Preface

Special Issue on Homogenization Theory and Related Topics

This special issue contains selected papers on Homogenization Theory and related topics. It is dedicated to Eugene Khruslov on the occasion of his seventieth birthday. Professor Khruslov made pioneering contributions into this field.

Homogenization problems were first studied in the late nineteenth century (Poisson, Maxwell, Rayleigh) and early twentieth century (Einstein). These studies were based on deep physical intuition allowing these outstanding physicists to solve several specific important problems such as calculating the effective conductivity of a two-phase conductor and the effective viscosity of suspensions. It was not until the early 1960s that homogenization began to gain a rigorous mathematical footing which enabled it to be applied to a wide variety of problem in physics and mechanics. A number of mathematical tools such as the asymptotic analysis of PDEs, variational bounds, heterogeneous multiscale method, and the probabilistic techniques of averaging were developed. Although this theory is a well-established area of mathematics, many fascinating problems remain open. Interesting examples of such problems can be found in the papers of this issue.

Homogenization theory is becoming a part of the applied mathematics curriculum. It draws the attention of many scientists because of various applications in natural sciences (physics, mechanics, biology) and engineering. On the other hand, homogenization theory has influenced the development of new mathematical tools and ideas. Examples of such developments are two-scale convergence for thin, singular periodic and random structures and theory of elliptic and parabolic PDEs with variable order of non-linearity (this type of PDEs attracted attention due to novel models of non-Newtonian fluids). Other important mathematical issues that were addressed in homogenization context include the so-called Lavrentiev phenomenon, discrete network approximation, and the limiting behavior of nonlinear divergent elliptic PDEs.

Many topics in homogenization remain at the frontiers of studies in applied mathematics. These include homogenization of elastic and electromagnetic problems in thin structures, two-scale operator convergence in the "photonic crystal" model, filtration problems in multiscale media and the rigorous justification of Biot type models. Homogenization of variational problems with non-convex integrands remains a very active research area motivated by possible applications to phase transitions.

We next mention some prospective trends in homogenization methods and techniques. Diffusion theory is one of the main sources of challenging homogenization problems. Homogenization along with pure probabilistic methods are key tools of asymptotic analysis of random walks in random environments. One may expect further progress in the derivation of the Central Limit Theorem for diffusion in random turbulent flows and for random walks in random and combined structures. In recent years progress has been achieved in operator approach to homogenization providing new so-called operator estimates on the rates of convergence. Such estimates are often more efficient that the classical ones, since they are formulated in the terms of uniform resolvent convergence. One expects further studies along these lines to appear. Finally we mention here homogenization in presence of a large number of non-separated scales that arises in various applications but has not been fully addressed due to lack of appropriate mathematical tools. The time is now ripe to attack this challenging problem.

We have mentioned above several recent trends in homogenization, but extrapolation based on the history often fails. Without a doubt, in the coming years entirely new applications as well as completely new mathematical tools will appear in homogenization, as has happened in the past.

In this issue the reader will find a variety of works encompassing the rapidly developing area of random and nonlinear homogenization, new approaches to linear homogenization, and modeling of some important physical problems.

Guest Editors:
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