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TOWARDS NONLINEAR DELAY-BASED CONTROL FOR CONVECTION-LIKE DISTRIBUTED SYSTEMS: THE EXAMPLE OF WATER FLOW CONTROL IN OPEN CHANNEL SYSTEMS

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Abstract. In this paper, the driving idea is to use a possible approximation of partial differential equations with boundary control by ordinary differential equations with *time-varying delayed input*, for a control purpose. This results in the development of a specific nonlinear control methodology for such delayed-input systems. The case of water flow control in open channel systems is used as a motivating and illustrative example, with corresponding simulation results.

1. Introduction.

Control purposes for systems governed by partial differential equations, or distributed systems, often need that the model be reduced to a finite dimension. Typically finite-difference or finite-element methods can be used, allowing to apply control techniques available for finite-dimension systems, including non linear ones [8, 9, ...].

Noting however that in systems with transport phenomena there naturally exists some delay in the dynamics, one can also think of using control methodologies for *time-delay systems* [12, 17, ...]. For a simple convection, this delay is a constant one, but for more complex phenomena, this delay can be varying, even depending on the system state itself. Such a model has for instance been emphasized in [11] in the case of river dynamics modelling. A control design then requires to face systems with delayed input, possibly nonlinear dynamics, and with a delay varying (in particular according to the system state). A methodology is thus here proposed in that respect, in the continuation of [5, 2].

A motivation for this approach is more particularly underlined in section 2, via the example of open channel water flow dynamics [3].

The proposed control principle for systems with delayed input is then presented in section 3. The corresponding results are subsequently illustrated in simulation with an example of irrigation-like canal in section 4.

Some conclusions and lines for future works are finally drawn in section 5.

2. Motivation and case study.

One-dimensional water flow dynamics in open channel systems with a rectangular section can classically be described by so-called

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Saint-Venant equations as follows [3]:

$$\begin{array}{rcl} W\dfrac{\partial h}{\partial t}+\dfrac{\partial Q}{\partial X} & = & 0 \\ \dfrac{\partial Q}{\partial t}+\dfrac{1}{W}\dfrac{\partial\left[Q^2/h\right]}{\partial X}+gWh\left[\dfrac{\partial h}{\partial X}-I+J(h,Q,f_c)\right] & = & 0, \end{array} \quad (1)$$

where $h = h(t, X)$ and $Q = Q(t, X)$ are the water levels and flow rates at each time t and each $X \in [0, L]$ (for a channel length equal to L), J stands for the friction along the channel (generally depending on h, Q and a friction coefficient f_c), while W and I respectively denote the flow width and the channel slope, as summarized on figure 1 below.

Typically, the control is achieved at one (or both) extremity, reacting to initial conditions on h and Q, as well as possible disturbances such as withdrawals or water additions. A simplified version of the above equations (1) under sub-critical

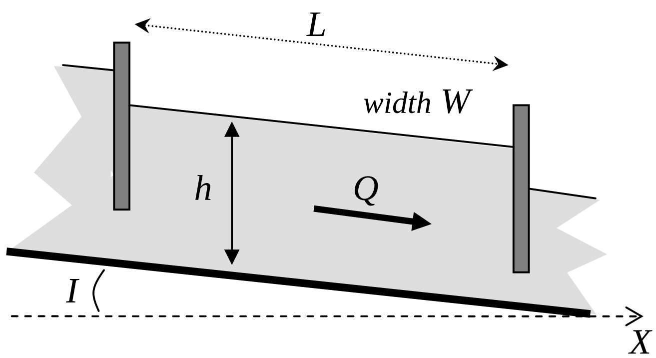

FIGURE 1. Water flow in some open channel reach.

flow assumption, and allowing to focus on the flow rate control, can be given by a diffusive wave model as follows (see e.g. [10] for details):

$$\frac{\partial Q}{\partial t}+\nu(Q)\frac{\partial Q}{\partial X}-\Delta(Q)\frac{\partial^2 Q}{\partial X^2}=0 \quad (2)$$

where ν and Δ are appropriate functions resulting from (1) (by neglecting inertia terms $\partial Q/\partial t + \partial[Q^2/(Wh)]/\partial X$ w.r.t. $gWh\partial h/\partial X$).

In this model, the control input becomes $u(t) = Q(t, 0)$ and the to-be-controlled output $y(t) = Q(t, L)$. It is clear that u will have a delayed effect on y, and in [11], the authors have shown how this effect can be represented by a simple second order model, nonlinear, with a delayed output and a delay nonlinearly varying according to the flow rate. In short, this is done by approximating the linearized transfer (the so-called *Hayami transfer*) by a delayed second order model, parameterized by the linearization point, and from this finally recovering a nonlinear model.

With the same arguments, but with a time shift, one can get a similar model but with a delayed *input* of the following form:

$$\begin{array}{rcl} \dot{x}(t) & = & A(x_2(t))x(t)+B(x_2(t))u(t-\tau(x_2(t))) \\ y(t) & = & x_2(t) \end{array} \quad (3)$$

where A, B are in a controllable form, with coefficients depending on ν and Δ, and the state vector x reduces to a couple of variables: $Q(t, L)$ and its time derivative (the full details on A, B and τ are given in the appendix).

The control of output y from input u thus raises the challenging problem of controlling a nonlinear delayed-input system, with a delay nonlinearly depending on the state. This is the problem which is addressed in next section.

Notice that such a problem has been very widely considered for *linear systems* with a *constant* delay, that the case of *nonlinear* systems with a delayed input has been only very scarcely addressed (see [15, 14] for few recent contributions), and that the here considered case of state-dependent delay goes even beyond this.

3. **Proposed control method for delayed-input systems.** Let us now consider systems described by a state equation of the following form:

$$\dot{x}(t) = F(x(t), u(t - \tau(t, x(t)))) \tag{4}$$

where $x(t) \in \mathbb{R}^n$ denotes the state vector, $u(t) \in \mathbb{R}^m$ the control input, F is smooth w.r.t. x, u, and $\tau(t, x(t))$ is a known (positive) delay, also smoothly depending on its arguments.

Let us further assume that the origin is an equilibrium for this system ($F(0,0) = 0$), and consider the problem of control design so as to make this equilibrium asymptotically stable.

To that end, the idea here will be that of using some 'predictor', extending the *finite spectrum assignment* approach already inspected for linear delay systems [13, 16]: in short, it consists in relying on a stabilizing control law for the non-delayed system, and combine it with some appropriate predictor for the delayed system.

In the case of a constant delay τ, one classically needs a prediction at time $t + \tau$ for a current time t .

In a case of a *time-varying delay* the required prediction time is a bit different, as originally noted in [19], and in the case of varying delay here considered which generalizes this previous one, one can check that this prediction time takes the form of $\delta(t)$ defined by:

$$\delta(t) = \tau(t + \delta(t), x(t + \delta(t))). \tag{5}$$

One can indeed show that if an exact prediction is available at that time, then the control based on this prediction achieves the desired purpose, as stated below:

Proposition 1. *If* $\exists \Phi : \mathbb{R}^n \to \mathbb{R}^m$*,* $\exists D \subset \mathbb{R}^n$ *containing* 0*, and* $V : D \to \mathbb{R}$ $\mathcal{C}^1$ *positive definite* $\forall x \in D$ *such that:*

(i) $c_1 \|x\|^2 \le V(x) \le c_2 \|x\|^2$

(ii) $\dfrac{\partial V(x)}{\partial x} F(x, \Phi(x)) \le -c_3 \|x\|^2$

then the following control law makes the origin locally asymptotically stable for the closed-loop system:

$$\begin{aligned} x_p(t, t + \delta(t)) &= x(t) + \int_t^{t+\delta(t)} F(x_p(t, \theta), u(\theta - \tau(x_p(t, \theta))))d\theta \\ u(t) &= \Phi(x_p(t + \delta(t))) \end{aligned} \tag{6}$$

This result can be established by setting:

$$z(t) := x(t + \delta(t)) \tag{7}$$

and considering $V(z)$ as a Lyapunov function for the resulting dynamics:

$$\dot{z}(t) = f(z(t)) \tag{8}$$

where f defines the closed loop dynamics (see appendix B for full details).

Remark 1. Notice that for a system without any finite escape time, if conditions of proposition 1 hold on $D = I\!R^n$ with V being radially unbounded, and if for any x the following conditions hold:

$$\frac{\partial \tau}{\partial x}(x)F(x, \Phi(x)) \quad < \quad 1 \tag{9}$$

$$\tau(x) \quad \geq \quad 0 \tag{10}$$

then one further gets a *global* convergence result.

Obviously conditions on τ are here rather strong, but they are necessary for some well-posedness of the model (since (9) is necessary for $t > \tau(x(t))$ to be satisfied at any time in closed loop, while (10) simply means causality). $\diamond$

In practise, the problem for some actual implementation of control (6) is to compute the predicted state x_p.

A simple method to do so can be to approximate it by some first order Euler approximation scheme. In that case however, the achieved closed loop dynamics, even considered in the shifted time variable z (as in (7)), will not only depend on the current time, via $z(t)$, *but also on* $z(t - \delta(t))$.

The stability of the origin for this system can nevertheless still be ensured, provided that some additional condition is required: roughly the fact that the approximation error introduced in those dynamics be 'small enough' w.r.t. the control performances, or conversely the control law 'can be tuned' appropriately w.r.t. it.

Denoting by $\Gamma(z(t), z(t - \delta(t)))$ this approximation error, in the sense that with the approximate computation of x_p:

$$\dot{z}(t) = f(z(t)) + \Gamma(z(t), z(t - \delta(t))), \tag{11}$$

this can more formally be stated as follows:

Proposition 2. *If* $\exists \Phi : I\!R^n \to I\!R^m$ *s.t.* $\hat{x}_p$ *can be computed by:*

$$\begin{array}{rcl} \hat{x}_p(t, t+\delta) & = & x(t) + \tau(\hat{x}_p(t, t+\delta)) \times F(\hat{x}_p(t, t+\delta), \Phi(\hat{x}_p(t, t+\delta))) \\ u(t) & = & \Phi(\hat{x}_p(t, t+\delta)) \end{array} \tag{12}$$

and $\exists D \subset I\!R^n$ *containing* 0 *with* $V : D \to I\!R$ $\mathcal{C}^1$ *positive definite* $\forall x \in D$ *s.t.:*

(i) $c_1\|x\|^2 \leq V(x) \leq c_2\|x\|^2$

(ii) $\dfrac{\partial V(x)}{\partial x} F(x, \Phi(x)) \leq -c_3\|x\|^2$

(iii) $\|\dfrac{\partial V(x)}{\partial x}\| \leq c_4\|x\|$

(iv) $\gamma \leq \dfrac{c_3}{4c_4}$ *with* $\|\Gamma(x, x_d)\| \leq \gamma\|x_d\|$ *for* $x_d \in D$

then the above control law (12) makes the origin locally asymptotically stable for the closed-loop system.

This result can be established by considering $W(z_t) := V(z) + \mu \int_{t-\delta}^{t} \|z(\theta)\|^2 d\theta$, as a Lyapunov-Krasovskii functional (see appendix C).

Notice that conditions (i) to (iii) are quite standard in Lyapunov theory for nonlinear systems [9], and that (iv) just expresses the required relationship between the 'approximation error' and the 'control performances'.

At this stage, it can be noticed that such a control law needs to solve at each time a nonlinear problem for the prediction computation. This can instead be done dynamically by a technique of *dynamic inversion*, as expressed below:

Proposition 3. *If conditions of proposition 2 are satisfied, then there exists* Λ *large enough such that the following control law makes the origin asymptotically stable for the closed-loop system:*

$$\begin{array}{rcl} \dot{X}_p & = & (I_d - \frac{\partial H}{\partial X_p})^{-1}[F(x(t), u(t-\tau(x(t)))) - \Lambda(X_p - H(X_p, x(t)))] \\ u(t) & = & \Phi(X_p(t)) \end{array} \tag{13}$$

where H *is defined by identity (12) on* $\hat{x}_p$*, re-written as:*

$$\hat{x}_p(t, t+\delta(t)) = H(x(t), \hat{x}_p(t, t+\delta(t))).$$

This follows from two facts:

- X_p will asymptotically approach $\hat{x}_p$ if not initialized on it, with a rate given by Λ (since equation (13) means that $\dot{G} = -\Lambda G$ where $G := X_p - H(x, X_p)$);
- By choosing Λ large enough one can recover the stability result of proposition 2, by applying Tikhonov's results for instance [9].

Notice that a direct dynamical computation scheme for a more exact prediction could even be thought of, in the spirit of observers: this has been inspected for special classes of delayed-input systems in [1], and will be more completely investigated in future developments.

4. **Simulation results.** The proposed methodology is illustrated in simulation in a case of water flow control in an open channel - or river, as mentioned in section 2. This provides an alternative approach to the various other studies towards automatic control of such systems (see e.g. [6] and references therein).
The considered numerical values are taken from [11] as follows:

Length	L	10 km
Width	W	8m
Slope	I	0.04%
Friction	f_c	0.05

and the considered control purpose is basically that of handling a setpoint change on the output flow rate: here it is chosen for a nominal change of about a 30%-variation, from $y = 0.93m^3/s$ to $y = 1.24m^3/s$, in about $5h$.

The control is designed as in the previously presented methodology, namely on the basis of some 'nominal' control law for the non delayed model, combined with a predictor.

Here the nominal control law is designed according to the so-called *exact feedback linearization* method [8], which can be easily used in view of the model structure of appendix A, and can be easily tuned according to the considered closed loop specifications, while the predictor is implemented as in (13) for some Λ chosen from simulation attempts.

This control law has been tested in simulation on a complete Saint-Venant model (equations (1)), simulated with the reference Preissmann finite-difference scheme [18].

The corresponding results are briefly illustrated by figure 2 below, where the achieved downstream water flow rate is represented (in solid line) versus the desired one (in dashed line), showing how the control purpose is indeed achieved. The corresponding input control behaviour (the upstream flow rate) is shown on figure 3.

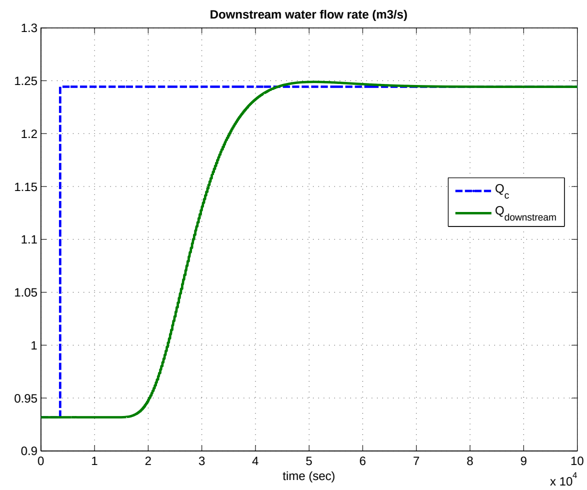

FIGURE 2. Step response with a state feedback law.

The control law has further been tested on much larger variations, still giving

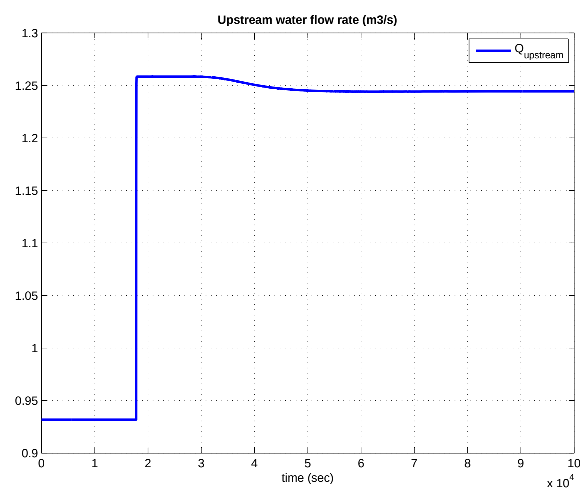

FIGURE 3. Input response to a step change with a state feedback law.

very admissible results: this is illustrated by figures 4-5 below, corresponding to the downstream and upstream flow rates for more than 100% change on the set-point.

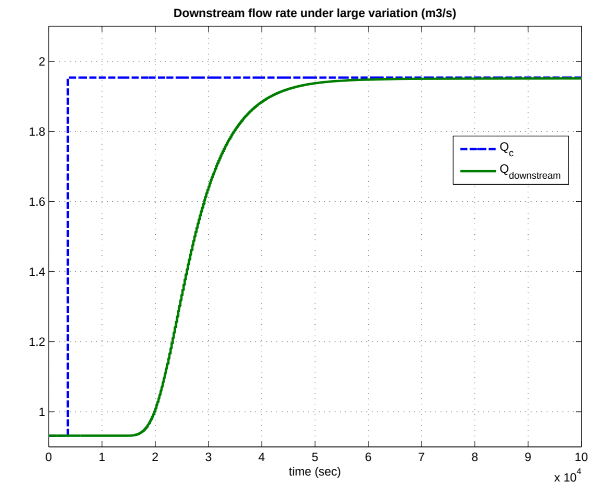

FIGURE 4. Step response under large variation.

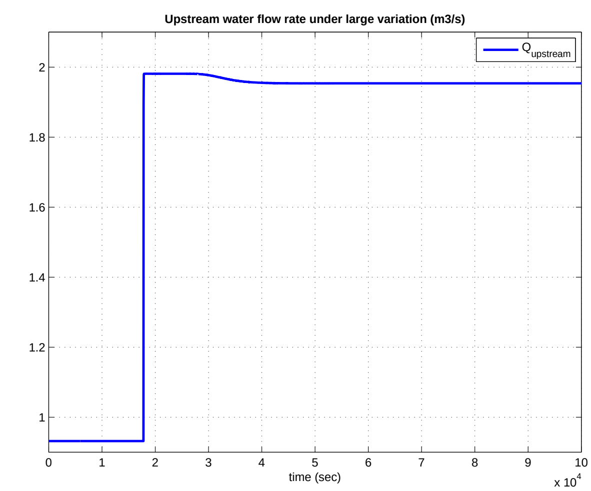

FIGURE 5. Input response to a large step change.

An additional interest of relying on some 'nominal' control design is that any possible tool for the considered nominal system can be used. For instance, in order to further take into account possible (constant) disturbances on the flow rate, a modified version of the control - typically including some integral action, has also been tested: the obtained results simulated in the presence of a lumped withdrawal on the basis of Saint-Venant equations are given by figure 6 (here the withdrawal is simulated as a step disturbance at time $t = 31h$, located close to the downstream end, and with a magnitude of about 10% of the nominal water flow rate).

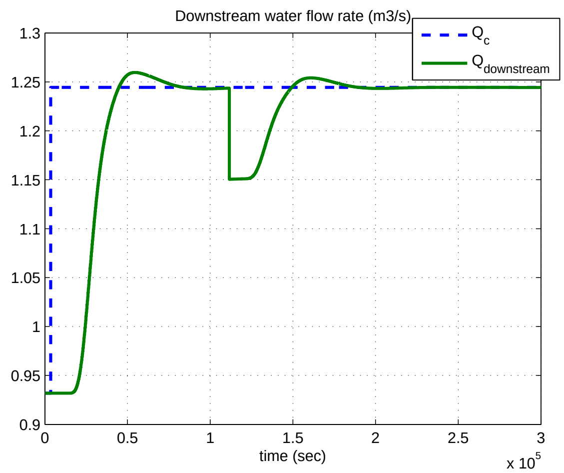

FIGURE 6. Step response with a disturbance and integral action.

Finally, noting that the implementation of the proposed control so far relies on a *full state measurement*, namely here $Q(t, L)$ and $\dot{Q}(t, L)$, while in practise only $Q(t, L)$ is likely to be measured, a problem of *state observer* arises. Once again here, it can be noticed that the system structure of appendix A allows to make use of available results in that respect: injecting indeed the measured output in the input delay function brings the system into a form appropriate for some so-called *high gain* design [4].

Although in general the introduction of an observer in a control loop for a *nonlinear* system may affect the closed-loop overall stability, it has been shown how the use of appropriate *high gains* can yield stabilization w.r.t. a priori given operation regions, in a sense of *semi-global stabilization* (see e.g. again [8]).

Figure 7 illustrates how here the introduction of an observer in the previously discussed control law allows to fairly recover the transient behaviour obtained without it.

5. **Conclusions and future works.** In this paper, a control methodology for nonlinear systems with a varying delay in the control input has been proposed, and it has been emphasized how such a method can be useful for the control of some distributed systems. In particular its performances have been illustrated in the case of some open channel water flow control. The extension of such an approach to more general infinite dimensional systems will be part of future works.

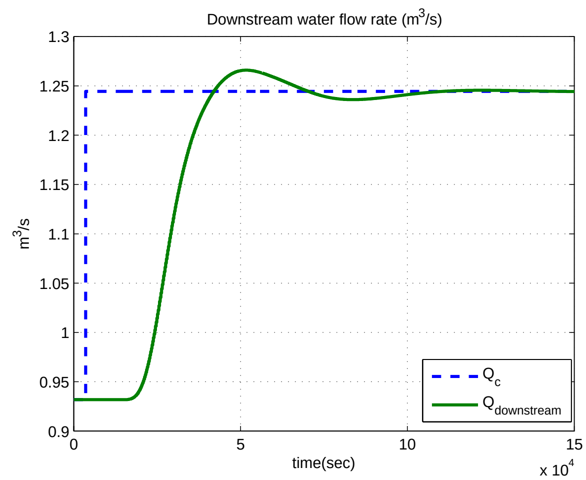

FIGURE 7. Step response with state feedback law and observer.

Appendix A. **Delayed model for water flow dynamics.** The full data for approximate model (3) of the full wave representation (2) is summarized as follows:

$$A(x_2) = \begin{pmatrix} -\alpha(x_2)\beta(x_2) & -\beta(x_2) \\ 1 & 0 \end{pmatrix};\ B(x_2) = \begin{pmatrix} \beta(x_2) \\ 0 \end{pmatrix};$$

where:

$$\begin{aligned} \alpha(x_2) &= 2\sqrt{\frac{2L\Delta(x_2)}{\nu(x_2)^3}}cos\left(\frac{\pi}{6}+\frac{1}{3}arctan\sqrt{\frac{9\Delta(x_2)}{2L\nu(x_2)-9\Delta(x_2)}}\right); \\ \beta(x_2) &= \frac{\nu(x_2)^3}{2L\Delta(x_2)}\frac{\alpha(x_2)\nu(x_2)^2}{\alpha(x_2)\nu(x_2)^2-3\Delta(x_2)}; \\ \tau(x_2) &= \frac{L-\alpha(x_2)\nu(x_2)}{\nu(x_2)}; \end{aligned}$$

with:

$$\begin{aligned} \nu(x_2) &= \frac{5}{3}\left(\frac{x_2}{W}\right)^{0.4}\left(\frac{I}{f_c^2}\right)^{0.3}; \\ \Delta(x_2) &= \frac{x_2}{2WI}; \end{aligned}$$

and L, W, I, f_c respectively the channel length, width, slope and friction coefficients.

Appendix B. **Proof of proposition 1.** First of all, define $f(x) := F(x, \Phi(x))$ and $z(t) := x(t+\delta(t))$. Then the closed-loop system (4)-(6) can be re-written w.r.t. z and f as:

$$\dot{z}(t) = (1+\dot{\delta}(t))f(z(t)). \tag{14}$$

where $\dot{\delta}$ can be computed from (5).

Let us then consider $V(z)$ as a candidate Lyapunov function for this system. Clearly:

$$\dot{V} = (1+\dot{\delta}(t))\frac{\partial V}{\partial x}f(z(t)).$$

Now notice that from the definition of δ we have:

$$[1-\frac{\partial\tau}{\partial x}(z)f(z)]\dot{\delta}=\frac{\partial\tau}{\partial x}(z)f(z) \tag{15}$$

where $\frac{\partial\tau}{\partial x}(z)f(z)$ vanishes when z goes to zero.

Hence there exists a domain $\bar{D}\subset I\!R^n$ of states z of D and containing the origin, such that $1-\frac{\partial\tau}{\partial x}(z)f(z)>0$, and consequently such that $\dot{\delta}>-1$. Then using condition (ii), we get on $\bar{D}$:

$$\dot{V}\leq-(1+\dot{\delta})c_3\|z\|^2.$$

Now using assumption (i) and integrating the above inequality from 0 to t, we obtain:

$$V(t)\leq V(0)e^{-\phi(t)},$$

where

$$\phi(t)=\frac{c_3}{c_1}\int_0^t(1+\dot{\delta}(\theta))d\theta=\frac{c_3}{c_1}(t+\delta(t)-\delta(0))$$

Again using assumption (i), the following inequality holds:

$$\|z(t)\|^2\leq\frac{c_2}{c_1}\|z(0)\|^2e^{-\phi(t)}.$$

Clearly $\delta\geq0$ since $\tau\geq0$ (at least locally), and thus $\phi(t)\to+\infty$. This yields that for any initial condition in some neighborhood of 0,

$$\lim_{t\longrightarrow\infty}\|x(t+\delta(t))\|=0$$

and finally $\lim_{t\longrightarrow\infty}\|x(t)\|=0$.

Appendix C. **Proof of proposition 2.** Let us consider the candidate Lyapunov-Krasovskii functional $W(z_t)=V(z)+\mu\int_{t-\delta}^t\|z(\theta)\|^2d\theta$, where $z_t=z(t+\theta),\theta\in[-\delta,0]$ as usual in Lyapunov-Krasovskii formalism (see e.g. [7]), and $\mu>0$ is to be specified later on.

Considering the closed-loop dynamics (11), the time derivative of W is given by:

$$\begin{aligned}\dot{W}\ &=\ (1+\dot{\delta})\frac{\partial V(z)}{\partial z}F(z,\Phi(z)) &(16)\\ &+\ (1+\dot{\delta})\frac{\partial V(z)}{\partial z}\Gamma(z(t),z(t-\delta(t)))\\ &+\ \mu[\|z(t)\|^2-(1-\dot{\delta}(t))\|z(t-\delta(t))\|^2]. &(17)\end{aligned}$$

Using again expression (15), we have that: $1-\dot{\delta}\geq\varepsilon$ whenever $\left|\frac{\partial\tau}{\partial x}F(z,\Phi(z))\right|\leq\frac{1-\varepsilon}{2-\varepsilon}$ for any $0<\varepsilon<1$. Moreover, this also guarantees that $1+\dot{\delta}>\tilde{\varepsilon}$ with $\tilde{\varepsilon}=\frac{2-\varepsilon}{3-2\varepsilon}>0$. Finally, it is also clear that in this case, $1+\dot{\delta}<2$.

Hence, given such an ε, and using conditions (ii) and (iii) of the theorem, we can obtain on some small enough neighborhood of the origin:

$$\begin{aligned}\dot{W}\ &\leq\ -c_3\tilde{\varepsilon}\|z(t)\|^2+2c_4\gamma\|z(t)\|\|z(t-\delta)\|+\mu[\|z(t)\|^2-\varepsilon\|z(t-\delta)\|^2] &(18)\\ i.e.\ \dot{W}\ &\leq\ -\begin{pmatrix}\|z(t)\|\|z(t-\delta)\|\end{pmatrix}\begin{pmatrix}c_3\tilde{\varepsilon}-\mu & -c_4\gamma\\ -c_4\gamma & \varepsilon\mu\end{pmatrix}\begin{pmatrix}\|z(t)\|\\ \|z(t-\delta)\|\end{pmatrix}. &(19)\end{aligned}$$

Hence choosing $\mu<c_3\tilde{\varepsilon}$, we get that for:

$$\gamma<\frac{\sqrt{(c_3\tilde{\varepsilon}-\mu)\varepsilon\mu}}{c_4} \tag{20}$$

the right-hand side of the above inequality is negative definite, and thus $\dot{W} \leq -\rho\|z(t)\|$ for some $\rho > 0$ which gives the local asymptotical stability of $z = 0$ by the Lyapunov-Krasovskii stability result, and finally that of $x = 0$ for the system in time t.

Notice that choosing e.g. $\mu = \frac{c_3\tilde{\varepsilon}}{2}$, condition (20) becomes $\gamma < \frac{c_3\tilde{\varepsilon}\sqrt{\varepsilon}}{2c4}$. Since the right-hand side can be made arbitrarily close to $\frac{c_3}{2c_4}$ by choosing ε close enough to 1 (by lower values), this can in turn make (20) to be satisfied whenever $\gamma \leq \frac{c_3}{4c_4}$, which ends the proof.

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