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INFINITE-DIMENSIONAL NONLINEAR PREDICTIVE CONTROL DESIGN FOR OPEN-CHANNEL HYDRAULIC SYSTEMS

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Abstract. A nonlinear predictive control design based on Saint Venant equations is presented in this paper in order to regulate both water depth and water flow rate in a single pool of an open-channel hydraulic system. Thanks to variational calculus, some necessary optimality conditions are given. The adjoint partial differential equations of Saint Venant partial differential equations are also derived. The resulting two-point boundary value problem is solved numerically by using both time and space discretization and operator approximations based on nonlinear time-implicit finite differences. The practical effectiveness of the control design is demonstrated by a simulation example. A extension of the predictive control scheme to a multi-pool system is proposed by using a decomposition-coordination approach based on two-level algorithm and the use of an augmented Lagrangian, which can take advantage of communication networks used for distributed control. This approach may be easily applied to other problems governed by hyperbolic PDEs, such as road traffic systems.

1. **Introduction.** In this paper, we consider the control of open-channel hydraulic systems. Such environmental systems are numerous: irrigation or drainage systems, dam-river systems, waste-water networks, etc ... In this paper we will focus our attention on the case of irrigation systems, but the here-proposed approach may be easily applied to other open-channel hydraulic systems. Regulation of irrigation channels has received an increasing interest over the last two decades. In Europe and in America, a lot of interconnected irrigation networks are already monitored and controlled by a distant human operator via some communication systems. Since water is becoming more and more a rare and expensive resource, the need for fully automatic regulation systems which would be able to satisfy the water demands, while guaranteeing both minimum water level and overflow avoidance in each canal, together with minimum wastes of water, is increasing. An irrigation network is usually made of a primary open-channel canal which deserves open-channel secondary canals. Canals themselves are made of several long reaches (most of the time, they are several kilometers long) separated by engineering works (like sliding gates or weirs for instance). The dynamics of open channels is characterized by important time lags (due to water transport), wave superposition effects and strong nonlinearities due to control gates or sweirs. A large number of control schemes are based on both linear modelling and linear control design: PID control [20], LQG controller [17, 15], linear predictive controller [19], linear robust controller [16] or fuzzy controller [21, 3]. Other researches have been devoted to nonlinear control design based

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on the Saint Venant partial differential equations governing the water flow dynamics. In [8], the control law is obtained from a Lyapunov function-based stability analysis in the infinite-dimensional framework. A internal model approach has been also proposed in this framework [10]. The semigroup theory has also been used for both control analysis and design [4, 22]. In [11], the design of a finite-dimensional nonlinear controller based on input-output feedback linearization has been proposed; this approach is based on the reduction of nonlinear partial differential Saint Venant equations under the form of a set of nonlinear ordinary differential equations of reduced order through the use of a collocation weighted residual method.

Our main goal in this paper is to propose a nonlinear predictive control design. This approach is a priori well suited for the control of nonlinear systems with delays. First of all we analyze the infinite-dimensional optimal control problem associated to the nonlinear predictive control problem. Some necessary optimality conditions are then derived thanks to variation calculus (see [5, 6] for an early example of this approach) and a numerical scheme is proposed in order to compute the solution of the related infinite-dimensional two-point boundary value problem. An adjoint method has been also proposed for the control of air traffic flow based on a network of conservation laws [2]. With this approach, both nonlinearities and the distributed nature of open-channel dynamics can be potentially taken into account (transport and diffusion phenomena in particular). Rather than first reducing the Saint Venant equations and then designing a finite-dimensional controller, we consider an infinite-dimensional control design which is finally approximated in order to be implementable.

The paper is now organized as follows: in section 2, some backgrounds on modelling of open-channel hydraulic systems based on the Saint Venant partial differential equations are given and a single pool control problem is described. In section 3, model predictive control is briefly recalled and an infinite-dimensional two-point boundary value problem of the related infinite-dimensional optimal control problem with receding horizon is then derived. First-order optimality conditions are provided for a class of nonlinear hyperbolic PDEs. In particular, the adjoint-state PDEs of the problem are defined. Then the description of the model predictive control scheme based on the infinite-dimensional two-point boundary value problem is provided. In section 4, some simulation results are presented which demonstrate the effectiveness of the approach. The section 5 is devoted to the extension of the predictive control scheme for multi-pool systems by using a distributed control scheme based on decomposition-coordination and the use of an augmented Lagrangian. Finally, the last section is devoted to some conclusions and perspectives.

2. **Open-channel hydraulic system modelling.**

2.1. **Saint Venant equations.**

The dynamics of open-channel hydraulic systems is governed by a set of two nonlinear hyperbolic partial differential equations known as the Saint Venant equations:

$$S: \quad \left\{ \begin{array}{l} \dfrac{\partial A}{\partial t} + \dfrac{\partial Q}{\partial x} = q \\ \dfrac{\partial Q}{\partial t} + \dfrac{\partial}{\partial x}(\dfrac{Q^2}{A}) + gA\dfrac{\partial z}{\partial x} - gA(I-J) = kq\dfrac{Q}{A} \end{array} \right. \tag{1}$$

where z is the water depth(m), Q is the water flow rate (discharge) $(\frac{m^3}{s})$, A is the wet area (m^2), I is the canal slope, g is the acceleration of gravity $(\frac{m}{s^2})$; J is the friction term ($\frac{m}{m}$) ; $q(x)$ is the withdrawal per length unit ($\frac{m^2}{s}$) ; $k = 0$, if $q > 0$, $k = 1$, if $q < 0$. The friction term J may be given by Manning-Strickler's formula (some other friction models exist):

$$J = \frac{Q^2}{K^2 A^2 R^{\frac{4}{3}}} \tag{2}$$

where R is the wet perimeter and K is Manning-Strickler's coefficient. In what follows, we will only consider that J is a function of both z and Q.

Without restriction and for simplification purpose, we will also consider that the section profile of the canal is rectangular: $A = Bz$, where B is the canal width, and $q(x) = 0$. It follows that:

$$S: \quad \begin{cases} \dfrac{B\partial z}{\partial t} + \dfrac{\partial Q}{\partial x} = 0 \\ \dfrac{\partial Q}{\partial t} + \dfrac{\partial}{\partial x}(\dfrac{Q^2}{Bz}) + gBz\dfrac{\partial z}{\partial x} - gBz(I - J) = 0 \end{cases} \tag{3}$$

In addition two boundary conditions and some initial state conditions have to be defined.

2.2. **The case study.** The regulation of a single pool irrigation system is considered. The problem is to regulate the pool around a equilibrium state by controlling the upstream flow gate. The downstream flow gate is acting on the pool as a disturbance source.

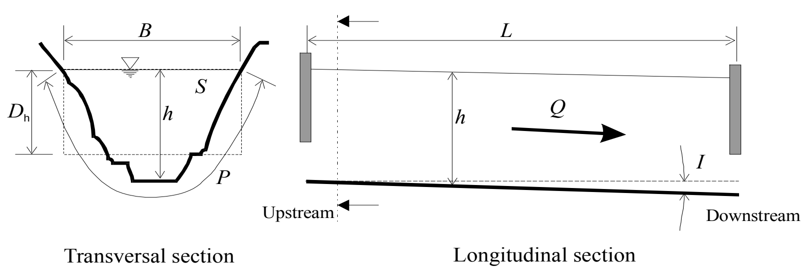

FIGURE 1. A single pool canal

Through the flow gates, which are dissipative elements, water levels and flow are linked by an expression of the form:

$$G(Q(t), z_{us}(t), z_{ds}(t), u(t)) = 0, \tag{4}$$

where z_{us} is the upstream water level at the gate and z_{ds} is the downstream water level at the same gate and u is the gate opening (control input).

In the case of underflow gates, we get the following model:

$$Q^2(t) = K_g^2 u^2 \times 2g(z_{us} - z_{ds}) \tag{5}$$

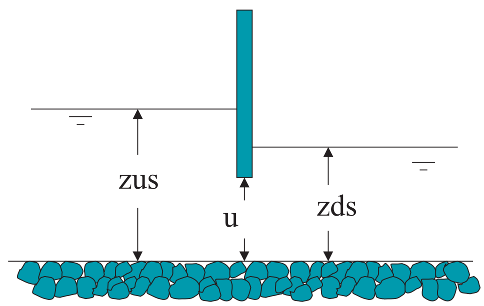

FIGURE 2. Control gate scheme

where K_g is the so-called gate coefficient.

In the present case where two gates are present, the following boundary conditions have been introduced:

$$B.C. \left\{ \begin{array}{l} G_{us}(Q(x=0,t), z_{us}(t), z(x=0,t), u(t)) = 0, \\ G_{ds}(Q(x=L,t), z(x=L,t), z_{ds}(t), v(t)) = 0, \end{array} \right. \tag{6}$$

where z_{us} is now the upstream water level at the upstream gate and z_{ds} is now the downstream water level at the downstream gate.

The control approach will be based on a boundary control of the time derivative of the upstream gate opening u, while the opening v of the downstream gate acts as a disturbance. It is of special interest to introduce integrator at the control input since in this case constant disturbance rejection will be guaranteed.

3. A nonlinear infinite-dimensional predictive control scheme.

3.1. Some backgrounds on predictive control.

A general predictive control scheme can be defined as follows [12]:

Let us consider a nonlinear system of the general form:

$$\dot{x} = F(x, u),\ x(0) = x_0 \tag{7}$$

with $F(0, 0) = 0$ (the origin is an equilibrium point).

1. At time t, obtain the current state $x(t) = x_t$ (through direct measurements or by mean of a state observer).
2. Compute the optimal solution of an optimal control problem (defined on a control horizon T by using the available state x_t and called in what follows "related optimal control problem"):

$$\min_{u(.)} \int_t^{t+T} L(x(\tau), u(\tau)) d\tau \tag{8}$$

$$s.t.\ \dot{x} = F(x, u),\ x(t) = x_t \tag{9}$$

$$and\ x(t+T) \in E \tag{10}$$

where E is a region of the state space in which the state $x(t+T)$ is assigned.
In practice, a numerical scheme has to be introduced through the discretization of the time interval $[t, t+T]$ into $N+1$ values $t+i\Delta t$, $i=0,...,N$. The optimal solution is to be computed as $N+1$ piecewise constant control inputs of the form $\{u(t), u(t+\Delta t), u(t+2\Delta t), ..., u(t+N\Delta t)\}$.

3. Apply the first control input $u(t)$ of the sequence $\{u(t), u(t+\Delta t), u(t+2\Delta t), ..., u(t+N\Delta t)\}$.
4. $t+\Delta \rightarrow t$ and go back to 1).

Remark: The constraint on the final state (10) reduces in many cases to $x(t+T)=0$. This "hard" constraint offers the advantage of guaranteeing the closed-loop stability under some mild assumptions and provided that a global optimal solution is available for each receding horizon optimal control problem. In this case, it can be shown that the optimal cost function of the predictive controller with this terminal constraint is a decreasing Lyapunov function along the trajectory of the closed-loop system [18, 12]. *The main idea here is to use the same approach in the case of a infinite-dimensional regulation problem of the Saint Venant equation.* In this paper we will only demonstrate the practical effectiveness of infinite-dimensional model predictive control applied to open-channel hydraulic systems. Derivation of a formal proof of stability remains an open problem.

3.2. **Formulation of the related optimal control problem.** A change of time coordinates is (temporarily) made: $t \rightarrow 0$ et $t+T \rightarrow T$ and the following optimal control problem is introduced:

$$\min_u \int_0^T m(u)dt + \int_0^T \int_0^L l(z(x,t), Q(x,t))dxdt \tag{11}$$

where $m(.)$ is positive definite and $l(.,.)$ is positive semidefinite, subject to the dynamics

$$S: \quad \begin{cases} B\dfrac{\partial z}{\partial t} + \dfrac{\partial Q}{\partial x} = 0 \\ \dfrac{\partial Q}{\partial t} + \dfrac{\partial}{\partial x}(\dfrac{Q^2}{Bz}) + gBz\dfrac{\partial z}{\partial x} - gBz(I-J) = 0 \\ \dot{\mu} = u \end{cases} \tag{12}$$

with the initial conditions

$$I.C. \begin{cases} Q(x,0) = \phi_1(x),\ x \in [0, L] \\ z(x,0) = \phi_2(x) \\ \mu(0) = \phi_3(0) \end{cases} \tag{13}$$

the boundary conditions

$$B.C. \begin{cases} G_{us}(Q(0,t), z_{us}(t), z(0,t), \mu(t)) = 0, \\ G_{ds}(Q(L,t), z(L,t), z_{ds}(t), v(t)) = 0, \end{cases} \tag{14}$$

and the terminal constraint:

$$\{Q(x,T), z(x,T)\} \in E,\ x \in [0, L] \tag{15}$$

The cost function l has now to be determined in order to comply with the control objective:

In this paper, **a regulation problem around an equilibrium state is considered:** $m(u) = \frac{r}{2}u(t)^2$, $r > 0$ and $l(z, Q) = \frac{1}{2}[q_1(z(x,t) - z_0(x))^2 + q_2(Q(x,t) -$

$Q_0(x))^2]$, $q_1, q_2 > 0$, with $z_0(x)$, $Q_0(x)$, $x \in [0, L]$ corresponding to a reference equilibrium state defined as the solution of the equilibrium PDEs:

$$S_{eq}: \quad \begin{cases} \dfrac{\partial Q}{\partial x} = 0, \\ \dfrac{\partial}{\partial x}(\dfrac{Q^2}{Bz}) + gBz\dfrac{\partial z}{\partial x} - gBz(I - J) = 0, \end{cases} \tag{16}$$

with $v_0, \mu_0, z_{us0}, z_{ds0}$ satisfying the boundary condition:

$$B.C. \begin{cases} G_{us}(Q_0(0), z_{us0}, z_0(0), \mu_0) = 0, \\ G_{ds}(Q_0(L), z_0(L), z_{ds0}, v_0) = 0, \end{cases} \tag{17}$$

and with the terminal constraint

$$T.C. \begin{cases} Q(x, T) = Q_0(x), \ x \in [0, L], \\ z(x, T) = z_0(x). \end{cases} \tag{18}$$

3.3. **Derivation of the first-order optimality conditions for nonlinear hyperbolic PDEs.** In this section, we will consider the formal derivation of the first-order optimality conditions for nonlinear hyperbolic PDEs. We will not consider the problem of existence of solutions to such optimal control problems. We only adopt the convenient differentiability hypotheses on the functions to give a sense to the optimality conditions.

Consider the following nonlinear hyperbolic PDEs + integrator described by

$$\begin{cases} \dfrac{\partial \xi}{\partial t} = \dfrac{\partial f}{\partial x}(\xi) + h(\xi) \\ L_b(\mu(t), \xi(0, t), \xi(L, t)) = 0, \\ \dot{\mu} = u, \end{cases} \tag{19}$$

where $\xi(x, t)$ is the state which is defined on the domain $\Omega = [0, L]$ with the boundary $\Gamma = \{0, L\}$. L_b denotes the boundary conditions. $u(t)$ is the integral boundary control to be determined (u is a 2-dimensional vector if we consider that the two boundaries are controlled).

We intend to find the optimal control $u(t)^*$, which minimizes the following cost integral:

$$J = \int_0^T \psi(u(t))dt + \int_0^T \int_0^L l(\xi(x, t))dxdt \tag{20}$$

where $t \in [0, T]$, $x \in [0, L]$, T and L are fixed. The terminal state $\xi(x, t = T)$ is imposed. $\psi(u(t))$ is the input control cost function, which is supposed to be positive definite. $l(\xi(x, t))$ is the state cost function, which is supposed to be positive semidefinite.

In order to derive the first-order necessary conditions for optimality, we introduce the following Lagrangian formulation:

$$\begin{aligned} L_f = \int_0^T \int_0^L [l(\xi(x, t)) + \lambda^T (\frac{\partial f}{\partial x}(\xi(x, t)) + h(\xi(x, t)) - \frac{\partial \xi(x, t)}{\partial t})]dxdt \\ + \int_0^L \phi(\xi(x, T))dx + \int_0^T \Psi(u(t), \mu(t), \xi(0, t), \xi(L, t))dt \end{aligned} \tag{21}$$

where λ is the vector of Lagrange multipliers associated to the PDEs (the adjoint state), and we denote

$$\Psi(u(t), \mu(t), \xi(0, t), \xi(L, t)) = \psi(u(t)) + \gamma^T L_b(\mu(t), \xi(0, t), \xi(L, t)) + \lambda_\mu^T (u - \dot{\mu}), \tag{22}$$

where γ is the vector of Lagrange multipliers associated to the boundary conditions $L_b(\xi,t)$ and λ_μ is the adjoint state of $\dot{\mu}=u$.

First of all we apply the Green-Riemann formula to the following double integral

$$\begin{aligned}
& \int_0^T \int_0^L [l+\lambda^T(\frac{\partial f}{\partial x}(\xi)+h(\xi)-\frac{\partial \xi}{\partial t})]dxdt \\
= & \int_0^T \int_0^L [l+\lambda^T h(\xi)+\lambda^T(\frac{\partial f}{\partial x}(\xi)-\frac{\partial \xi}{\partial t})]dxdt \\
= & \int_0^T \int_0^L [l+\lambda^T h(\xi)+(\frac{\partial \lambda^T f(\xi)}{\partial x}-\frac{\partial \lambda^T \xi}{\partial t}) \\
- & ((\frac{\partial \lambda}{\partial x})^T f(\xi)-(\frac{\partial \lambda}{\partial t})^T \xi)]dxdt \\
= & \int_0^T \int_0^L [l+\lambda^T h(\xi)-((\frac{\partial \lambda}{\partial x})^T f(\xi)-(\frac{\partial \lambda}{\partial t})^T \xi)]dxdt \\
+ & \int_0^L [\lambda^T \xi]_0^T dx+\int_0^T [\lambda^T f(\xi)]_0^L dt. \qquad (23)
\end{aligned}$$

Then we introduce the following functional:

$$H(\xi,\lambda)=l(\xi)+\lambda^T h(\xi)-(\frac{\partial \lambda}{\partial x})^T f(\xi) \qquad (24)$$

and by replacing H in (21), we get

$$\begin{aligned}
L_f(\xi,\lambda,\gamma,u,\mu) \quad = & \int_0^T \Psi(u,\mu,\xi(0,t),\xi(L,t))dt \\
+ & \int_0^L [\lambda^T \xi]_0^T dx+\int_0^T [\lambda^T f(\xi)]_0^L dt+\int_0^T \int_0^L [H+(\frac{\partial \lambda}{\partial t})^T \xi]dxdt
\end{aligned} \qquad (25)$$

The first variation of L_f is given by

$$\begin{aligned}
\delta L_f \quad = & \; L_f(\xi+\delta\xi,\mu+\delta\mu,u+\delta u)-L_f(\xi,\mu,u) \\
= & \int_0^L [\lambda^T \delta\xi]_0^T dx+\int_0^T [\lambda^T \frac{\partial f}{\partial \xi}(\xi)\delta\xi]_0^L dt+\int_0^T \frac{\partial \Psi}{\partial \xi(0,t)}\delta\xi(0,t)dt \\
+ & \int_0^T \frac{\partial \Psi}{\partial \xi(L,t)}\delta\xi(L,t)dt+\int_0^T [\frac{\partial \Psi}{\partial u}^T+\lambda_\mu]^T \delta u dt+\int_0^T [\frac{\partial \Psi}{\partial \mu}^T+\dot{\lambda}_\mu]^T \delta\mu dt \\
+ & \int_0^T \int_0^L [\frac{\partial H}{\partial \xi}^T+\frac{\partial \lambda}{\partial t}]^T \delta\xi dxdt-[\lambda_\mu^T \delta\mu]_0^T. \qquad (26)
\end{aligned}$$

Since the first order variation of L_f must be equal to zero, we impose that

$$\frac{\partial \psi}{\partial u}^T+\lambda_\mu=0. \qquad (27)$$

We get also the following adjoint equations:

$$\frac{\partial \lambda}{\partial t}+\frac{\partial l}{\partial \xi}^T+\frac{\partial h}{\partial \xi}^T(\xi)\lambda-\frac{\partial f}{\partial \xi}^T(\xi)\frac{\partial \lambda}{\partial x}=0, \qquad (28)$$

$$\dot{\lambda}_\mu+\frac{\partial L_b}{\partial \mu}^T\gamma=0. \qquad (29)$$

The remaining terms, which have to be canceled, provide the boundary and terminal time conditions as well as the vector of multipliers γ, as a function of the adjoint state defined at the boundaries:

$$-\frac{\partial f(\xi(0,t))}{\partial \xi}^T \lambda(0,t) + \frac{\partial L_b}{\partial \xi(0,t)}^T (\xi(0,t))\gamma = 0, \quad (30)$$

$$\frac{\partial f(\xi(L,t))}{\partial \xi}^T \lambda(L,t) + \frac{\partial L_b}{\partial \xi(L,t)}^T (\xi(L,t))\gamma = 0, \quad (31)$$

$$\lambda(x,0), \lambda(x,T),\ free, \quad (32)$$

$$\lambda_\mu(T) = 0. \quad (33)$$

3.4. **First-order necessary conditions of the related optimal control problem.** The adjoint method is now applied to the related optimal control problem, which has been previously defined.

Proposition 1. Necessary conditions for optimality. *In order the boundary control input $u(t) \in [0,T]$ and the related trajectory, solution of the problem:*

$$\min_{u(.)} \int_0^T \frac{r}{2} u(t)^2 dt + \int_0^L \int_0^T \frac{1}{2}[q_1(z(x,t) - z_0(x))^2 + q_2(Q(x,t) - Q_0(x))^2] dxdt, \quad (34)$$

subject to

$$S: \quad \begin{cases} B\frac{\partial z}{\partial t} + \frac{\partial Q}{\partial x} = 0, \\ \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}(\frac{Q^2}{Bz}) + gBz\frac{\partial z}{\partial x} - gBz(I-J) = 0, \\ \dot{\mu} = u, \end{cases} \quad (35)$$

with the initial conditions

$$I.C. \begin{cases} Q(x,0) = \phi_1(x),\ x \in [0,L] \\ z(x,0) = \phi_2(x) \\ \mu(0) = \phi_3(0) \end{cases} \quad (36)$$

the boundary conditions

$$B.C. \begin{cases} G_{us}(Q(0,t), z_{us}(t), z(0,t), \mu(t)) = 0, \\ G_{ds}(Q(L,t), z(L,t), z_{ds}(t), v(t)) = 0, \end{cases} \quad (37)$$

and the terminal constraint:

$$T.C. \begin{cases} Q(x,T) = Q_0(x),\ x \in [0,L], \\ z(x,T) = z_0(x). \end{cases} \quad (38)$$

are optimal, it is necessary that there exists an adjoint vector $\lambda = (\lambda_1(x,t), \lambda_2(x,t), \lambda_\mu)$, $\forall t \in [0,T]$, $\forall x \in [0,L]$, solution of the adjoint equations:

$$S_{adj}: \quad \begin{cases} \frac{\partial \lambda_1}{\partial t} + q_1(z - z_0) - \frac{\partial \lambda_2}{\partial x}(\frac{Q^2}{Bz^2} - gBz) + \lambda_2(gB(I-J) - gBz\frac{\partial J}{\partial z}) = 0, \\ \frac{\partial \lambda_2}{\partial t} + q_2(Q - Q_0) + \frac{1}{B}\frac{\partial \lambda_1}{\partial x} + 2\frac{\partial \lambda_2}{\partial x}\frac{Q}{Bz} - \lambda_2(gBz\frac{\partial J}{\partial Q}) = 0, \\ \dot{\lambda}_\mu + \frac{\partial G_{us}}{\partial \mu}(\frac{\partial G_{us}}{\partial z(0,t)})^{-1}(\frac{Q^2}{Bz^2} - gBz)\lambda_2(0,t) = 0, \end{cases} \quad (39)$$

with the boundary conditions

$$B.C. \begin{cases} \dfrac{1}{B}\lambda_1(0,t) + (\dfrac{2Q}{Bz} + \dfrac{\partial G_{us}}{\partial Q(0,t)}(\dfrac{\partial G_{us}}{\partial z(0,t)})^{-1}(\dfrac{Q^2}{Bz^2} - gBz))\lambda_2(0,t) = 0, \\ \\ \dfrac{1}{B}\lambda_1(L,t) + (\dfrac{2Q}{Bz} + \dfrac{\partial G_{ds}}{\partial Q(L,t)}(\dfrac{\partial G_{ds}}{\partial z(L,t)})^{-1}(\dfrac{Q^2}{Bz^2} - gBz))\lambda_2(L,t) = 0, \end{cases} \tag{40}$$

and with the additional transversality conditions

$$Tr.C. \begin{cases} \lambda_1(x,0), \lambda_1(x,T),\ free,\ x \in [0,L] \\ \lambda_2(x,0), \lambda_2(x,T),\ free \\ \lambda_\mu(T) = 0. \end{cases} \tag{41}$$

For all $t \in [0,T]$, the optimal control, if it exists, is given by $u = u^*$:

$$u^*(t) = -\frac{\lambda_\mu(t)}{r}.$$

Proof. Immediate by using the previous derivations and the following definitions $\xi = (z,Q)^T$,

$$h(\xi) = \begin{pmatrix} 0 \\ gBz(I - J(z,Q)) \end{pmatrix}, \tag{42}$$

$$f(\xi) = \begin{pmatrix} -\dfrac{Q}{B} \\ -\dfrac{Q^2}{Bz} - \dfrac{1}{2}gBz^2 \end{pmatrix}, \tag{43}$$

$$\psi(u) = \frac{r}{2}u^2, \tag{44}$$

and

$$l(\xi) = \frac{1}{2}[q_1(z(x,t) - z_0(x))^2 + q_2(Q(x,t) - Q_0(x))^2]. \tag{45}$$

□

Finally a two-point boundary value problem is obtained. One can emphasize the fact that regularity conditions are needed to ensure existence of an optimal solution to this problem, since both $(\frac{\partial G_{us}}{\partial z(0,t)})^{-1}$ and $(\frac{\partial G_{ds}}{\partial z(L,t)})^{-1}$ must be defined.

The here-proposed computational method differs from the one proposed in [5]) where a gradient method were used to compute a solution, since a method based on both time and space discretization of the canonical equations is derived.

3.5. **Computation of the two-point boundary value problem.** The computation is defined in two stages:

1. Perform both space and time discretization of the canonical equations by using the Preissman numerical scheme: applied to both S and S_{adj};
2. Compute the solution of the nonlinear algebraic equations derived from a two-dimensional grid via a Newton-Raphson method.

The Preissmann scheme [9] is based on the following approximation of functions f and their derivatives:

$$\begin{aligned} f(x,t) &= \frac{1-\theta}{2}[f_{i+1}+f_i]+\frac{\theta}{2}[f^+_{i+1}+f^+_i] \\ \frac{\partial f}{\partial x}(x,t) &= \frac{1-\theta}{\Delta x}[f_{i+1}-f_i]+\frac{\theta}{\Delta x}[f^+_{i+1}-f^+_i] \\ \frac{\partial f}{\partial t}(x,t) &= \frac{1}{2\Delta t}[f^+_i-f_i+f^+_{i+1}-f_{i+1}] \end{aligned} \tag{46}$$

where i is the space index, $+$ corresponds to $t+\Delta t$ and $0 \leq \theta \leq 1$ is a relaxation coefficient.

If $\theta \geq 0,5$, we get an unconditionally stable integration scheme.

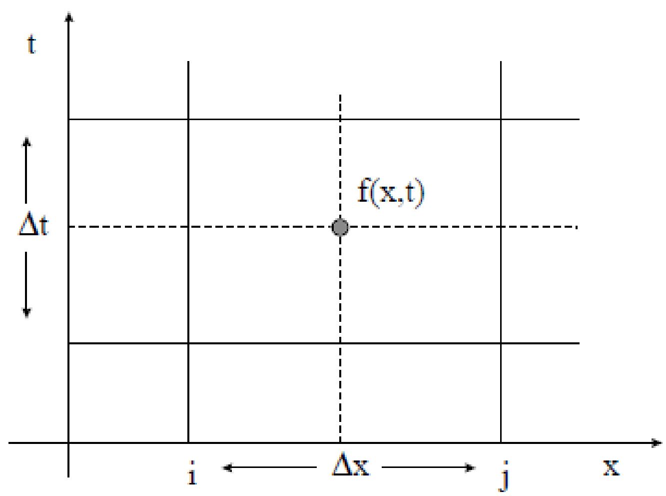

FIGURE 3. The Preissmann scheme

3.6. **The overall model predictive control scheme.** The overall control scheme can be now defined as follows:

1. At each sampling time t: Get the current state $z_t(x) = z(x,t)$, $Q_t(x) = Q(x,t)$ and compute the two-point boundary value problem defined by

$$\begin{aligned} &B\frac{\partial z}{\partial t}+\frac{\partial Q}{\partial x}=0, \\ &\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}(\frac{Q^2}{Bz})+gBz\frac{\partial z}{\partial x}-gBz(I-J)=0, \end{aligned} \tag{47}$$

$$\begin{aligned} &\dot{\mu}=-\frac{\lambda_\mu(t)}{r}, \\ &\frac{\partial \lambda_1}{\partial t}+q_1(z-z_0)-\frac{\partial \lambda_2}{\partial x}(\frac{Q^2}{Bz^2}-gBz)+\lambda_2(gB(I-J)-gBz\frac{\partial J}{\partial z})=0, \\ &\frac{\partial \lambda_2}{\partial t}+q_2(Q-Q_0)+\frac{1}{B}\frac{\partial \lambda_1}{\partial x}+2\frac{\partial \lambda_2}{\partial x}\frac{Q}{Bz}-\lambda_2(gBz\frac{\partial J}{\partial Q})=0, \\ &\dot{\lambda}_\mu+\frac{\partial G_{us}}{\partial \mu}(\frac{\partial G_{us}}{\partial z(0,t)})^{-1}(\frac{Q^2}{Bz^2}-gBz))\lambda_2(0,t)=0, \end{aligned} \tag{48}$$

with the boundary conditions, defined $\forall \tau \in [t, t+T]$, by

$$B.C. \begin{cases} G_{us}(Q(0,\tau), \hat{z}_{us}(\tau), z(0,\tau), \mu(\tau)) = 0, \\ G_{ds}(Q(L,\tau), z(L,\tau), \hat{z}_{ds}(\tau), \hat{v}(\tau)) = 0, \\ \frac{1}{B}\lambda_1(0,\tau) + (\frac{2Q}{Bz} + \frac{\partial G_{us}}{\partial Q(0,\tau)}(\frac{\partial G_{us}}{\partial z(0,\tau)})^{-1}(\frac{Q^2}{Bz^2} - gBz))\lambda_2(0,\tau) = 0, \\ \frac{1}{B}\lambda_1(L,\tau) + (\frac{2Q}{Bz} + \frac{\partial G_{ds}}{\partial Q(L,\tau)}(\frac{\partial G_{ds}}{\partial z(L,\tau)})^{-1}(\frac{Q^2}{Bz^2} - gBz))\lambda_2(L,\tau) = 0, \end{cases} \tag{49}$$

where "$\hat{.}$" denotes a predicted value of the variable, and with the transversality conditions

$$Tr.C. \begin{cases} Q(x,t) = Q_t(x), \; x \in [0,L] \\ z(x,t) = z_t(x), \\ \mu(t) = \phi_3(0), \\ Q(x,t+T) = Q_0(x), \; x \in [0,L], \\ z(x,t+T) = z_0(x), \\ \lambda_1(x,t), \lambda_1(x,t+T), \; free, \; x \in [0,L] \\ \lambda_2(x,t), \lambda_2(x,t+T), \; free \\ \lambda_\mu(t+T) = 0. \end{cases} \tag{50}$$

2. Apply $u(t)$ defined as the first optimal control input sampling of the sequence computed on $[t, t+T]$: the system reaches state $(z(x, t+\Delta t), Q(x, t+\Delta t))$, $\forall x \in [0, L]$.
3. $t + \Delta t \to t$ and go to 1).

4. **A simulation example.** In order to illustrate the effectiveness of this infinite-dimensional control scheme, we consider the following example:

- The simulation is based on Preissmann's numerical scheme;
- A 5 km long canal divided into 10 sections of 500 m each is considered;
- The problem consists in controlling the pool around a uniform equilibrium state corresponding to a constant relative water level z_0 of 1.05 m, along the pool;
- The initial condition is a uniform equilibrium of 1 m;
- At each time sampling Δt, we consider $N = 11$ spatial samplings and $M = 6$ time samplings, with $\Delta t = 100s$: we have to solve a problem defined by $4N \times M + 2M = 288$ equations for $4N \times M + 2M = 288$ unknown variables.
- The computation time were < 60 s on a Pentium 1,8 Mhz, 512 Mo Laptop: this means *real-time control is possible.*

The figures 4 et 5 show the dynamics of the different states of the pool corresponding to the values of both water flow and depth at each space discretization point of the Preissmann scheme. From the left to the right of the figure 4, we get the system behavior from the downstream end to the upstream end.

5. **The multi-pool case: A decomposition-coordination scheme.** Now we consider an extension to the case of a N pools cascaded via $N+1$ regulation gates. Each pool i is defined on a domain $\Omega_i = [L_i^+, L_{i+1}^-]$.

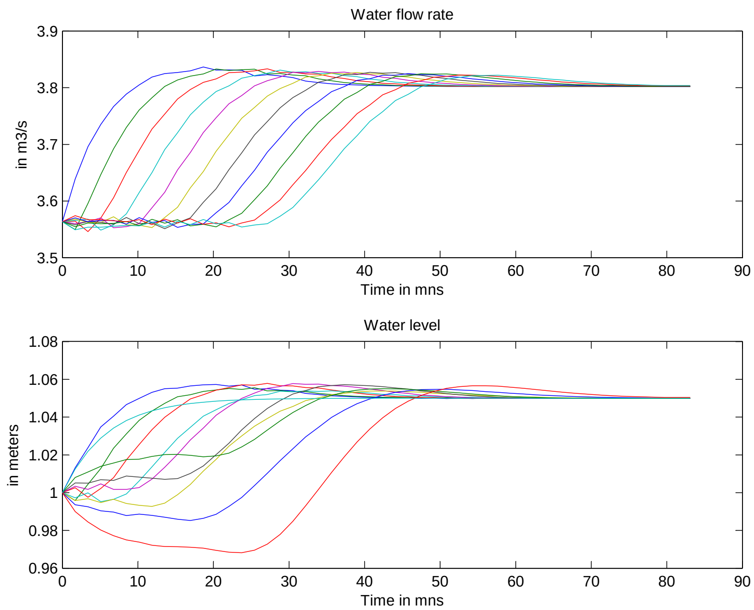

FIGURE 4. Closed-loop response

According to the notations introduced by the figure 6, the multi-pool system can be modeled by the following set of conservation laws:

$$S_{pool_i}, i = 1, ..., N \quad \left\{ \begin{array}{l} B\dfrac{\partial z_i}{\partial t} + \dfrac{\partial Q_i}{\partial x} = 0, \\ \dfrac{\partial Q_i}{\partial t} + \dfrac{\partial}{\partial x}(\dfrac{Q_i^2}{Bz_i}) + gBz_i\dfrac{\partial z_i}{\partial x} - gBz_i(I - J(Q_i, z_i)) = 0, \end{array} \right. \tag{51}$$

with the boundary conditions (based on the regulator gate models)

$$(B.C.)_i, i = 1, ..., N \left\{ \begin{array}{l} G_i(Q_i(L_i^+, t), z_{i-1}(L_i^-, t), z_i(L_i^+, t), \mu_i(t)) = 0, \\ G_{i+1}(Q_i(L_{i+1}^-, t), z_i(L_{i+1}^-, t), z_{i+1}(L_{i+1}^+, t), \mu_{i+1}(t)) = 0, \end{array} \right. \tag{52}$$

and the integrator associated to each gate control

$$\dot{\mu}_i = u_i, \ i = 1, ..., N + 1. \tag{53}$$

We give sense to the variables $z_0(L_1^-, t)$ and z_{N+1}^+ which will correspond to the water level at the upstream end and the downstream end of the multi-pool system, respectively.

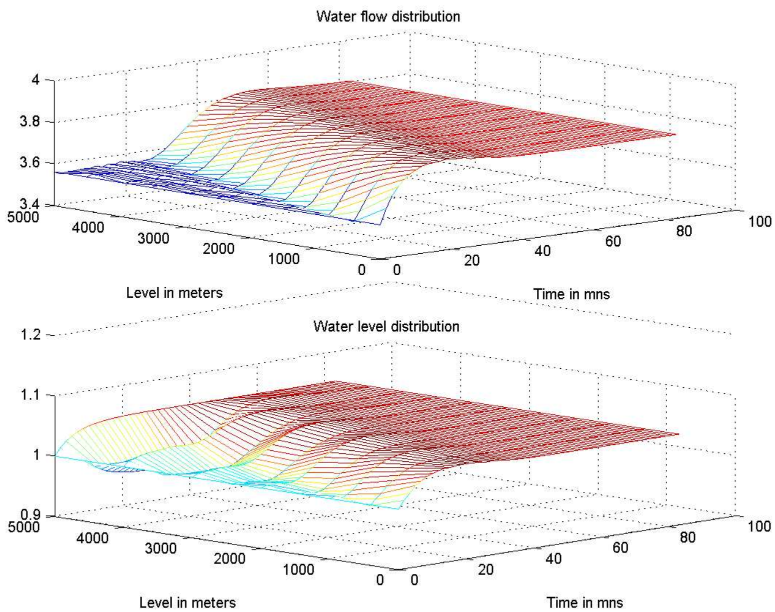

FIGURE 5. 3D plot of the closed-loop response

We intend to design a model predictive control scheme based on the receding horizon $[t, t+T]$ defined by

$$\min_u \sum_{i=1}^{N+1} \int_0^T m_i(u_i)dt + \sum_{i=1}^{N} \int_0^T \int_{L_i^+}^{L_{i+1}^-} l_i(z_i(x,t), Q_i(x,t))dxdt \tag{54}$$

s.t. S_{pool_i} (51) + (52)+ (53).

The first-order optimality conditions can be derived by using the adjoint method. However the computation of a two-point boundary value problem for such a large system becomes a very complex task. In this paper, two main issues are faced:

- How to reduce the computational complexity?
- How to take advantage of distributed control architecture (supervisory control and data acquisition: SCADA) used in large-scale water distribution systems?

Here we propose to use of a decomposition-coordination algorithm based on Lagrangian relaxation. To that purpose, we introduce the following augmented Lagrangian formulation where the regulator gate model G_i (52) are the constraints

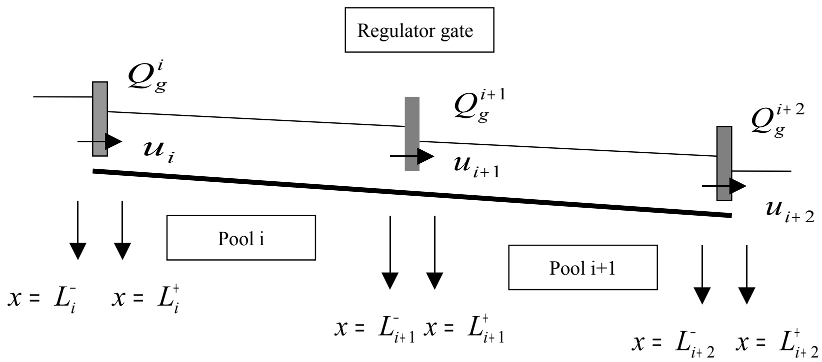

FIGURE 6. Two interconnected pools in a multi-pool system

introduced in the Lagrangian functional

$$
\begin{aligned}
L_c(u, Q_g, p) = \min_{u, Q_g} \sum_{i=1}^{N+1} \int_0^T m_i(u_i)dt + \sum_{i=1}^{N} \int_0^T \int_{L_i^+}^{L_{i+1}^-} l_i(z_i(x,t), Q_i(x,t))dxdt \\
+ \sum_{i=1}^{N+1} \int_0^T (p_i + \frac{c}{2} G_i(Q_g^i(t), z_{i-1}(L_i^-, t), z_i(L_i^+, t), \mu_i(t)))^T \\
\times G_i(Q_g^i(t), z_{i-1}(L_i^-, t), z_i(L_i^+, t), \mu_i(t))
\end{aligned} \tag{55}
$$

s.t. S_{pool_i}, $i = 1, ..., N$ (51), with the new boundary conditions

$$
\begin{aligned}
Q_i(L_i^+, t) = Q_g^i(t), \\
Q_{i+1}(L_{i+1}^-, t) = Q_g^{i+1}(t),
\end{aligned} \tag{56}
$$

and (53), where $c > 0$, Q_g is the vector of the Q_g^i's, u is the vector of the u_i's, p denotes the vector of the Lagrange multipliers associated to the regulator gate models G_i, $i = 1, ..., N+1$ 52.

Notice that some "slack" variables Q_g^i corresponding to the water flow rate at each regulator gate i have been introduced.

An augmented Lagrangian differs from an ordinary Lagrangian by an additional term proportional to the square of the norm of the constraints. The motivation for using an augmented Lagrangian is found in the fact that *for nonconvex problems and for $c > 0$ large enough, there exists at least one saddle-point of L_c under essentially the assumption that the second-order Kuhn-Tucker conditions hold for the constrained optimization (see* [7] *for example).* As a consequence, the convergence of dual algorithms (such as the Uzawa algorithm) is ensured (absence of duality gaps).

5.1. **A decomposition-coordination algorithm for the related optimal control problem solution.** In order to motivate the decomposition-coordination scheme proposed in this paper, we consider the following optimization problem:

$$\min_{u_i} \sum_{i=1}^{N} J_i(u_i) \tag{57}$$

$$s.t.\ \theta(u) = 0 \tag{58}$$

with $u = (u_1, ..., u_N)$.

The augmented Lagrangian associated to the problem is given by

$$L_c(u,p) = \sum_{i=1}^{N} J_i(u_i) + < p, \theta(u) > + \frac{c}{2}\|\theta(u)\|^2 \tag{59}$$

where $< ., >$ denotes the inner product and $\|.\|$ is the associated norm.

Computation of a saddle-point of L_c may be obtained thanks to the classical Uzawa algorithm (that is based on a gradient ascent method for maximizing the dual functional $w(p) = \min_u L_c(u,p)$):

1. At iteration $k = 0$: Choose p^0
2. At (dual) iteration k: Solve $\min_u J(u) + < p^k, \theta(u) > + \frac{c}{2}\|\theta(u)\|^2 \Rightarrow u^{k+1}$
3. $p^{k+1} = p^k + \rho\theta(u^{k+1})$
4. if $\|p^{k+1} - p^k\| < \alpha$, sufficiently small: stop, otherwise $k+1 \to k$ and go to 2).

By using linearization of the square of the constraint norm, Cohen (in [7], pp. 234-236) has proposed an algorithm which is closely related to Uzawa algorithm. This algorithm offers the major advantage of allowing decomposition of stage 2) of Uzawa algorithm into N independent subproblems, *which can be solved in parallel*:

1. At iteration $k = 0$: Choose p^0 and u_i^0, $i = 1, ..., N$
2. At iteration k: Solve each primal subproblem i, $i = 1, ..., N$: $\min_{u_i} J_i(u_i) + \|u_i - u_i^k\|^2/2\epsilon + < p^k + c\theta(u^k), \theta_i'(u^k).u_i > \Rightarrow u_i^{k+1}$
3. $p^{k+1} = p^k + \rho\theta(u^{k+1})$
4. if $\|p^{k+1} - p^k\| < \alpha$, sufficiently small: stop, otherwise $k+1 \to k$ and go to 2).

where θ_i' denotes the Jacobian matrix of θ with respect to u_i.

Convergence of this algorithm is obtained for convex problems with $0 < \epsilon < 1/c\tau^2$ and $0 < \rho < 2c$, where τ is the Lipschitz constant of θ and some mild additional conditions (such as constraint qualification condition and simple convexity).

This algorithm is a so-called two-level algorithm: Level 1 is devoted to the solution of N independent subproblems (decomposition level), while level 2 is a coordination level whose goal is to compute the Lagrangian multipliers of the constraint $\theta(u) = 0$. This feature can be exploited in the context of networked control systems where N optimization agents can be coordinated by a simple coordination agent whose main task is to evaluate the interconnection constraints linking each sub-system i related to each agent i to update the Lagrangian multipliers and to broadcast the update of the Lagrangian multipliers to each optimization agent. One can also imagine that the coordination task is carried out by each agent i by computing a part $p_i^{k+1} = p_i^k + \rho\theta_i(u^{k+1})$ of the coordinator task.

Following this approach, we get the following algorithm for solving the related optimal control problem of the predictive control scheme:

1. At iteration k: for all $i = 1, ..., N$, **solve a two-point boundary value problem for the pool i (which is denoted $TBVP_i$ and can be associated to the control system of each gate i, $i = 1, ..., N$):**

$$\min_{u_i, Q_g^i} \int_0^T [m_i(u_i) + \frac{1}{2\epsilon}(u_i - u_i^k)^2 + \frac{1}{2\epsilon}(Q_g^i - {Q_g^i}^k)^2]dt$$
$$+ \int_0^T \int_{L_i^+}^{L_{i+1}^-} l_i(z_i(x,t), Q_i(x,t))dxdt$$
$$+ \int_0^T [p_i^k + cG_i({Q_g^i}^k, {z_{i-1}^-}^k, {z_i^+}^k, \mu_i^k]^T [\frac{\partial G_i}{\partial Q_k^i}^k Q_g^i + \frac{\partial G_i}{\partial z_i^+}^k z_i^+ \frac{\partial G_i}{\partial \mu_i}^k \mu_i]dt$$
$$+ \int_0^T [p_{i+1}^k + cG_{i+1}({Q_g^{i+1}}^k, {z_i^-}^k, {z_{i+1}^+}^k, \mu_{i+1}^k]^T \frac{\partial G_{i+1}}{\partial z_i^-}^k z_i^- dt \tag{60}$$

s.t. S_{pool_i}, (56) and $\dot{\mu}_i = u_i$,
$\implies u_i^{k+1}(.)$, ${Q_g^i}^{k+1}(.)$, ${z_i^-}^{k+1}(.)$, ${z_i^+}^{k+1}(.)$, ${\mu_i}^{k+1}(.)$,

and **compute the control of the gate $N+1$:**

$$\min_{u_{N+1}, Q_g^{N+1}} \int_0^T [m_{N+1}(u_{N+1}) + \frac{1}{2\epsilon}(u_{N+1} - u_{N+1}^k)^2 + \frac{1}{2\epsilon}(Q_g^{N+1} - {Q_g^{N+1}}^k)^2]dt$$
$$+ \int_0^T [p_{N+1}^k + cG_{N+1}({Q_g^{N+1}}^k, {z_N^-}^k, {z_{N+1}^+}^k, \mu_{N+1}^k]^T$$
$$\times [\frac{\partial G_{N+1}}{\partial Q_k^{N+1}}^k Q_g^{N+1} + \frac{\partial G_{N+1}}{\partial \mu_{N+1}}^k \mu_{N+1}]dt \tag{61}$$

s.t. $\dot{\mu}_{N+1} = u_{N+1}$, $\implies u_{N+1}^{k+1}(.)$, ${Q_g^{N+1}}^{k+1}(.)$, ${\mu_{N+1}}^{k+1}(.)$,

2. **Coordination:**

$$p_i^{k+1} = p_i^k + \rho G_i({Q_g^i}^{k+1}, {z_{i-1}^-}^{k+1}, {z_i^+}^{k+1}, {\mu_i}^{k+1}), \ i = 1, ..., N+1 \tag{62}$$

$k + 1 \to k$ and go to 1)

where z_i^- and z_i^+ denote the variable z_i evaluated at $x = L_i^-$ and $x = L_i^+$ respectively.

The solution of each problem $TPBVP_i$ can be obtained with an adjoint method very similar to the one described in the sections 3.3 and following, while the subproblem (61) is a simple LQ optimal control problem if m is a quadratic functional.

5.2. **A distributed predictive control scheme.** The overall predictive control scheme may be described as follows, at each sampling time t (see also the figure 7):

1. Each agent i (corresponding to the control system of each gate i) broadcasts the current state $(Q_i(., t), z_i(., t))$.
2. A local receding horizon problem is solved by each agent i on horizon $[t, t+T]$, using last information sent by the other agents.
3. Each agent i broadcasts through the communication network its local information $Q_g^i(.)$, $z_i^-(.)$, $z_i^+(.)$, $\mu_i(.)$, which correspond to the solution of the local receding horizon problem.
4. The coordination agent updates the Lagrangian multipliers p_i by using (62): if the norm of the constraints is less than α, go to 5), otherwise go to 2).

5. Each agent i applies to the system the first optimal control input for the current instant t of the last computed optimal sequence u_i, $t \rightarrow t + \Delta t$ and go to 1).

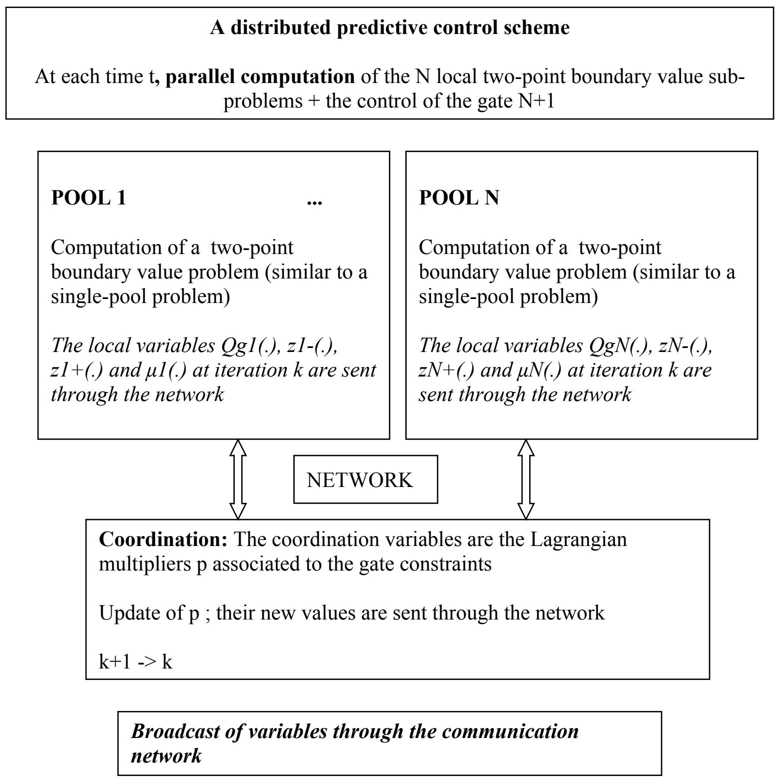

FIGURE 7. Distributed model predictive control

6. **Conclusions and perspectives.** We can state what follows:

1. The practical extension of finite-dimensional predictive control techniques to the infinite-dimensional framework for the control of open-channel hydraulic systems is possible, provided that no shock wave appears.
2. However it is well known that nonlinear conservation laws may exhibit shock wave propagation (depending on both the initial conditions and the boundary conditions), leading to nonsmoothness of the solutions. In the presence of shocks, traditional variational calculus is not appropriate, since the linearized equations have Dirac masses. A method was proposed in [1] in order to derive linearizations of nonlinear hyperbolic equations based on Frechet derivation in the distributional interpretation. Such an approach was used for the optimal control of traffic conservation laws in [14]. It is important to point out that

the here-proposed approach may be adapted without restriction to consider shock waves in the sense of distributions.

3. A decomposition-coordination technique has been proposed for the computation of the model predictive control of a multi-pool system, which takes advantage of communication networks used for control purpose. This approach remains to be implemented to evaluate its practical effectiveness. However a similar two-level algorithm has been successfully applied to a finite-dimensional model predictive control problem intended for the load-frequency control of a power system [13].
4. This approach may be easily applied to other problems governed by hyperbolic PDEs, such as road traffic systems.
5. The computation of the related two-point boundary value problem by using a discretization method based on Preissmann integration scheme has been validated.
6. A theoretical analysis has still to be performed to provide assumptions guaranteeing existence of optimal solutions and closed-loop stability of the predictive control scheme.
7. For practical implementation, a state observer (which can be derived by using variational calculus as a "dual control problem") is needed.

REFERENCES

[1] C. Bardos and O. Pironneau, *Derivatives and control in the presence of shocks*, Computational Fluid Dynamics Journal, **11** (2003), 383–392.

[2] A. M. Bayen, R. L. Raffard and C. J. Tomlin, *Adjoint-based control of a new eulerian network model of air traffic flow*, IEEE Trans. on Control System Technology, **14** (2006), 804–818.

[3] O. Begovich, J. Salinas and V. M. Ruiz, *Real-time implementation of a fuzzy gain scheduling control in a multi-pool open irrigation canal prototype*, IEEE International Symposium on Intelligent Control (ISIC2003), October 5-8, 2003, Houston, Texas, USA.

[4] H. Bounit, H. Hammouri and J. Sau, *Regulation of an irrigation channel through the semi-group approach*, Proc. of the Workshop on Regulation and Irrigation Canals RIC'97, Marrakesh, Maroc, (1997), 261–267.

[5] M. L. Chen and D. Georges, *Nonlinear optimal control of an open-channel hydraulic system based on an infinite-dimensional model*, in "Proceedings of IEEE Conference on Decision and Control," Phoenix, USA, 1999.

[6] M. L. Chen and D. Georges, *Nonlinear closed-loop robust control of an open-channel hydraulic system based on an infinite-dimensional model*, in "Proceedings of European Control Conference," Porto, Portugal, 2001.

[7] G. Cohen and D. L. Zhu, "Decomposition Coordination Methods in Large Scale Optimization Problems. The Nondifferentiable Case and the Use of Augmented Lagrangians," JAI Press inc., 1984.

[8] J. M. Coron, B. d'Andrea-Novel and G. Bastin, *A strict Lyapunov function for boundary control of hyperbolic systems of conservation laws*, IEEE Trans. on Automatic Control, **52** (2007), 2–11.

[9] J. A. Cunge, Jr. F. M. Holloy and A. Verwey, "Practical Aspects of Computational River Hydraulics," Oitman, London, 1980.

[10] V. Dos Santos and Y. Toure, *Regulation of irrigation canals: Multivariable boundary control approach by internal model*, Second IFAC Conference on Control Systems Design (CSD'03), Poland, september 7-10 2003, abs-056-35-35.

[11] J. F. Dulhoste, D. Georges and G. Besançon, *Nonlinear control of open-channel water flow based on a collocation control model*, Journal of Hydraulic Engineering, **130** (2004), 254–266.

[12] R. Findeisen, L. Imsland, F. Allgower and B. A. Foss, *State and output feedback nonlinear model predictive control: An overview*, European Journal of Control, fundamental issues in control, **9**, (2003).

[13] D. Georges, *Distributed model predictive control via decomposition-coordination techniques and the use of an augmented Lagrangian*, IFAC Workshop on Nonlinear Model Predictive Control for Fast Systems, Grenoble, october 9-11 2006.

[14] D. Jacquet, "Macroscopic Freeway Modelling and Control," Ph.D thesis, Grenoble Institute of Technology, 2006, http://tel.archives-ouvertes.fr/tel-00150434/en/.

[15] X. Litrico and D. Georges, *Robust optimal control of a dam-river system with intermediate measurements*, in "Proceedings of European Control Conference," Karlsruhe, Germany, 1999.

[16] X. Litrico and V. Fromion, *Advanced control politics and optimal performance for an irrigation canal*, in "Proceedings of European Control Conference," Cambridge, UK, 2003.

[17] P. O. Malaterre, "Modélisation, Analyse et Commande Optimale LQR d'un Canal D'irrigation," Ph.D thesis, LAAS-CNRS-ENGREF-Cemagref, ISBN 2-85362-368-8, 1994.

[18] D. Q. Mayne and H. Michalska, *Receding horizon control of nonlinear systems*, IEEE Trans. on Automatic Control, **35** (1990), 814–824.

[19] S. Sawadogo, R. Faye, P. O. Malaterre and F. Mora Camino, *Decentralized predictive controller for delivery canals*, in "proceedings of IEEE International Conference on Systems Man and Cybernetics (SMC'98)," San Diego (USA), 1998, 3880–3884.

[20] M. Skertchly and J. P. Miles, *Control of an irrigation canal*, Journal of Hydraulical Engineering, **122** (1996), 403–410.

[21] B. Voron and A. P. Bouillot, *Application of the fuzzy set theory to the control of large canals*, in "Proceedings of the Workshop on Regulation and Irrigation Canals RIC'97," Marrakech, Morocco, 1997, 317–331.

[22] C. Z. Xu, *Linear symmetric hyperbolic systems and application to boundary PI-control of the Saint-Venant equations*, in "Proceedings of MTNS'2000," Perpignan, France, 2000.

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