduce the afore-said empirical data. concerning both fundamental velocity diagrams and emerging behaviors.

– The modeling corresponding to jam densities $\rho \in [\rho_s, 1]$ simply implements that pedestrians reduce to zero their velocity.

Finally, mathematical models are obtained by inserting the above transition probability densities modeling interactions at the microscopic scale into Eq. (11). The simple approach that has been presented above can be further simplified or enriched. For instance simplifications can restrict to one or two the velocity modules or suppose that the activity variable is the same for all pedestrians. On the other hand, the dynamics over the activity variable can be included in a more general structure:

$$\begin{aligned}
&\big(\partial_t + \mathbf{v}_{ij}\cdot\partial_{\mathbf{x}}\big) f_{ijw}(t,\mathbf{x}) = \mathcal{J}[\mathbf{f}](t,\mathbf{x}) \\
&= \sum_{h,p=1}^{n}\sum_{k,q=1}^{m}\sum_{r,s=1}^{z}\int_\Lambda \eta[\rho(t,\mathbf{x}^*)]\sigma(\mathbf{x},\mathbf{x}^*)\mathcal{A}_{hkr,pqs}^{ijw}[\rho(t,\mathbf{x}^*)] f_{hkr}(t,\mathbf{x})\, f_{pqs}(t,\mathbf{x}^*)\, d\mathbf{x}^* \\
&\quad - f_{ijw}(t,\mathbf{x})\sum_{p=1}^{n}\sum_{q=1}^{m}\sum_{s=1}^{z}\int_\Lambda \eta[\rho(t,\mathbf{x}^*)]\,\sigma(\mathbf{x},\mathbf{x}^*) f_{pqs}(t,\mathbf{x}^*)\, d\mathbf{x}^*, \qquad (14)
\end{aligned}$$

where $\mathbf{f} = \{f_{ijw}\}$, while the constraint for the term $\mathcal{A}_{hkr,pqs}^{ijw}$ is

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{w=1}^{z}\mathcal{A}_{hkr,pqs}^{ijw} = 1\,,$$

for all conditioning local density, and $\forall\, h,p \in \{1,\ldots,n\}$, $\forall\, k,q \in \{1,\ldots,m\}$, and $\forall\, rs \in \{1,\ldots,z\}$. This issue is not treated here, however the interested reader is referred to Subsection 8.1 of [15], to recover some hints on this matter in analogy to vehicular traffic.

3.3. **From a critical analysis to modeling panic conditions.** The class of mathematical models presented in the preceding section is based on the mathematical structure Eq. (11) or Eq. (14) which have been proposed to take into account the five complexity features reported in Section 2. In details, the ability to express a strategy is modeled by the variable u, which is heterogeneously distributed among pedestrians; interactions are modeled by stochastic games, namely by the transition probability density $\mathcal{A}$, which transfers the input related to the probability densities of the interacting particles into the output, in probability, into a cell of the space of microscopic states. On the other hand, large deviations due to the onset panic conditions do not appear explicitly in the mathematical structure, however the critical analysis developed in the following shows that their modeling can be inserted into Eq. (11) or Eq. (14). Therefore we trust that the afore-said equations are appropriate to act as a background structure for the derivation of specific models, which can be obtained by modeling interactions at the microscopic scale.

Panic conditions modify, as already mentioned, the dynamics of interactions in various ways. For instance by increasing quantitatively the interaction rate and by disregarding the exit zone in favor of clustering, where the crowd feels to be, in various cases wrongly, out of danger [43]. Therefore, when these conditions appear, the model has to take into account a different way of depicting interactions at the microscopic level.

It is difficult indicating general rules valid for all specific cases considering that these special conditions somehow differ from case to case. The following indications are given looking at research perspectives on this interesting topic:

i) The critical density ρ_s tends to zero, which means that in full packing conditions the trend towards the exit is substituted by the action of the flow;

ii) The values of the activity u needs to be increased considering that pedestrians accelerate their dynamics despite the environmental conditions;

iii) The assumption that the weight σ decays with the distance is not a general rule. Indeed, different contexts lead to different pedestrian behavior, who may be, in some cases, attracted by distant rather that near areas;

iv) The search of optimal paths, corresponding to the smallest density gradients, may not be pursued considering that pedestrians tend to clustering rather than moving to the exit zone. Therefore, one can put $\varepsilon_1 \cong 0$ to neglect the trend towards the exit zone, increase the value of ε_2 to take into account the greater action of the flow and modify the selection of the best path weighted by ε_3. In fact pedestrian try to reach zones where the crowd aggregates.

v) The geometry of the domain containing the crowd can have a great influence on the emerging collective behaviors. In fact, pedestrians may chose irrational paths rather than the usual ones.

The various indications that have been given above do not yet provide a precise guideline towards modeling. Rather, they can be regarded as hints to implement the mathematical structure with the appropriate models of interactions at the microscopic scale,

4. **From crowds to swarm dynamics.** Modeling of swarms is an attractive research perspective which is also motivated by the observation of the beauty of the shapes formed by birds which appear in the sky during spring and autumn periods. Analogous phenomena are, however, observed in other systems such as fishes which try to escape the attack of a predator, or cells which aggregate forming particular patterns. Generally, the expression of a strategy of the individuals forming a swarm is finalized to their fitness or even survivance.

The existing literature offers different modeling approaches. Among others, macroscopic equations derived from stochastic perturbation of individual dynamics [27], [33], modeling swarming patterns, [20], [63], and flocking phenomena [31, 57, 62]. A deep insight on emerging strategies needs to be specifically referred to the type of individuals composing the swarm [4, 5, 26]. A specific characteristic is that the swarm has the ability to express a collective intelligence, somehow related to the environmental conditions [22], which can evolve by learning processes. This dynamics is used to drive learning processes in modern technology of robots [52].

The aim of this section consists in understanding how the mathematical structures proposed in the preceding section needs to be modified to tackle the higher order complexity of swarm dynamics. Some differences are purely technical and simply need additional notations. For instance, interactions between active particles of a swarm are in three space coordinates, while those of particles of a crowd are defined over two-space coordinates.

Moreover, mathematical problems are stated in unbounded domains with initial conditions with compact support. The solution of problems should provide the evolution in time of the domain of the initial conditions. Say, if Ω_0 is the domain containing the swarm at $t = 0$, the solution of mathematical problems should compute the map $\Omega_0 \to \Omega_t$, where Ω_t contains the swarm for $t > 0$.

However, some of the specific characteristics need substantial modifications of the mathematical structure Eq. (11). Therefore, additional reasonings are proposed in the following to be viewed as research perspectives:

i) Generally, swarms refer to animal behaviors, which differ from population to population and that can be modified by external actions that can induce panic. Namely, a swarm in normal conditions has a well defined objective, for instance reaching a certain zone starting from a localization. However, panic conditions can modify the overall strategy to pursue this objective, which is consequently modified.

ii) The swarm has the ability to express a common strategy, which is a nonlinear elaboration of all individual contributions, generated by each individuals based on the microscopic state of all other individuals. Namely, this collective intelligence is generated by a cooperative strategy [22, 54]. This strategy includes a clustering ability (flocking) that prevents the fragmentation of Ω_t. Moreover, when a fragmentation of Ω_t occurs, the clustering ability induces an aggregation.

iii) Recent studies [4, 26] conjecture, on the basis of empirical data, that some systems of animal world develop a common strategy based on interactions depending on topological rather than metric distances. In general, the insight on emerging strategies needs to be specifically referred to the type of individuals composing the swarm [40], and the specific applications considered in [5, 37].

iv) The dynamics of interactions differs in the various zones of the swarm. For instance, from the border to the center of Ω_t. More precisely, the modeling of stochastic fluctuations should be inserted for individuals on the border of the domain,

v) If the concept of swarm is extended to other types of micro organisms, ultimately to cells in a multicellular system, additional difficulties have to be treated. For instance, the strategy expressed by the interacting entities depends on their specific phenotypes and related biological functions. Moreover, the modeling approach should include proliferative and/or destructive events [17].

Of course the mathematical structure presented in the preceding section cannot be straightforwardly used to model all above phenomena. The approach to deal with some of the above technical difficulties will be developed in a forthcoming research project, while some hints are here anticipated. For instance, a different conceptual approach to modeling the encounter rate and the transition probability density have to be developed. More precisely, the visibility zone has to be substituted by an interaction domain, which depends on a fixed critical number of individuals that are located into the visibility zone. Moreover, the transition density should be modeled by the overall action of the other individuals in their interaction domain. This means extending to the full space of microscopic states the ideas that were simply referred in [11] to the activity variable. Additional force fields can be included to model interactions between individuals of the swarm [25]

Finally, let is mention that the link microscopic and macroscopic in the case of swarms of microorganisms have been obtained by asymptotic analysis and moment closure. Among others [1, 8, 61]. This approach has been able to derive models, such as the celebrated Keller and Segel model [50] that have been heuristically derived to model pattern formation. This remark completes the variety of challenging open problems presented in this last section. Some results are known in the case of vehicular traffic, see [18, 3] and there in cited bibliography. This approach can be possibly extended to the case of crowd.

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